Online Appendix

to accompany

Intangible Capital and the Investment-q Relation

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Contents

1. Measuring Tobin's q as the market-to-book-assets ratio

The following table shows that the market-to-book assets ratio produces lower R^2 values for physical, intangible, total, and R&D investment, whether we scale each investment measure by total capital (Panel A) or book assets (Panel B).

Table A1
Counterpart of paper's Table 1 using the market-to-book-assets ratio

Details are the same as in Table 2 in the main paper, except Panel A measures Tobin's q as the market-to-book-assets ratio, measured as the firm's market value (Compustat items $prcc\_f * csho + dltt + dlc - act$) divided by total book assets (Compustat item at). Panel B shows regressions that scale both q and investment by total assets. The purpose of Panel B is to check whether Panel A's low R^2 values result from scaling investment and q by different values. For comparison, Panels C and D reproduce the results from the paper's Table 2, Panels A and B.

	Investment scaled by total capital (K^{tot})				
	Physical (ι^{phy})	Intangible (ι^{int})	Total (ι^{tot})	R&D	CAPX/PPE (ι^{*})
Panel A: Regressions with market-to-book-assets ratio					
Market/Book assets	0.037	0.024	0.060	0.010	0.078
	(0.001)	(0.001)	(0.001)	(0.000)	(0.002)
R^2	0.158	0.198	0.240	0.106	0.176
Panel B: Same as Panel A, but scale all investment measures by book assets, not total capital					
Market/Book assets	0.025	0.014	0.039	0.006	0.025
	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)
R^2	0.126	0.083	0.168	0.047	0.126
Panel C: Regressions with Total q (same as Paper Table 2, Panel A)					
Total q	0.029	0.020	0.049	0.013	0.062
	(0.001)	(0.000)	(0.001)	(0.000)	(0.001)
R^2	0.209	0.279	0.327	0.270	0.244
Panel D: Regressions with standard q (same as Paper Table 2, Panel B)					
Standard q	0.006	0.005	0.011	0.003	0.017
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
R^2	0.139	0.266	0.250	0.250	0.233

2. Results using the fourth-order cumulant estimator

Table A2

Counterpart of paper's Table 3 using the fourth-order cumulant estimator

Details are the same as in Table 3 in the main paper, except here we use the fourth-order rather than the third-order cumulant estimator.

	Investment scaled by total capital (K^{tot})				
	Physical (ι^{phy})	Intangible (ι^{int})	Total (ι^{tot})	R&D	CAPX/PPE (ι^{*})
Panel A: Regressions without cash flow					
Total q (q^{tot})	0.056	0.039	0.069	0.026	
	(0.001)	(0.000)	(0.001)	(0.000)	
Standard q (q^{*})					0.029
					(0.000)
OLS R^2	0.209	0.279	0.327	0.270	0.244
	(0.004)	(0.006)	(0.005)	(0.009)	(0.005)
ρ^2	0.288	0.409	0.337	0.436	0.299
	(0.007)	(0.007)	(0.006)	(0.013)	(0.005)
τ^2	0.544	0.536	0.749	0.528	0.611
	(0.010)	(0.007)	(0.009)	(0.015)	(0.009)
Panel B: Regressions with cash flow					
Total q (q^{tot})	0.065	0.040	0.080	0.026	
	(0.001)	(0.000)	(0.002)	(0.000)	
Total cash flow (c^{tot})	0.035	0.043	0.158	-0.009	
	(0.008)	(0.004)	(0.011)	(0.004)	
Standard q (q^{*})					0.030
					(0.000)
Standard cash flow (c^{*})					0.022
					(0.004)
OLS R^2	0.235	0.326	0.374	0.281	0.238
	(0.004)	(0.005)	(0.005)	(0.009)	(0.005)
ρ^2	0.349	0.464	0.460	0.435	0.312
	(0.006)	(0.005)	(0.006)	(0.011)	(0.005)
τ^2	0.452	0.480	0.575	0.529	0.590
	(0.008)	(0.008)	(0.009)	(0.014)	(0.008)
Observations	141,800	141,800	141,800	74,326	141,800

3. Horse race between total q and standard q

Table A3
Horse race between total q and standard q

We use the cumulant estimator to regress total investment on either total or standard q. Details are the same as in Table 3 in the main paper.

	(ι^{tot})	(ι^{tot})
Total q (q^{tot})	0.086	
	(0.001)	
Standard q (q^{*})		0.023
		(0.000)
OLS R^2	0.327	0.250
ρ^2	0.423	0.314
τ^2	0.597	0.497
Observations	141,800	141,800

As we explain in Section 4.2 in the paper, this horse race illustrates why we need a new q proxy that account for intangibles, i.e., why the cumulant estimator alone cannot solve the problem of omitted intangibles. This table also provides the following useful consistency check: Total q produces a higher τ^2, which is consistent with its having less measurement error than standard q.

4. Bias in the investment-cash flow sensitivity?

Section 4.2 in the main paper explains that the slope of SG&A investment on total cash flow c^{tot} may be biased upwards. The reason is that measurement error in SG&A investment appears mechanically in the c^{tot} measure, because we add back intangible investment to compute c^{tot}. Put differently, c^{tot} equals profits gross of SG&A and R&D investment. Making this measure gross of SG&A investment can introduce measurement error from SG&A investment into c^{tot}. If the same measurement error is on both sides of the regression, the slope is biased upwards.

We now construct an alternate cash-flow measure that is immune to this concern about bias. The new measure is cash flows net of SG&A investment but gross of R&D investment. Since SG&A investment is not added back this cash-flow measure, the measure contains no measurement error in SG&A. Results are in the table below.

Table A4

Bias in the investment-cash flow sensitivity?

Details are the same as in Table 3 in the main paper. Results in Panel A match those in Table 3 Panel B, except we add the last column, which uses SG&A investment as the dependent variable. SG&A investment equals 0.3 times SG&A, scaled by lagged total capital. Panel B matches Panel A, except it uses an alternate cash-flow measure equal to $(IB_{it} + DP_{it} + RD_{it}(1 - \kappa))/K^{tot}_{i,t-1}$, which equals cash flow gross of R&D but net of SG&A investment.

	Investment scaled by total capital (K^{tot})				
	Physical (ι^{phy})	Intangible (ι^{int})	Total (ι^{tot})	R&D	0.3× SG&A
Panel A: Results using cash flows gross of R&D and SG&A investment (c^{tot})					
Total q (q^{tot})	0.069	0.038	0.086	0.024	0.085
	(0.001)	(0.001)	(0.002)	(0.001)	(0.002)
Total cash flow (c^{tot})	0.024	0.050	0.140	0.000	0.115
	(0.008)	(0.004)	(0.009)	(0.004)	(0.011)
OLS R^2	0.235	0.326	0.374	0.281	0.278
ρ^2	0.361	0.447	0.481	0.405	0.380
τ^2	0.435	0.502	0.544	0.568	0.385
Panel B: Results using cash flows gross of R&D, net of SG&A investment					
Total q (q^{tot})	0.069	0.038	0.086	0.024	0.084
	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)
Alternate cash flow	0.023	0.012	0.094	-0.003	0.008
	(0.007)	(0.004)	(0.008)	(0.004)	(0.010)
OLS R^2	0.228	0.29	0.349	0.276	0.225
ρ^2	0.359	0.407	0.452	0.401	0.329
τ^2	0.438	0.539	0.567	0.574	0.432
Observations	141,800	141,800	141,800	75,426	141,800

As expected, when we compare Panels A and B, we see that making cash flows gross rather than net of SG&A invesment increases SG&A investment's cash-flow slope, from 0.008 to 0.115. In panel B, SG&A investment's cash-flow slope is not statistically different from zero.

This exercise places a useful upper and lower bound on SG&A investment's cash-flow slope. Due to the measurement-error bias discussed above, the 0.115 slope in Panel A is too large, providing an upper bound. The statistically insignificant 0.008 slope in Panel B is arguably too low, providing a lower bound. It is too low because netting SG&A from cash flow depresses the cash-flow slope, and an economically meaningful cash-flow measure should be gross of all investment, including SG&A investment.

5. A More General Theory of Intangible Capital, Investment, and q

This appendix presents a more general version of the model in Section 2 of the main paper. The goal is to understand whether and how departures from Section 2's assumptions could explain our empirical results. We relax four main assumptions: perfect competition, constant returns to scale, equal depreciation rates for the two capital types, and quadratic adjustment costs.

5.1. Setup

The model in Section 2 assumes the profit function Π is linearly homogenous in K^{tot}, meaning the profit function can be written $\Pi\left(K^{tot},\varepsilon\right)=\varepsilon K^{tot}=\varepsilon\left(K^{phy}+K^{int}\right)$. We now use a more flexible profit function equal to

$$\Pi\left(K^{phy},K^{int},\varepsilon\right)=\varepsilon\left(\left(K^{phy}\right)^{\phi^{phy}}+\left(K^{int}\right)^{\phi^{int}}\right)^{\theta}, \tag{1}$$

where parameters ϕ^{phy}, ϕ^{int}, and θ are all strictly positive and less than or equal to one. This specification collapses to our main specification in Section 2 when $\theta=\phi^{phy}=\phi^{int}=1$. Equation (1) can be interpretted as the maximized profit function from a more general model that features endogenously chosen labor, either constant or decreasing returns to scale, and an exogenously given demand function for the firm's output. Abel and Eberly (2011) solve exactly such a model, albeit with one type of capital, and they show that the maximized profit function has the same form as equation (1).[1] According to Abel and Eberly's (2011) result, our parmeter θ summarizes all relevant information about returns to scale (in total capital) and the degree of competition. If the firm faces constant returns to scale and perfect competition, then $\theta=1$. If the firm instead either has some monopoly power or faces decreasing returns to scale, then $\theta<1$. In other words, relaxing our assumption that $\theta=1$ can be interpretted as letting the firm face either decreasing returns to scale or imperfect competition. Instead of reproducing Abel and Eberly's (2011) more general model, we make the reduced-form assumption in equation (1) to keep our analysis as simple and transparent as possible. We allow $\phi^{int}\neq\phi^{phy}$ to investigate the situation in which physical and intangible capital face different returns to scale.

The model in Section 2 assumes both capital types depreciate a the same rate δ. We now allow them to depreciate at different rates, denoted δ^{phy} and δ^{int}.

The model in Section 2 assumes quadratic adjustment costs. We now use the more

[1] The maximized profit function is equation (3) in Abel and Eberly (2011). Parameter α in their model corresponds to θ in ours. K in their model corresponds to K^{tot} in ours. R in their model corresponds to Π in ours.

general cost function

$$c_i^m = p^m I^m + K^{tot}\left(\zeta_i^m \frac{I^m}{K^{tot}} + \frac{\gamma_i^m}{2}\left|\frac{I^m}{K^{tot}}\right|^{\nu^m}\right), \quad m = phy,\ int, \quad \nu^m > 1 \tag{2}$$

This cost function collapses to one in our main model when $\nu^{phy} = \nu^{int} = 2$. We need the absolute value above to allow negative investment while avoiding imaginary numbers.

We cannot solve the generalized model in closed form. We switch from continuous to discrete time so we can solve the model numerically using value-function iteration on discretized state-variable grids. We choose parameter values to match those in Strebulaev and Whited (2012) where possible, except we choose γ to more closely match our estimated q-slopes.

We compare the baseline model from Section 2 with several variations. The baseline model and variations all share the same following inputs. One period corresponds to a year. The discount rate is 0.04. Physical and intangible capital share the same adjustment-cost parameters, $\gamma^{phy} = \gamma^{int} = 100$. The log of ε and p^{phy} follow

$$\begin{aligned} \log \varepsilon_{it} &= 0.7 \log \varepsilon_{i,t-1} + u_{\varepsilon,i,t}, \quad u_\varepsilon \text{ is i.i.d. } N(0, 0.15) \\ \log p_t^{phy} &= 0.7 \log p_{t-1}^{phy} + u_{pt}, \quad u_p \text{ is i.i.d. } N(0, 0.2). \end{aligned}$$

The baseline model has the following additional inputs. We normalize intangible capital's price p^{int} to one, so that p^{int} equals the unconditional median of p^{phy}. Depreciation rates equal $\delta^{phy} = \delta^{int} = 0.15$. There are constant returns to scale ($\theta = \phi^{phy} = \phi^{int} = 1$), and there are quadratic adjustment costs ($\nu^{phy} = \nu^{int} = 2$).

5.2. Predictions

We obtain predictions about investment regressions by solving the model, simulating a large panel of data, and running regressions of investment (either ι^{phy}, ι^{int}, or ι^{tot}) on q^{tot}, cash flow (c^{tot}), and time fixed effects. In our theory, cash flows correspond to profits (gross of investment costs) scaled by total capital, i.e., Π/K^{tot}.

5.2.1. Effects of alternate assumptions on the investment-q relation

First, we use the general model to explore how violations of Section 2's assumptions affect the predicted investment-q relation. Simulation results are in Table A5.

Table A5

Effects of alternate assumptions on the investment-q relation

This table shows results from OLS panel regressions estimated using simulated data. Panel A shows the slope coefficients from a regression of investment on lagged total q and time fixed effects. Panel B shows those regressions' R^2 values. Panel C shows results from regressions that are identical except include cash flow as an additional regressor. Panel D shows the increase in R^2 that results from including cash flow. The baseline model features constant returns to scale ($\theta = \phi^{phy} = \phi^{int} = 1$) and quadratic adjustment costs $\nu^{phy} = \nu^{int} = 2$). The next two columns show results from models with decreasing returns to scale ($\theta < 1$). The last columns show results from models with non-quadratic adjustment cost functions ($\nu \neq 2$). We simulate 3,000 firms for 250 years, then discard the first 200 years.

	Baseline Model	Decreasing Returns to Scale		Non-Quadratic Adjustment Costs	
Specification	($\theta = 1,\ \nu = 2$)	($\theta = 0.75$)	($\theta = 0.5$)	($\nu = 1.75$)	($\nu = 1.5$)
Panel A: Slopes on q^{tot}					
(1) ι^{phy} on q^{tot}	0.0067	0.0041	0.0023	0.0039	0.0010
(2) ι^{int} on q^{tot}	0.0067	0.0040	0.0023	0.0039	0.0011
(3) ι^{tot} on q^{tot}	0.0133	0.0082	0.0045	0.0077	0.0021
Panel B: R^2 values					
(1) ι^{phy} on q^{tot}	0.999	0.924	0.836	0.999	0.994
(2) ι^{int} on q^{tot}	0.999	0.919	0.832	1.000	1.000
(3) ι^{tot} on q^{tot}	0.999	0.924	0.836	0.999	0.994
Panel C: Slopes on cash flow					
(4) ι^{phy} on q^{tot}, c^{tot}	-0.0121	0.0044	0.0039	-0.0041	0.0008
(5) ι^{int} on q^{tot}, c^{tot}	-0.0121	0.0042	0.0038	-0.0039	0.0008
(6) ι^{tot} on q^{tot}, c^{tot}	-0.0242	0.0087	0.0077	-0.0082	0.0016
Panel D: Incremental R^2 from adding cash flow to regression					
(4) ι^{phy} on q^{tot}, c^{tot}	0.001	0.025	0.099	0.000	0.000
(5) ι^{int} on q^{tot}, c^{tot}	0.001	0.024	0.099	0.000	0.000
(6) ι^{tot} on q^{tot}, c^{tot}	0.001	0.025	0.099	0.000	0.000

The first column shows results from the baseline model. Because these results are from a discrete-time model, three of the baseline model's predictions deviate from those in Section 2's continuous-time model. First, whereas Section 2 predicts $q-$slopes of $1/\gamma^{phy} = 1/\gamma^{int} = 0.01$, we find $q-$slopes here of 0.0067. The reason for this discrepancy is that the investment-q relation is contemporaneous in the continuous-time model, but in discrete time, investment depends on $1/\gamma$ times the expectation of next period's q. In our simulated and empirical regressions, we use lagged q, not expected future q. The slope of expected q on lagged q is roughly 0.67, which explains the gap between $1/\gamma$=0.0100 and our estimated $q-$slope, 0.0067. Second, we find an R^2 of 99.9% rather than 100% in our main investment regressions. This deviation may simply reflect numerical error or the gap between lagged and expected future q. Third, Section 2 predicts no slopes on cash flow, but we find sizeable negative slopes on cash flow in Panel C. These slopes may result from numerical error or from the gap between expected and lagged q. It is comforting that adding cash flow to the regression increases its R^2 by only 0.001 (Panel D).

The second and third columns show how results change if there are decreasing returns to scale. These models are the same as the baseline model, except we reduce θ from 1 to either 0.75 or 0.5. The two capital types are still perfect substitutes in all ways, but there are now decreasing returns to scale in total capital (K^{tot}). We find that the more strongly returns decrease with scale, the lower are the predicted $q-$slopes (panel A), the lower are the R^2 values in the main investment-q regressions (Panel B), and the more explanatory power cash flows have for investment (Panel D). Decreasing returns to scale also flip cash flow's slope coefficient from negative to positive (Panel C), although the strength of this relation is not monotonic in θ. Abel and Eberly (2011) also show that cash flow is positively related to investment when there are decreasing returns to scale. When $\theta = 0.5$, $q-$slopes are roughly 1/3 their values in the baseline model, R^2 values are down to 84%, and cash flows explain 10% of the variation in investment.

The last two columns show how results change if there are non-quadratic capital adjustment costs. We reduce the adjustment cost function's curvature ν from 2 to either 1.75 or 1.5.[2] The lower the curvature in the adjustment-cost function (i.e., lower ν), the smaller are the predicted $q-$slopes (Panel A). We still find R^2 values near 100% in the investment-q regressions, even though these models do not predict a linear relation between investment and q when $\nu \neq 2$.[3] The regression's linear specification is apparently a very good approximation for the theory's nonlinear relation. As a result, cash flows provide virtually no additional explanatory power for investment (Panel D).

[2]The model does not converge for exponents above 2.

[3]For example, when ι^{phy} is positive, ι^{phy} depends nonlinearly on expected q as follows:

$$\iota_t^{phy} = \left(\frac{2}{\upsilon^{phy}} \frac{\beta E_t \left[q\left(\varepsilon_{t+1}, p_{t+1}\right)\right] - p_t^{phy}}{\gamma^{phy}} \right)^{\frac{1}{\upsilon^{phy}-1}}.$$

5.2.2. Differences between physical and intangible capital

Next, we examine potential sources of biases in our test of the theory's last prediction. This last prediction states that the intangible intensity (the ratio of intangible to total capital) equals the ratio of adjustment-cost parameters $\gamma^{phy}/\left(\gamma^{phy}+\gamma^{int}\right)$, which can be measured using the ratio of estimated q-slopes $\beta^{int}/\left(\beta^{int}+\beta^{phy}\right)$. This prediction relies on the strong assumptions in our baseline model, along with the additional assumptions that the capital types have the same price ($p^{int}=p^{phy}$) and the same linear adjustment cost $\left(\zeta^{int}=\zeta^{phy}\right)$. Section 5.2 from the main paper reports empirical evidence consistent with this prediction: Firms with higher intangible intensities have a higher ratio $\beta^{int}/\left(\beta^{int}+\beta^{phy}\right)$.

A concern here is that alternate models could produce this same empirical pattern. Specifically, the capital prices, linear adjustment costs, depreciation rates, and economies of scale could be quite different for physical and intangible capital. These differences could determine firms' optimal mix of capitals, and they could also influence their investment-q slopes in a way that produces a pattern like the one we report in Section 5.

To explore this concern, we now consider models in which physical and intangible capital differ in these ways, and we judge whether these differences can explain our empirical result. In all the results below, the two capital types have the same adjustment cost parameters $\gamma^{phy}=\gamma^{int}$, so we shut down the channel highlighted in the last prediction. Simulation results are in Table A6.

Table A6

Simulations with differences between capital types

This table shows results from OLS panel regressions estimated using simulated data. Panel A shows the average intangible intensity in simulated data. Panel B shows the slope on q^{tot} from a regression of either physical investment (regression 1) or intangible investment (regression 2) on total q and time fixed effects. The third row of Panel B show the ratio of specification (2)'s q-slope to the sum of slopes from specifications (1) and (2). Panel C shows the two regressions R^2 values. In the baseline model, physical and intangible capital's prices are equal on average, their depreciations rates are equal ($\delta^{phy} = \delta^{int} = 15\%$), there are constant returns to scale ($\theta = \phi^{phy} = \phi^{int} = 1$), and both capital types face quadratic capital adjustment costs ($\nu^{phy} = \nu^{int} = 2$). The last three columns show results from alternate models that are identical to the baseline model except for the change noted in the column header. We simulate 3,000 firms for 250 years, then discard the first 200 years.

Specification	Baseline Model	Different Purchase Prices ($p^{phy} < p^{int}$)	Different Depreciation Rates ($\delta^{phy} = 15\%$, $\delta^{int} = 20\%$)	Different Returns to Scale ($\phi^{phy} = 0.5$, $\phi^{int} = 0.9$)	Different Adjustment Cost Curvatures ($\nu^{phy} = 2$, $\nu^{int} = 1.75$)
Panel A: Average intangible intensity (K^{int}/K^{tot})					
	0.497	0.302	0.324	0.501	0.318
Panel B: Slopes on q^{tot}					
(1) ι^{phy} on q^{tot}	0.00665	0.00665	0.00684	0.00317	0.00666
(2) ι^{int} on q^{tot}	0.00665	0.00665	0.00635	0.00318	0.00403
$\beta^{int}/(\beta^{int} + \beta^{phy})$	0.500	0.500	0.481	0.501	0.377
Panel C: R^2 values					
(1) ι^{phy} on q^{tot}	0.999	0.999	0.995	0.932	0.999
(2) ι^{int} on q^{tot}	0.999	0.999	0.982	0.812	1.000

The first column of Table A6 shows results from our baseline model. Since the two capital types are alike in all ways, we find equal slopes in regressions of physical and intangible investment on q^{tot}, and we find an average intangible intensity of 50%.

Column two shows results from a model that is identical to the baseline model, except p^{int}, the price of intangible capital, is normalized to four instead of one. This variation on the baseline model conforms to all the assumptions in the paper's simple model, except for the additional assumption that is imposed to obtain the theory's last prediction. This variation also sheds light on differences in linear adjustment costs: Prediction 2 in the paper's model shows that the linear adjustment costs ζ play the same role as prices p in explaining investment. We can therefore interpret the model with unequal purchase prices as a model with unequal linear adjustment costs. As expected, increasing the relative purchase price of intangible capital makes firms buy relatively less intangible capital, and the average intangible intensity decreases from 50% to 30%. The estimated $q-$slopes are the same as in the baseline model, which is also expected given Prediction 3 in the paper. To summarize, differences in purchase prices or linear adjustment costs help explain differences in the optimal capital mix, but they do not explain differences in $q-$ slopes.

Column three shows results from a model in which intangible capital's depreciation rate is increased from 15% to 20%, so that the two capital types now have unequal depreciation rates. Firms now use relatively less intangible capital (32% compared to 50%), since its user cost is relatively higher. The ratio of $q-$slopes, $\beta^{int}/\left(\beta^{int}+\beta^{phy}\right)$, decreases slightly from 50% to 48%. Differences in depreciation rates are therefore an alternate explanation for our empirical result that firms using less intangible capital have a lower ratio $\beta^{int}/\left(\beta^{int}+\beta^{phy}\right)$.

Column four shows results from a model in which both capital types face diseconomies of scale, yet physical capital faces more severe diseconomies. Firms still use 50% intangible capital, and the ratio of q-slopes is still 50%. In other words, we find that differences in diseconomies do not necessarily produce unequal capital mixes or different q-slopes.

Column five shows results from a model in which physical capital still faces quadratic adjustment costs ($\nu^{phy}=2$), but intangible capital faces adjustment costs to the power 1.75 ($\nu^{int}=1.75$). The intangible intensity drops from 50% to 32%. The reason firms invest less in the capital type with a lower ν is that both the level and slope of adjustment costs with respect to investment are higher when ν is lower and investment rates are in their simulated rate. We also see that intangible investment's $q-$slope is now lower than that of physical investment, making the ratio $\beta^{int}/\left(\beta^{int}+\beta^{phy}\right)$ drop from 50% to 38%. In other words, we find that differences in curvature for the adjustment cost function are an alternate explanation for our empirical result that using less intangible capital have a lower ratio $\beta^{int}/\left(\beta^{int}+\beta^{phy}\right)$.

6. Intangibles, economies of scale, and competition

Section 5.1 in the main paper shows that the classic q theory fits the data better in firms with more intangibles. One potential explanation is that firms with more intangibles face fewer diseconomies of scale or more competition. In this section, we empirically compare the characteristics of firms with different amounts of intangible capital. Specifically, Table A7 below shows the economies of scale and competition faced by firms in the four intangible-intensity subsamples that we examine in the paper's Table 4.

We use two methods from the literature to estimate the economies of scale in each subsample. Both methods estimate the curvature of firms' production functions, with a curvature of one implying constant returns to scale, and a value less than one implying diseconomies of scale.

The first method is from Cooper and Haltiwanger (2006), which we review here. This method estimates the parameter θ in the profit function $\Pi = \varepsilon(K^{tot})^{\theta}$, using lagged instruments to account for the endogeneity of K^{tot}. Results are qualitatively similar if we use K^{phy} in place of K^{tot}. We assume ε depends on a common shock φ_t and a firm-specific shock ϵ_{it}:

$$\log \varepsilon_{it} = \varphi_t + \epsilon_{it}. \tag{3}$$

The dynamics of ϵ_{it} are given by

$$\epsilon_{it} = \rho_\varepsilon \epsilon_{it-1} + \omega_{it}, \tag{4}$$

where ω_{it} is an i.i.d. shock. Using the notation $\pi = \log(\Pi)$ and $k = \log(K^{tot})$, quasi-differencing yields

$$\pi_{it} = \beta_1 \pi_{it-1} + \beta_2 k_{it} + \beta_3 k_{it-1} + \varphi_t^* + \omega_{it}. \tag{5}$$

$$\beta_1 = \rho_\varepsilon, \quad \beta_2 = \theta, \quad \beta_3 = -\rho_\varepsilon \theta, \quad \varphi_t^* = \varphi_t - \rho_\varepsilon \varphi_{t-1}. \tag{6}$$

We follow Cooper and Haltiwanger (2006) in using log revenues in place of log profits. We estimate (5) in each subsample by GMM with time dummies, and we instrument for k_{it} using $k_{i,t-2}$ and $\pi_{i,t-2}$. Variables $\pi_{i,t-1}$ and $k_{i,t-1}$ do not require instruments, because they do not depend on ω_{it}. We use $\widehat{\beta}_2$ as our estimate of θ, and we confirm that the restriction $\beta_1\beta_2 = -\beta_3$ is not rejected.

The second method for estimating returns to scale is from Olley and Pakes (1996), which we review next. This method assumes a production function implying log revenues that follow

$$y_{it} = \beta_0 + \beta_l l_{it} + \beta_k k_{it} + u_{it}, \tag{7}$$

$$u_{it} = \Omega_{it} + \eta_{it}, \tag{8}$$

where l_{it} is the log value of variable inputs, k_{it} is the log of capital, Ω_{it} is a productivity shock that is observed by the decision-maker in the firm but not by the econometrician, and η_{it} is an unexpected productivity shock observed by neither the econometrician or the decision-maker. Our goal is to measure β_k, which measures the revenue function's curvature with respect to capital. This method uses the observed level of investment to infer the value of Ω_{it}, thereby allowing us to control for the correlation between the error term and inputs in the equation above. We implement this method using log total capital for k, log COGS for l, and total investment to infer Ω. Results are qualitatively similar if we use physical capital in place of total capital.

Next, we define three proxies for the degree of product-market competition a firm faces.

The first proxy is the Herfindahl index in the given firm's industry and year. We compute this index using revenues, and we use four alternate industry classifications: the Fama-French 5 and 49 industries, and also Hoberg and Phillips' (2010, 2015) 25 and 100 text-based Fixed Industry Classifications (FIC).

The second proxy is profitability, which we measure two ways. The logic is that high competition is associated with low profitability. The first profitability measure is EBITDA scaled by lagged revenues. Note that EBITDA is net of intangible investment. The second measure is c^{tot}, which equals firm profits gross of intangible investment, scaled by lagged total capital (details in section 3.4 in our paper).

The third proxy is firm size. The logic is that a small firm, and especially a firm that is small relative to its competitors, is more of a price-taker. We measure firm size four ways: the log of revenues, log of market cap, revenues scaled by median revenues within the same FIC-100 industry, and market cap scaled by median market cap within the same FIC-100 industry.

Table A7
Characteristics of firms with different amounts of intangible capital

We compute characteristics in the four subsamples formed using each year's quartile breakpoints for intangible intensity. These subsamples match those in our paper's Table 4. We define the characteristics in detail in the text immediately above.

	Quartile 1 (8% intan.)	Quartile 2 (33% intan.)	Quartile 3 (56% intan.)	Quartile 4 (76% intan.)	Quartile 4–1 Diff.	Quartile 4–1 (StdErr.)
Panel A: Production-function curvature estimates						
Cooper-Haltiwanger method	0.413	0.427	0.480	0.401	-0.012	0.038
Olley-Pakes method	0.250	0.466	0.581	0.386	0.136	0.091
Panel B: Mean Herfindahl index						
Fama-French 5 industries	0.0215	0.0230	0.0236	0.0244	0.0029	(0.0049)
Fama-French 49 industries	0.0966	0.0879	0.0816	0.0775	-0.0190	(0.0095)
FIC 25 industries	0.0765	0.0728	0.0758	0.0768	0.0002	(0.0041)
FIC 100 industries	0.1435	0.1585	0.1688	0.1742	0.0307	(0.0082)
Panel C: Mean profitability						
EBITDA/Sales	0.112	0.088	0.054	-0.213	-0.325	(0.028)
Total cash flow (c^{tot})	0.116	0.158	0.180	0.155	0.039	(0.004)
Panel D: Mean firm size						
Log(Revenues)	5.55	5.69	5.56	4.88	-0.68	(0.07)
Log(Market Cap)	5.45	5.43	5.55	5.43	-0.02	(0.06)
Sales / industry median	0.948	0.994	1.072	0.697	-0.251	(0.070)
Market cap / industry median	0.896	0.958	1.043	0.748	-0.148	(0.051)

7. Results from IV estimators

Table A8

Counterpart of paper's Table 3 using Arellano and Bond's (1991) IV estimator

Details are the same as in Table 3 in the main paper, except here we use Arellano and Bond's (1991) IV estimator. This estimator takes first differences of the investment-q model, uses twice-lagged q and investment as instruments, and weights these instruments optimally using GMM. We do not present R^2 values, which are poorly defined for the IV estimator.

	Investment scaled by total capital (K^{tot})				
	Physical (ι^{phy})	Intangible (ι^{int})	Total (ι^{tot})	R&D	CAPX/PPE (ι^{*})
Panel A: Regressions without cash flow					
Total q (q^{tot})	0.010	0.005	0.014	0.004	
	(0.001)	(0.001)	(0.002)	(0.001)	
Standard q (q^{*})					0.010
					(0.001)
Panel B: Regressions with cash flow					
Total q (q^{tot})	0.016	0.009	0.023	0.005	
	(0.001)	(0.001)	(0.002)	(0.001)	
Total cash flow (c^{tot})	0.090	0.049	0.142	0.020	
	(0.005)	(0.003)	(0.007)	(0.003)	
Standard q (q^{*})					0.013
					(0.001)
Standard cash flow (c^{*})					0.010
					(0.001)
Observations	99,553	99,553	99,553	53,841	99,553

Table A9
Counterpart of paper's Table 3 using Biorn's (2000) IV estimator

Details are the same as in Table 3 in the main paper, except here we use Biorn's (2000) IV estimator. This estimator takes first differences of the investment-q model and uses three lags of cash flow and Tobin's q as instruments for the first difference of Tobin's q. We do not present R^2 values, which are poorly defined for the IV estimator.

	Investment scaled by total capital (K^{tot})				
	Physical (ι^{phy})	Intangible (ι^{int})	Total (ι^{tot})	R&D	CAPX/PPE (ι^*)
Panel A: Regressions without cash flow					
Total q (q^{tot})	0.024	0.004	0.028	-0.002	
	(0.003)	(0.002)	(0.004)	(0.002)	
Standard q (q^*)					0.012
					(0.002)
Panel B: Regressions with cash flow					
Total q (q^{tot})	0.021	0.002	0.023	-0.003	
	(0.003)	(0.002)	(0.004)	(0.002)	
Total cash flow (c^{tot})	0.088	0.066	0.155	0.036	
	(0.005)	(0.004)	(0.007)	(0.004)	
Standard q (q^*)					0.011
					(0.002)
Standard cash flow (c^*)					0.026
					(0.003)
Observations	88,700	88,700	88,700	47,482	88,700

8. Placebo Analysis

We perform a placebo analysis to show that our main results would not obtain if our intangible measures were pure noise. Note that we can write our variables as

$$
\begin{aligned}
q_{i,t-1}^{tot} &= q_{i,t-1}^{*} A_{i,t-1} && (9)\\
\iota_{i,t}^{phy} &= \iota_{i,t}^{*} A_{i,t-1} && (10)\\
\iota_{i,t}^{tot} &= \iota_{i,t}^{*} A_{i,t-1} B_{i,t} && (11)\\
\iota_{i,t}^{int} &= \iota_{i,t}^{tot} - \iota_{i,t}^{phy} && (12)\\
A_{i,t-1} &\equiv \frac{K_{i,t-1}^{phy}}{K_{i,t-1}^{phy} + K_{i,t-1}^{int}} && (13)\\
B_{i,t} &\equiv \frac{I_{i,t}^{phy} + I_{i,t}^{int}}{I_{i,t}^{phy}}. && (14)
\end{aligned}
$$

We simulate intangible investment $\widetilde{I}^{int}$ that has same mean, persistence, and volatility as actual intangible investment, but is otherwise pure noise. Next, we compute simulated intangible capital stocks $\widetilde{K}^{int}$ by applying the perpetual-inventory method to $\widetilde{I}^{int}$, just as we do in the actual data. We use these simulated values of $\widetilde{I}^{int}$ and $\widetilde{K}^{int}$, along with actual values of I^{phy}, K^{phy}, q^{*}, and ι^{phy}, to compute the placebo variables $\widetilde{\iota}^{phy}$, $\widetilde{\iota}^{int}$, $\widetilde{\iota}^{tot}$, and $\widetilde{q}^{tot}$ using formulas (9)-(14) above. We use $\widetilde{\iota}^{tot}$ and $\widetilde{q}^{tot}$ in OLS and cumulants regressions similar to those in Tables 2 and 3.

The specific steps of the placebo analysis are as follows:

1. Define $x_{it} = I_{it}^{int} / K_{i,t-1}^{phy}$. Using actual data, estimate the panel regression

$$x_{it} = a_i + a_t + \theta x_{i,t-1} + \varepsilon_{it}. \qquad (15)$$

   Collect the estimates $\widehat{a}_i, \widehat{a}_t$, $\widehat{\theta}$, and $var\left(\widehat{\varepsilon}_{it}\right)$.

2. For each firm i in our sample, randomly select some other firm j. Collect $\widehat{a}_j$ and the initial values x_{j1} and A_{j0}.

3. Create simulated values $\widetilde{x}_{it}$ assuming $\widetilde{x}_{i1} = x_{j1}$ and

$$\widetilde{x}_{it} = \widehat{a}_j + \widehat{a}_t + \widehat{\theta} \widetilde{x}_{i,t-1} + \widetilde{\varepsilon}_{it}, \ t > 1, \qquad (16)$$

   where $\widetilde{\varepsilon}_{it}$ is drawn independently from $N\left(0, var\left(\widehat{\varepsilon}_{it}\right)\right)$. We set any negative values of $\widetilde{x}_{it}$ to zero, since x_{it} is never negative in our data.

4. Compute simulated values of intangible investment according to $\widetilde{I}_{i,t}^{int} = \widetilde{x}_{i,t} K_{i,t-1}^{phy}$.

5. Compute the simulated intangible capital stock assuming firm i's initial intensity is $A_{j,0}$. Firm i's simulated starting intangible stock therefore equals $\widetilde{K}_{i,0}^{int} = K_{i,0}^{phy}/A_{j,0} - K_{i,0}^{phy}$. Compute future periods' $\widetilde{K}^{int}$ applying the perpetual-inventory method to $\widetilde{I}_{i,t}^{int}$ with a 20% depreciation rate.

Regression results using simulated data are in the following table:

Table A10
Counterpart of Paper's Table 3 using simulated data

Details are the same as in Table 3 in the main paper, except Panel A uses simulated data on intangible investment to construct placebo values of ι^{phy}, ι^{int}, ι^{tot}, and q^{tot}. Results in Panel B match the results in the paper's Table 3, Panel A.

	Investment scaled by total capital (K^{tot})			
	Physical (ι^{phy})	Intangible (ι^{int})	Total (ι^{tot})	CAPX/PPE (ι^{*})
Panel A: Placebo regressions using simulated data				
Placebo total q (q^{tot})	0.050	-0.089	0.046	
Standard q (q^{*})				0.036
OLS R^2	0.267	0.033	0.247	0.244
ρ^2	0.280	-0.094	0.331	0.372
τ^2	0.458	-0.022	0.553	0.492
Panel B: Results using actual data (from Paper Table 3, Panel A)				
Total q (q^{tot})	0.070	0.037	0.086	
	(0.001)	(0.001)	(0.001)	
Standard q (q^{*})				0.036
				(0.001)
OLS R^2	0.209	0.279	0.327	0.244
	(0.004)	(0.006)	(0.005)	(0.005)
ρ^2	0.358	0.392	0.423	0.372
	(0.007)	(0.007)	(0.008)	(0.007)
τ^2	0.437	0.559	0.597	0.492
	(0.008)	(0.009)	(0.008)	(0.010)

Since simulated intangible investment is pure noise, it is not explained well by q: The OLS R^2 for $\widetilde{\iota}^{int}$ in panel A is just 0.033. Values of ρ^2 and τ^2 are negative for placebo regressions using $\widetilde{\iota}^{int}$, indicating a misspecified model. The model is misspecified because the q-slope is effectively zero, which violates one of the cumulant estimator's assumptions.

Next, we compare the literature's standard regression (last column) to the placebo regression of $\widetilde{\iota}^{tot}$ on $\widetilde{q}^{tot}$. The placebo regression produces an R^2 of 0.247, which is slightly higher than the 0.244 R^2 from literature's standard regression, but is well below the 0.327 R^2 from using actual data on total capital (Panel B). The placebo regression's ρ^2 is 0.331, lower than the total-capital ρ^2 (0.423) and even the physical-capital ρ^2 (0.372). The placebo regression's τ^2 is 0.553, roughly halfway between the standard regression's τ^2 (0.492) and the total-capital τ^2 (0.597). The placebo regression produces a bias-corrected q-slope of 0.046, higher than the physical-capital slope (0.036), but much lower than the total-capital slope (0.086). To summarize, our total-investment results would not obtain if our intangible-capital measures were pure noise. Such noise could explain only half of the observed increase in τ^2. It explains very little of the observed increases in R^2 and q-slopes. Noise could explain none of the observed increase in ρ^2.

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