# Embrace or Fear Uncertainty: Growth Options, Limited Risk Sharing, and Asset Prices*

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**Abstract**

The impact of uncertainty shocks on asset prices and macroeconomic dynamics depends on the degree of risk sharing in the economy and the origin of uncertainty. We develop a general equilibrium model with imperfect risk sharing and two sources of uncertainty shocks: (i) productivity uncertainty shocks, which affect the idiosyncratic volatility of firms' productivity, and (ii) investment uncertainty shocks, which affect the idiosyncratic variability of firms' investment opportunities. My model deviates from the neoclassical setting in two respects: first, firms' investment policies are set by the managers who are subject to a moral hazard problem and thus must maintain an undiversified ownership stake in the firm; and second, costly hedging can be obtained through intermediaries. As a result, risk sharing capacity between managers and other investors is limited, which is governed by the intermediary condition. Limited risk sharing distorts equilibrium investment choices, firm valuation, and prices of risk in equilibrium relative to the frictionless benchmark. In the calibrated model, the risk premium on investment uncertainty shocks is negative when risk sharing capacity is low and positive otherwise. Moreover, the cross-sectional spread in valuations between value and growth stocks loads positively on the investment uncertainty shocks under poor risk sharing conditions and negatively otherwise. The calibrated model provides quantitative implications that help understand empirical patterns. Empirical tests also support the predictions of the model.

*Keywords: Productivity uncertainty; Investment uncertainty; Idiosyncratic risks; Investment lumps; Managerial incentive; Market Incompleteness; Risk management; Value premium.

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# 1 Introduction

The volatility of idiosyncratic shocks can affect agents' economic behaviors when markets are incomplete or technologies contain optionality features. The literature refers to the aggregate shock to the common component of idiosyncratic volatilities as an uncertainty shock, since it alters agents' information sets about future economic outcomes altogether.[1] Uncertainty shocks have proven useful in explaining macroeconomic fluctuations and have been adopted as a standard feature of dynamic stochastic general equilibrium (DSGE) models for policy analysis.[2]

Despite substantial advances in understanding the economic impact of uncertainty shocks, two central and fundamental questions remain unsolved: first, how to reconcile the mixed empirical evidence about the effect of uncertainty shocks on asset prices and investment in one coherent framework; second, what are the factors underpinning the impact of uncertainty shocks. In particular, whether a positive uncertainty shock benefits or harms growth firms relative to value firms remains debatable, as does whether a rise in uncertainty boosts or curtails aggregate investment. The stylized facts are summarized in Figure 1.[3]

To address these questions, I develop a tractable investment-based general equilibrium model of asset prices with heterogeneous firms and agents in incomplete markets. Using the model as a guide, I revisit the link between asset prices and uncertainty with an explicit emphasis on the interaction with risk sharing conditions in the economy. The model not only provides a theoretical framework that quantitatively makes sense of these seemingly contradictory empirical findings; its main contribution is to do so by providing a fundamental economic mechanism through the explicit modeling of endogenous imperfect risk sharing. The model recognizes two key elements shaping the impact of uncertainty shocks: (i) the risk sharing condition of the economy and (ii) the sources of uncertainty shocks.

Let me describe the main features of my model, starting with the characteristics of firms' technologies then turning to the characteristics of the agents. In the model, firms produce consumption goods using

[1] There has been a fast growing literature studying the aggregate effects of such uncertainty shocks (e.g., Pástor and Veronesi, 2006, 2009; Bloom, 2009; Arellano, Bai and Kehoe, 2011; Bloom et al., 2013; Bachmann and Bayer, 2014; Christiano, Motto and Rostagno, 2010, 2014; Bundick and Basu, 2014; Gilchrist, Sim and Zakrajsek, 2014; Herskovic et al., 2014). Here, the use of the term *uncertainty* is different from *Knightian uncertainty*, which emphasizes the situations where agents cannot know all the information they need to set accurate odds in the first place (e.g., Knight, 1921; Hansen and Sargent, 2008). Also, uncertainty here is different from aggregate volatility, which has also been extensively studied in the literature (Bansal and Yaron, 2004; Drechsler and Yaron, 2011; Shaliastovich, 2015; Campbell, Giglio and Polk, 2013; Campbell et al., 2015; Fernandez-Villaverde et al., 2011; Nakamura, Sergeyev and Steinsson, 2014; Segal, Shaliasovich and Yaron, 2015; Gourio, Siemer and Verdelhan, 2015; Ai and Kiku, 2015).

[2] Policy authorities, including the Federal Reserve Board and the European Central Bank have claimed that uncertainty has an adverse effect on economy, and they have built uncertainty shocks into their core DSGE models as a main driver of the aggregate fluctuations (Christiano, Motto and Rostagno, 2010, 2014). For example, at the 2013 Causes and Macroeconomic Consequences of Uncertainty conference, Federal Reserve Bank of Dallas President Richard Fisher gave a formal speech titled "Uncertainty matters. A lot." It emphasized that uncertainty could worsen the Great Recession and the ongoing recovery.

[3] I use the average idiosyncratic volatility across U.S. public firms' stock returns as a proxy for the total uncertainty. In Panels A and B of Figure 1, the high uncertainty in the late 1980s occurs with positive value spreads (i.e., cross-sectional spreads between value and growth stock returns) and high investment, and the high uncertainty in the late 1990s occurs with negative value spreads and high investment. However, the high uncertainty in the early 1990s and the late 2000s accompanies negative value spreads and low investment. Panel C shows that aggregate market volatility is almost perfectly correlated with total uncertainty over the period 1980 - 2014. The mixed empirical evidence on total uncertainty's effects illustrated in Panels A and B are thus linked to the ambiguous impacts of market volatility on asset prices and macroeconomic dynamics documented by Bansal et al. (2014) and Campbell et al. (2015), among others.

production units, which are building blocks of assets in place. The existing assets in place depreciate over time. Firms invest to create new assets in place using growth options. Growth options are intangible assets associated with innovations such as blueprints and research and development (R&D) projects. The investment decision is an option that the firm exercises optimally only when it receives an investment opportunity. Their investment opportunities arrive randomly over time and are subject to firm-specific shocks.

Firms' technologies feature two sources of uncertainty shocks: the cash-flow uncertainty shock and the growth uncertainty shock. Cash-flow uncertainty captures the variation in idiosyncratic volatility of assets-in-place productivity; growth uncertainty captures the variation in idiosyncratic volatility of investment-opportunity quality. Growth uncertainty can have a very different effect than cash-flow uncertainty due to the optionality embedded in growth options. This optionality arises from the flexibility in the innovation process. Simply put, if the quality of the investment opportunity turns out to be exceedingly good, the firm has the flexibility to dial up investment to exploit the beneficial realization of the investment shock; alternatively, the firm can tune down investment to insure against the adverse realization of the investment shock. The optionality makes the benefit of growth options a convex function of the underlying shock. As a result, growth uncertainty increases the value of growth options and the aggregate investment. Effectively, the growth uncertainty shock affects the economy in the same way as a simple aggregate investment-specific technological (IST) shock, which directly alters the economy's real investment environment.[4]

By their nature, the two uncertainty shocks cause different impacts in complete markets; moreover, their effects on the economy can be altered by the interactions between imperfect risk sharing and uncertainty shocks. To understand the interactions, I introduce financial frictions that endogenously arise from a standard moral hazard problem. Specifically, agents in my economy are either experts or households. Each expert is a representative agent for a team of managers and active insiders, who are usually financial intermediaries. Within each team, the managers and active insiders perfectly insure each other's consumption risks.[5] Each expert uses her unique skills to manage a particular firm's assets. In other words, the expert is the key talent without whose efforts the particular firm would cease to perform. To invest, each expert raises funds from the capital markets by issuing equity. However, experts face a moral hazard problem that imposes a co-investment or skin-in-the-game constraint: each expert must retain an undiversified ownership stake in the firm as a commitment not to make managerial decisions that maximize private benefits at the cost of reduced firm value. This incentive constraint limits an expert's capacity to insure against idiosyncratic cash flow and investment risks. On the other hand, households cannot run firms or trade assets, but they can invest in financial securities and therefore partially share

[4] The aggregate investment-specific technological shock has become a standard feature of real business cycle models (Greenwood, Hercowitz and Krusell, 1997, 2000; Fisher, 2006; Justiniano, Primiceri and Tambalotti, 2011). Moreover, recent papers show that the aggregate investment-specific technological shock can help explain asset pricing puzzles (Christiano and Fisher, 2003; Papanikolaou, 2011; Kogan and Papanikolaou, 2013, 2014; Garlappi and Song, 2014; Kogan, Papanikolaou and Stoffman, 2015).

[5] This is a simplification assumption widely adopted in the macroeconomic models with financial sectors (e.g., Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011; Brunnermeier and Sannikov, 2014). In other words, like mine here, these models focus on the financial frictions between households versus insiders of the corporate and financial sectors.

risks with experts.

My model therefore deviates from the neoclassical setting in one key respect: firms' investment policies are set by experts who are subject to background risks imposed by incentive constraints.[6] This friction distorts experts' investment decisions and portfolio allocation from the first-best benchmark; households provide risk sharing to experts through financial markets trying to mitigate the distortion and smooth their own consumption. When experts' balance sheets are well capitalized, they bear a small amount of implied idiosyncratic wealth risks and thus have a higher capacity to share risks with households; otherwise, they bear a large amount of implied idiosyncratic wealth risks and thus have a lower capacity to share risks with households, because they require more insurance from households to keep up real investment. The theoretical concept of the risk sharing condition can be interpreted as the condition of the financial sector in the data; in reality, the financial sector plays the largest role in determining the degree of risk sharing in the economy.

Due to experts' endogenous background risks, the impact of uncertainty shocks on asset prices and investment depends on the degree of risk sharing. When risk sharing is limited, positive uncertainty shocks dramatically increase the severity of background risks to experts, who become implicitly more risk averse. More precisely, in response to a positive cash-flow uncertainty shock, experts require higher risk premia on assets in place; for a rise in growth uncertainty, experts require higher risk premia on growth options. Yet whether the implied higher risk premia eventually increase or curtail experts' willingness to invest depends on the specification of preferences. More precisely, it depends on whether the intertemporal substitution effect dominates the wealth effect. In general, the intertemporal substitution effect dominates when the elasticity of intertemporal substitution (EIS) is larger than one.[7] In such cases, a rise in uncertainty induces experts to invest less in firms' assets today and more in the future, since their desire for a better investment environment dominates that for consumption smoothing. The interest rate tends to decline due to the fly-to-quality effect, and yet it remains stable because of the high EIS coefficient. As a result, assets' prices have to drop to provide higher risk premia. Specifically, a rise in cash-flow uncertainty always decreases the prices of assets in place. However, a rise in growth uncertainty can have an ambiguous impact on growth option's prices. The net effect of higher growth uncertainty depends on the competition between the positive force of the optionality and the negative force of the precautionary saving motive. In times when risk sharing is limited, the precautionary saving motive becomes strong enough to dominate the option effect.

Compared to cash-flow uncertainty shocks, higher growth uncertainty causes an additional risk to experts. It is the risk of increasing inequality in the distribution of innovation benefits from growth options. The skewness in the distribution of innovation benefits matters when the risk sharing of idiosyncratic investment shocks is limited. Such a high-moment risk thus becomes particularly devastating when the

[6]The implications of background risks for asset prices and firms' financing and investment behavior have been investigated by Heaton and Lucas (1997, 2000*a*,*b*); Miao and Wang (2007); Chen, Miao and Wang (2010), among others. In my model, the background risks are endogenously derived from a moral hazard problem, in an explicit and coherent way, within a general equilibrium macroeconomic framework. I focus on investigating the general equilibrium implications of the endogenous background risks.

[7]The discussion on the relationship between the EIS coefficient and the dominance of intertemporal substitution effect can be found in Weil (1990) and Bhamra and Uppal (2006).

risk sharing condition is poor. The intuition is further elaborated as follows. Most of the benefits from innovation accrue to a small fraction of experts, while the majority of experts bear the cost of creative destruction since they need to pay for the new assets in place to keep up their production levels. Wealth is reallocated from the experts who do not invest to those who receive high-quality investment opportunities. This reallocation becomes more skewed when growth uncertainty becomes higher, since growth-option benefits are asymmetric. Technically speaking, each expert faces a more skewed idiosyncratic investment risk.[8] For a risk averse expert, the higher skewness in the idiosyncratic risk leads to lower certainty equivalent wealth. Therefore, the growth uncertainty shock contributes to an adverse redistribution risk: the displacement risk.[9]

The risk sharing condition, moreover, is endogenous and affected by uncertainty shocks. When the intertemporal substitution effect dominates, experts charge higher risk premia and want to sell assets to reduce their exposure to idiosyncratic risks, in response to a rise in uncertainty. This leads to a plunge in asset prices. Experts are atomistic, so they do not take into account the general equilibrium effect of their own asset sales on asset prices, even though they are aware of the adverse effect of plunging asset prices on their risk sharing conditions. This pecuniary externality arises from financial constraints, together with competitive asset markets.[10] Due to such a pecuniary externality, the adverse feedback loop between plunging asset prices (soaring risk premia) and deteriorating risk sharing conditions characterizes the equilibrium.

This fundamental economic mechanism has important asset pricing implications. To understand them, it is necessary to establish how these shocks affect the marginal investor's utility and determine how uncertainty shocks affect the cross section of firms.

The cash-flow uncertainty shock always carries a negative market price of risk,[11] because the cash-flow uncertainty decreases experts' current and future consumption. However, when growth uncertainty rises, experts face three endogenous risks: the investment risk, the endogenous financial risk, and the displacement risk. When the EIS coefficient is sufficiently large, the latter two contribute to a negative market price of risk for growth uncertainty shocks, whereas the investment risk contributes to a positive market price of risk. The risk sharing condition determines the net effect between the two countervailing forces. The positive force of investment risk dominates when risk sharing is efficient; the negative force of the endogenous financial risk and the displacement risk dominates otherwise.

Uncertainty shocks do not affect all firms equally in the cross section. The heterogeneous impacts are time varying; they depend on risk sharing conditions. A positive cash-flow uncertainty increases the value of growth options relative to assets in place. This is because a higher cash-flow uncertainty immediately increases the riskiness of assets in place. As a result, experts gravitate to safer assets, including growth

[8] In contrast, the idiosyncratic cash flow risk is always symmetric.

[9] There is more discussion and a literature review on the asset pricing implications of displacement risks in Section 1.1.

[10] This particular pecuniary externality has been explicitly investigated and highlighted by Lorenzoni (2008). The models studying financial stability and its macroeconomic implications are mainly built on this basic mechanism (e.g., Bernanke and Gertler, 1989; Kiyotaki and Moore, 1997; Bernanke, Gertler and Gilchrist, 1999; He and Krishnamurthy, 2011, 2013; Brunnermeier and Sannikov, 2014; Di Tella, 2014).

[11] Recall that the formal (technical) definition of market price of risk for a shock is the negative contemporaneous response of a marginal investor's marginal utility to a unit increase in such a shock.

options. This portfolio rebalancing tendency increases the price of growth options. Meanwhile, the price of investment goods decreases, which provides a hedge for growth options against the drop in the value of assets in place. A rise in growth uncertainty can increase or decrease the value of growth options relative to assets in place; the sign depends on the risk sharing condition. When risk sharing is efficient, the growth uncertainty increases the investment risk attached to the growth options, which increases the value of growth options relative to assets in place; otherwise, the growth uncertainty shock increases the endogenous financial risk and the displacement risk attached to growth options, which pushes experts to safer assets, including assets in place. The portfolio rebalancing tendency has an increasing effect on the value of assets in place. In summary, uncertainty shocks affect firms differently depending on whether they derive most of their value from growth options or assets in place, and depending on whether the risk sharing condition is good or poor.

Using state-of-the-art techniques, I solve the model globally to capture the nonlinearity of economic dynamics and the endogenous fluctuations of risk sharing conditions. I calibrate the model to match the moments of macroeconomic variables and check whether the calibrated model can provide reasonable asset pricing moments and cross-sectional dynamics of firms. In my calibration, the model has a reasonable quantitative performance, which is summarized as follows. First, the model reproduces a sizable equity premium, mainly attributed to the market incompleteness; it also reproduces a large value premium, mainly attributed to the heterogeneous effects of cash-flow uncertainty shocks. Second, in the model as in the data, the sales dispersion is countercyclical, while the investment dispersion is pro-cyclical. This empirical pattern is highlighted in Bachmann and Bayer (2014) as an important cross-equation restriction for the macroeconomic models with uncertainty shocks. My model provides a novel reconciliation for the two dispersion processes within a unified framework. In this framework, the sales dispersion is driven by the cash-flow uncertainty shock, but not by the growth uncertainty shock; it is countercyclical because the cash-flow uncertainty leads to economic downturns. On the other hand, the investment dispersion is driven by the growth uncertainty shock, but not by the cash-flow uncertainty shock; it is pro-cyclical because the impact of growth uncertainty shocks on the investment dispersion is asymmetric: the effect is larger when the risk sharing condition is good. These connections between uncertainty and dispersion are verified in the data using estimated uncertainty shocks.

I empirically test the model's main predictions. I first set up a regime-switching model in which the exposure of value spreads to growth uncertainty shocks is time-varying and characterized by a latent Markovian state variable. My theory implies that the latent state in which the exposure is higher should correspond to the state in which risk sharing is limited. I use the credit spread (e.g., Gilchrist and Zakrajsek, 2012) and the chronologies of financial crisis constructed by Reinhart and Rogoff (2009) as proxies for the risk sharing condition in the data. The empirical evidence is consistent with the model: the estimated likelihood of being in the latent state of higher growth uncertainty shock exposure is significantly, positively associated with the proxies of risk sharing conditions. I also provide additional empirical tests verifying this particular prediction; the results of statistical tests are significant. Then, I verify the predictions of the market price of risk for uncertainty shocks in the data.

In summary, this paper casts light on the recent debate on the role of uncertainty shocks in explaining asset pricing phenomena and macroeconomic dynamics, and on how the cross section of asset returns can identify uncertainty shocks from different sources. Moreover, the time-varying cross-sectional moments of asset prices, depending on the degree of risk sharing, impose additional cross-equation restrictions on the properties of uncertainty shocks used in macroeconomic models and thus can provide extra insights on the origins of aggregate fluctuations. Further, as both the model and empirical evidence highlight the importance of sources and risk sharing conditions for determining how the economy reacts to uncertainty shocks and the endogeneity of aggregate volatilities driven by different underlying uncertainty shocks, this paper provides a cautionary note to empirical studies using one aggregate volatility index to draw conclusions on the economic impact of uncertainty.

### 1.1 Related Literature

The idea that uncertainty shocks affect investment and asset prices dates back at least to the literature exploring the (implicit) optionality associated with production and investment technologies (e.g., Oi, 1961; Hartman, 1972; Abel, 1983; Caballero, 1991; Dixit and Pindyck, 1994; Bar-Ilan and Strange, 1996; Abel et al., 1996). Since then, many different dynamic structural models have been developed based on these ideas trying to quantify the relevance of uncertainty shocks in the data.

The technical challenge of analyzing stochastic dynamic general equilibrium models with structural links between uncertainty shocks and the data is well known. The existing literature tries to make progress by focusing on a single isolated channel in each model. One strand of literature investigates the wait-and-see effect by introducing decreasing-scale-to-return production, sizable adjustment costs, and irreversibility into the dynamic setting (e.g., Bloom, 2009; Bloom et al., 2013; Bachmann and Bayer, 2014). The asymmetric effect of uncertainty on benefits and costs of waiting captures the essence of the waiting option effect. This is referred to as the *bad news principle* by Bernanke (1983). However, the waiting option effect can be mitigated or even turned over when some environmental variables shift. For example, this idea has been demonstrated in Miao and Wang (2007) and Bolton, Wang and Yang (2013) under partial equilibrium frameworks. For investors bearing uninsurable idiosyncratic risks and firms being financially constrained, the uncertainty shock can have both a positive and a negative effect on investment and financing decisions. My model deliberately brings the idea of financial friction and imperfect risk sharing into a general equilibrium framework in which the opposite impacts of uncertainty shocks emerge endogenously.

Another strand of literature explores the credit risk premium channel (e.g., Christiano, Motto and Rostagno, 2010, 2014; Arellano, Bai and Kehoe, 2011; Gilchrist, Sim and Zakrajsek, 2014). The key idea is that in an economy with corporate debt and costly default, higher uncertainty lifts the default probability for firms that are already near default boundaries, and hence the cost of debt financing increases. This in turn reduces the investment and increases the default probabilities for firms that are originally not so close to the default boundaries. As a result of the ripple effect, aggregate hiring decreases, which leads to lower household consumption and thus feeds back to a higher credit risk premium. This adverse feedback loop reinforces the ripple effect, dragging the whole economy into recessions and creating high credit spreads. It is clear that if the financial sector is strong and very few firms are close to financially binding constraints, the adverse risk premium effect will be largely dampened.

A third strand of literature investigates the interaction between learning and uncertainty shocks. One interaction is the learning-by-doing mechanism, which assumes that investors have imperfect information about the underlying state and that the only way to achieve extra signals about the true state is through a sequence of real investments. Naturally, in a high uncertainty environment, investors conduct earlier and more intensive investment to learn the underlying state (e.g., Roberts and Weitzman, 1981; Pindyck, 1993; Pavlova, 2002). Moreover, in Pástor and Veronesi (2006, 2009), the authors show that the uncertainty shock increases the value of growth options relative to assets in place, and this effect is particularly large when uncertainty shocks are convolved with Bayesian learning. On the other hand, uncertainty shocks, interacting with learning, can also depress asset prices and investment. In Van Nieuwerburgh and Veldkamp (2006), if acquiring information becomes slower and belief uncertainty becomes higher during economic downturns, the learning mechanism generates slow recoveries and countercyclical asset pricing dynamics. Moreover, Fajgelbaum, Schaal and Taschereau-Dumouchel (2013) show that low activity and slower learning can form an unpleasant feedback loop. The fixed point for this feedback loop is the equilibrium that displays uncertainty traps: self-reinforcing episodes of high uncertainty and low activity. The uncertainty trap can substantially worsen recessions and increase their duration.

A main contribution of this paper is to introduce two sources of uncertainty shocks into one unified theoretical framework in which the impact of uncertainty shocks varies endogenously, governed by a macroeconomic condition: the degree of risk sharing in the economy. Importantly, the theoretical framework is tractable, which allows for accurate global solutions. This model is motivated by several strands of literature. Basically, I incorporate the models of heterogeneous agents bearing undiversified idiosyncratic risks and the macroeconomic models of financial stability into an investment-based general equilibrium model for asset prices. Therefore, my paper is also deeply connected to the following three strands of literature.

The asset pricing literature on heterogeneous agents with undiversified idiosyncratic risks explores the possibility of solving the equity premium puzzle based on market incompleteness. This literature goes back to Mankiw (1986) and Constantinides and Duffie (1996). The key idea is that the time-varying cross-sectional dispersion of consumption can increase the volatility of the stochastic discount factor (e.g., Constantinides and Duffie, 1996; Storesletten, Telmer and Yaron, 2007; Herskovic et al., 2014; Ghosh and Constantinides, 2015), and the undiversified idiosyncratic investment risks increase the correlation between the individual consumption growth and the asset return (e.g., Heaton and Lucas, 1997, 2000*a*,*b*). In my model, both effects arise endogenously from a moral hazard problem. The resulting effects are further amplified by endogenous financial frictions. Most importantly, a key difference of my model is that the marginal investors of the aggregate equity have fully diversified portfolios. Here, the undiversified idiosyncratic shocks affect the economy initially through the real investment channel; then, the distorted real investment deteriorates agents' risk sharing on aggregate shocks due to the limited market participation. More broadly, my model is connected to the papers trying to rationalize the volatile stochastic discount factors through market incompleteness, such as Alvarez and Jermann (2000, 2001), Chien and Lustig (2010), Chien, Cole and Lustig (2012), and Dou and Verdelhan (2015).

The idea of undiversified idiosyncratic risks has also been adopted in dynamic structural corporate models (or partial equilibrium dynamic macroeconomic models) to study firm's investment and financing behavior (e.g., Miao and Wang, 2007; Chen, Miao and Wang, 2010; Panousi and Papanikolaou, 2012; Glover and Levine, 2015). My model incorporates these partial equilibrium mechanisms, together with asset pricing channels, into a general equilibrium model to study their aggregate implications.

The macroeconomic literature on financial stability builds financial frictions into otherwise standard neoclassical models. This literature started from Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke, Gertler and Gilchrist (1999). Recent advances explore the concentration of aggregate risk and its role in creating systemic risks and nonlinear risk premia dynamics through the balance sheet channel (e.g., Adrian and Boyarchenko, 2012; He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Di Tella, 2014; Haddad, 2014; Drechsler, Savov and Schnabl, 2014). One contribution of my paper to this literature is to quantitatively examine the asset pricing implications of financial frictions in the cross section of various types of assets.

My model fits within the literature studying asset prices in investment-based general equilibrium models. It is most closely related to the papers explicitly modeling assets in place and growth options and focusing on the cross section of asset prices. Gomes, Kogan and Zhang (2003) study a model in which book-to-market ratios are positively associated with average returns. But, growth options are riskier than assets in place in the model. Papanikolaou (2011) presents a model with aggregate investment shocks, which by nature affect assets in place and growth options differently. In this calibrated model, the aggregate investment shock benefits growth options relative to assets in place, and carries a negative market price of risk if the late resolution of uncertainty is preferred by investors. Pástor and Veronesi (2009) and Gârleanu, Panageas and Yu (2012) study the asset pricing dynamics in models with episodes of endogenous technology adoption. Ai and Kiku (2013) study a model in which the cost of option exercise is pro-cyclical and thus the assets in place are riskier. Ai, Croce and Li (2012) study a model in which the younger vintages of assets in place have lower exposure to aggregate productivity shocks and thus growth options are less risky. These papers all assume perfect risk sharing. However, Gârleanu, Kogan and Panageas (2012) and Kogan, Papanikolaou and Stoffman (2015) rationalize the negative price of risk for the aggregate investment shock by introducing displacement risks that arise from market incompleteness. Moreover, Opp (2014) explicitly incorporates the venture capital intermediation into an otherwise standard dynamic general equilibrium macroeconomic model of asset prices and focuses on the asset pricing phenomenon of venture capital cycles. Despite perfect risk sharing, the informational friction causes costly external financing for new ventures and hence distorts investment; the venture capital firms alleviate such information frictions in the economy. Also, the displacement risk of technological innovations plays an important role in determining the risk premia in the model. My model studies the unequal effects of uncertainty shocks on asset returns in the cross section and the time variation of these effects driven by endogenous imperfect risk sharing. In my model, growth uncertainty shocks endogenously cause displacement risks, especially when the risk sharing condition is poor.

# 2 Model

In this section, I develop a continuous-time general equilibrium model with two sectors: the consumption goods sector and the investment goods sector. I summarize the model's features as follows. First, there are two types of agents in the economy: experts and households. Experts with population $\varkappa$ indexed by $f \in \mathbb{F} \equiv [-\varkappa, 0]$ are the only ones who can manage and trade firm's assets; households with population 1 indexed by $h \in \mathbb{H} \equiv [0, 1]$ provide labor. Second, firms hold two classes of assets: assets in place generate consumption goods; growth options create new assets in place. Although assets are irreversible at the aggregate level, they can be continuously traded among firms. Third, outputs are affected by firm-specific idiosyncratic shocks, which are unobservable to agents except the expert who literally manages the assets. The information asymmetry makes it possible for the expert to take hidden actions such as shirking efforts or stealing for the private benefit at the expense of diffused shareholders. To deal with the agency problem, the expert is restricted to become a blockholder who owns a significant fraction of the firm's equity. Fourth, the volatilities of idiosyncratic shocks are time varying and driven by aggregate shocks. These are the uncertainty shocks. Experts respond optimally to the uncertainty shocks in making decisions on investment and hiring for the firm. Fifth, all agents can trade financial contracts in capital markets where a full set of Arrow-Debreu securities are available. Sixth, I deliberately cast the model in continuous time, because the continuous-time formation allows me to characterize the key equilibrium relationships by cleaner expressions and conveniently summarize the equilibrium conditions by a set of coupled ordinary differential equations.

## 2.1 Firms and Technologies

There is a continuum of infinitely-lived firms in the consumption goods sector. Each firm is managed by an expert and indexed by $f \in \mathbb{F}$. Existing assets in place depreciate with a constant rate $\delta$, and new assets in place are built based on a combination of existing growth options and investment goods newly produced in the investment goods sector. Growth options can be used to create new assets in place when investment opportunities arrive.

**Consumption goods firms.** Each firm's assets consist of assets in place and growth options. The equity of the firm $f$ is freely traded, and it is the claim on the dividends generated by the assets in place and the value added by the creation of new assets in place from growth options.

*Assets in place.* Denote by $k_t$ the aggregate amount of assets in place in the economy and by $k_{f,t}$ the amount of assets in place held by the individual firm $f$, where $t \in [0, \infty)$ is the time index. Assets in place $k_{f,t}$ held by the firm $f$ generates output at rate $y_{f,t}$, over the period $[t, t + \mathrm{d}t]$,

$$y_{f,t} = k_{f,t}^{\varphi} \ell_{f,c,t}^{1-\varphi}, \tag{1}$$

where $\varphi \in (0,1)$ captures the capital share in production and $\ell_{f,c,t}$ represents the labor input for production. When held by the expert $f$, the old existing assets in place evolves according to

$$\frac{\mathrm{d}k_{f,t}}{k_{f,t}} = -\delta \mathrm{d}t + \sigma \mathrm{d}Z_t + \mathrm{d}A_{f,t},$$

where $\delta$ is the constant depreciation rate, $Z_t$ is a Brownian motion describing an aggregate shock in the economy, and $A_{f,t}$ is a cumulative firm-specific process describing the idiosyncratic cash flows. The shocks $\mathrm{d}Z_t$ and $\mathrm{d}A_{f,t}$ can be interpreted as the aggregate and the idiosyncratic (short-term) cash flow shocks, respectively.[12] The cash-flow uncertainty is defined as the volatility of the idiosyncratic shock $\mathrm{d}A_{f,t}$:

$$\nu_{c,t} \equiv \mathrm{vol}\left(\mathrm{d}A_{f,t}\right).$$

The exposure to the aggregate shock is constant $\sigma$; however, the exposure to the idiosyncratic shock, denoted by $\nu_{c,t}$, is stochastic. The idiosyncratic volatility $\nu_{c,t}$ represents an aggregate economic condition because the prospects of short-term cash flows become blurred when $\nu_{c,t}$ increases.

*Growth options, investment opportunities, and new assets in place.* Growth options allow the firm to create new assets in place when investment opportunities arrive. Specifically, the growth options are intangible assets associated with ideas of technological innovations such as R&D projects, blueprints, and patents; however, these innovative ideas are necessary but not alone sufficient to realize the final commercial benefits. The investment opportunities are business opportunities or ideas to commercialize the technological innovations and turn them into commercial benefits through making real investment. The arrivals of investment opportunities are firm-specific, so the model has the feature that investment is lumpy at the firm level but smooth at the aggregate level, which is consistent with the data. This modeling feature is crucial since it allows me to study the time-series properties of cross-sectional investment dispersions.[13]

I denote it by $s_{f,t}$ the amount of growth options held by the firm $f$ and denote it by $p_t$ the unit price of growth options. Although the aggregate amount of growth options is assume to be constant $s_t = \overline{s}$,[14] the firms can freely trade growth options with each other at the price $p_t$. The existing stock of growth options stay constant over period $[t, t + \mathrm{d}t]$; that is, $\mathrm{d}s_{f,t} = 0$.

Let $M_{f,t}$ be the firm-specific point process that describes the number of investment opportunities obtained by firm $f$ up to time $t$. Upon the reception of a new investment opportunity at time $t$ (i.e.,

[12] This way of modeling capital accumulation and production is actually equivalent to the conventional TFP shock method, where the adjustment cost function is not only homogeneous with respect to capital stock $k_t$ but also the TFP shock $a_t$, i.e., the adjustment cost, is $\iota(g_t)a_t k_t$, if there is an adjustment cost function.

[13] This follows the standard modeling in the literature on the implications of lumpy investment for aggregate macroeconomic dynamics, such as Khan and Thomas (2008).

[14] The exogenous stationarity in the relative-growth option scale is standard in the asset pricing literature and growth literature (e.g., Gomes, Kogan and Zhang, 2003; Ai and Kiku, 2013). In order to avoid tracking an extra endogenous state variable, models either assume constant growth options or assume that the growth options grow proportionally to the total assets in place. In my model, the value of growth options is linear in $k_t$. Then, the relative growth option scale is $p_t s_t/(q_t k_t) = \overline{s}\hat{p}_t/\hat{q}_t$, where $p_t$ and $q_t$ are prices of growth options and assets in place, respectively. In equilibrium, $\overline{s}\hat{p}_t/\hat{q}_t$ is a stationary process.

$\mathrm{d}M_{f,t} = 1$), the firm $f$ decides whether to invest or not. This is similar to Khan and Thomas (2008), which explicitly accounts for the micro-level investment spikes and the fluctuation of the extensive margin of investments. The firm can undertake an investment only upon payment of its fixed adjustment cost $\varpi$, specifically by forfeiting $\varpi p_t s_{f,t}$ of current consumption goods. The fixed adjustment cost denominated in the units of growth options captures the essence of the real-option model of investment in Jovanovic (2009) and Ai and Kiku (2013), among others.[15] Denote by $u_{f,t}$ the variable characterizing whether the firm $f$ undertakes an investment or not. If it is undertaken, $u_{f,t} = 1$; otherwise, $u_{f,t} = 0$. Upon $u_{f,t} = 1$, the firm creates new assets in place $k_{f,t}^{new}$ using the technology:

$$k_{f,t}^{new} = \underbrace{\varepsilon_{f,t}}_{\text{idio. IST shock}} \times \underbrace{m\left(s_{f,t}, g_{f,t}\right) k_t}_{\text{inputs and scaling}}, \tag{2}$$

where $\varepsilon_{f,t}$ is the idiosyncratic investment-specific (IST) shock to capture the idiosyncratic shock on the quality of investment opportunities, $s_{f,t}$ is the amount of existing growth options, and $g_{f,t}$ is the input of investment goods. To create new assets in place with the amount of $\varepsilon_{f,t}m(s_{f,t}, g_{f,t})k_t$, the capital stock of growth options $s_{f,t}$ is prefixed (i.e., not adjustable at time $t$ after the realization of $\varepsilon_{f,t}$); however, the firm can choose the investment goods input $g_{f,t}$ optimally conditional on the realization of $\varepsilon_{f,t}$. The cost of purchasing investment goods is $\tau_t g_{f,t}$, where $\tau_t$ is the equilibrium market price of investment goods.[16] The production function $m(s, g)$ is a constant-elasticity-of-substitute (CES) function. In particular, I assume that $m(s, g)$ has the Cobb-Douglas functional form with the share of capital to be $\alpha$; that is, $m(s, g) \equiv s^{1-\alpha} g^{\alpha}$.[17]

Once the firm $f$ receives an investment opportunity at time $t$ (i.e. $\mathrm{d}M_{f,t} = 1$) and implements it (i.e. $u_{f,t} = 1$), the new assets in place $k_{f,t}^{new}$ are created from the growth options with the rate:

$$i_{f,t} \equiv k_{f,t}^{new}/k_t = \varepsilon_{f,t} m\left(s_{f,t}, g_{f,t}\right).$$

*Growth uncertainty.* The idiosyncratic IST shock $\varepsilon_{f,t}$ in (2) is assumed to be independently distributed over time and across firms, to avoid having to keep track of the distribution of $\varepsilon_{f,t}$ as an infinitely-dimensional state variable. The assumption of idiosyncratic investment risks have been adopted by the

[15] In Jovanovic (2009) and Ai and Kiku (2013), the growth options fully depreciate after being used for investment. In macroeconomic models studying the role of micro-level nonconvex costs of investment adjustment in generating nonlinear aggregate investment dynamics, the fixed adjustment costs are usually denominated by profits (e.g., Bloom, 2009) or denominated by labor (e.g., Khan and Thomas, 2008).

[16] It should be noted that, similar to Gomes, Kogan and Zhang (2003), although the tangible assets is complementary to the intangible assets investment at the aggregate level, each individual expert cannot really internalize the aggregate impact of their tangible asset holdings, and hence in the decentralized economy the assets in place investment has zero complementarity for the R&D investments. Therefore, there is an externality in the economy, which makes the allocations in a competitive equilibrium not necessarily identical to those solved by the social planner's problem.

[17] Similar to Gomes, Kogan and Zhang (2003), I assume that the scale of new assets in place created from growth options is linear in the aggregate assets in place. This guarantees that the ratios of the aggregate new to the aggregate existing assets in place and of the aggregate value of growth options to the aggregate value of assets in place are both stationary over time. Other examples include Ai, Croce and Li (2012) where the aggregate investment is assumed to be a deterministic function of the aggregate investment goods by restricting the cross-sectional distribution of idiosyncratic investment shocks, and Ai and Kiku (2013) where the aggregate assets in place and the aggregate growth options are assumed to follow an exogenous common stochastic trend which is the arrival intensity of new growth options.

macroeconomics literature (e.g., Khan and Thomas, 2008; Bachmann and Bayer, 2014), and by the asset pricing literature (e.g., Gomes, Kogan and Zhang, 2003; Ai, Croce and Li, 2012). However, a key difference in this model is that the variance of the distribution of idiosyncratic growth opportunity quality shocks is time varying. More precisely, I assume that $\varepsilon_{f,t}$ has a symmetric distribution $\varepsilon_{f,t} \sim \mathbf{N}\left(0, \nu_{g,t}^2\right)$. The growth uncertainty is the standard deviation of the IST shock

$$\nu_{g,t} \equiv \mathrm{std}\left(\varepsilon_{f,t}\right)$$

where the growth uncertainty $\nu_{g,t}$ evolves randomly over time. The idiosyncratic volatility $\nu_{g,t}$ represents an aggregate economic condition because the prospects of investment opportunities become blurred when $\nu_{g,t}$ increases.

*Optimal investment.* Here, I describe the investment decision of an expert when an investment opportunity arrives. Because experts can choose the variable utilization rate of growth options $u_{f,t}$, the optimal investment decision-making can be decomposed into two steps. First, conditioning on the full utilization (i.e., $u_{f,t} = 1$), the expert maximizes the net present value $\Pi_{f,t}$ by choosing investment goods input $g_{f,t}$. Given the price of assets in place, denoted by $q_t$, and the price of investment goods $\tau_t$, the optimization problem and the net present value $\Pi_{f,t}$ can be expressed as

$$\max_{g_{f,t}} \Pi_{f,t} \equiv q_t k_{f,t}^{new} - \tau_t g_{f,t}, \quad \text{with} \;\; k_{f,t}^{new} \equiv i_{f,t} k_t \;\; \text{and} \;\; i_{f,t} \equiv \varepsilon_{f,t} s_{f,t}^{1-\alpha} g_{f,t}^{\alpha}. \tag{3}$$

In other words, the net present value $\Pi_{f,t}$ is the market value of the new assets in place $k_t^{new}$ minus its investment cost $\tau_t g_{f,t}$. The optimal input of investment goods is strictly convex in the idiosyncratic investment shock $\varepsilon_{f,t}$ and is linear in the stock of existing growth options $s_{f,t}$:

$$g_{f,t} = o_g s_{f,t} \varepsilon_{f,t}^{\frac{1}{1-\alpha}} \left(\frac{q_t k_t}{\tau_t}\right)^{\frac{1}{1-\alpha}}, \quad \text{with constant } o_g \equiv \alpha^{\frac{1}{1-\alpha}}. \tag{4}$$

This is the result of a simple intratemporal optimization based on (3). The optimal investment condition (4) is similar to the standard q-theory of investment developed by Hayashi (1982) where the optimal investment is directly linked to the marginal q of assets in place ($\frac{q_t k_t}{\tau_t}$), denominated by the investment goods. Yet there is one key difference. Because $0 < \alpha < 1$, the optimal investment goods demand $g_{f,t}$ is a convex function of marginal q instead of a concave function, which is a direct result of the Oi-Hartman-Abel-Caballero channel. Given the price of growth options, denoted by $p_t$, the optimal present value of newly created assets in place can be expressed as $\Pi_{f,t} \equiv \overline{\pi}_{f,t} s_{f,t} p_t$ where the optimal net present value rate $\overline{\pi}_{f,t}$ has the analytical expression:

$$\overline{\pi}_{f,t} = o_\pi \varepsilon_{f,t}^{\frac{1}{1-\alpha}} \left(\frac{q_t k_t}{p_t^{1-\alpha} \tau_t^{\alpha}}\right)^{\frac{1}{1-\alpha}}, \text{ where } o_\pi \equiv (1-\alpha) o_g \text{ is a constant.} \tag{5}$$

And the optimal investment rate is $i_{f,t} = o_\iota \varepsilon_{f,t}^{\frac{1}{1-\alpha}} \left(\frac{q_t k_t}{\tau_t}\right)^{\frac{\alpha}{1-\alpha}} s_{f,t}$, where $o_\iota = o_g^\alpha$ is a constant. In the second step, the expert chooses the utilization rate $u_{f,t} \in \{0,1\}$ to maximize the profits from creating new assets in place. It is clear that a firm will absorb its fixed cost $\varpi p_t s_{f,t}$ to undertake the investment opportunity if the investment profit rate $\overline{\pi}_{f,t}$ is at least $\varpi$. It follows immediately that a firm will undertake the investment opportunity if its idiosyncratic IST shock $\varepsilon_{f,t}$ lies at or above some threshold values. Because all agents face the same option-exercising problem, the threshold value only depends on the aggregate state variables. I denote the exercising boundary by $\xi_t$, which is characterized as follows:

$$\varepsilon_{f,t} \geq \xi_t \quad \text{if and only if} \quad \overline{\pi}_{f,t} = o_\pi \varepsilon_{f,t}^{\frac{1}{1-\alpha}} \left(\frac{q_t k_t}{p_t^{1-\alpha} \tau_t^\alpha}\right)^{\frac{1}{1-\alpha}} \geq \varpi.$$

From (5), it leads the analytical expression for the exercising threshold $\xi_t$:

$$\xi_t = o_\xi \varpi^{1-\alpha} \left(\frac{q_t k_t}{p_t^{1-\alpha} \tau_t^\alpha}\right)^{-1}, \quad \text{where} \quad o_\xi \equiv o_\pi^{\alpha-1} \quad \text{is a constant.} \tag{6}$$

Thus, the profit rate of growth options for the firm $f$ is

$$\pi_{f,t} = (\overline{\pi}_{f,t} - \varpi)\mathbf{1}_{\{\varepsilon_{f,t} \geq \xi_t\}}.$$

**Investment goods firms.** There is a representative firm in the investment goods sector. It uses the labor of households to produce the investment goods needed to create new assets in place in the consumption goods sector. More precisely, the production function for the investment goods output rate over the infinitesimal interval $[t, t + \mathrm{d}t)$ is

$$g_t = z_\iota \ell_{\iota,t}, \tag{7}$$

where $z_\iota$ is the average total productivity factor in the investment goods sector and $\ell_{\iota,t}$ is the total labor demand to produce investment goods $g_t$. I assume constant return to scale for labor input for simplification.[18]

**Spot markets.** The outputs (consumption goods and investment goods) and the firm's assets (assets in place and growth options) are traded in perfectly competitive spot markets. There is one spot price in each market, and this spot price is only determined by the aggregate state of the economy, even though the participants are heterogeneous. The spot prices are market-clearing prices for which each single participant is a price taker.

[18]Similar to Papanikolaou (2011) and Kogan, Papanikolaou and Stoffman (2015), the production function of the investment goods only works with fixed amount of capital input. But, to guarantee profits on the capital input and thereby generate meaningful share prices of investment goods firms, Papanikolaou (2011) assumes decreasing returns to scale for the labor input. Like Kogan, Papanikolaou and Stoffman (2015), my focus is not to link investment-minus-consumption (IMC) portfolio returns to aggregate shocks in the economy. So, I also assume constant return to scale for the labor input.

## 2.2 Uncertainty Shocks

The cash-flow uncertainty $\nu_{c,t}$ and the growth uncertainty $\nu_{g,t}$ move stochastically. The uncertainty shocks are large shocks driving the state variable $\nu_t$, which has a one-to-one correspondence to the 2-tuple $(\nu_{g,t}, \nu_{c,t})$. I assume that the growth uncertainty $\nu_{g,t}$ follows a 2-state homogeneous continuous-time Markov chain taking values in the set $\mathcal{V}_g \equiv \left\{\nu_g^L, \nu_g^H\right\}$, where $\nu_g^L < \nu_g^H$. Similarly, I assume that the cash-flow uncertainty $\nu_{c,t}$ follows a 2-state homogeneous continuous-time Markov chain taking values in the set $\mathcal{V}_c \equiv \left\{\nu_c^L, \nu_c^H\right\}$ where $\nu_c^L < \nu_c^H$. For simplicity, the growth uncertainty process and the cash-flow uncertainty process are assumed to move independently with the transition rate matrices $\mathcal{Q}_g$ and $\mathcal{Q}_c$, respectively,

$$\mathcal{Q}_g \equiv \begin{bmatrix} \lambda^{(\nu_g^L,\nu_g^H)} & -\lambda^{(\nu_g^L,\nu_g^H)} \\ -\lambda^{(\nu_g^H,\nu_g^L)} & \lambda^{(\nu_g^H,\nu_g^L)} \end{bmatrix} \quad \text{and} \quad \mathcal{Q}_c \equiv \begin{bmatrix} \lambda^{(\nu_c^L,\nu_c^H)} & -\lambda^{(\nu_c^L,\nu_c^H)} \\ -\lambda^{(\nu_c^H,\nu_c^L)} & \lambda^{(\nu_c^H,\nu_c^L)} \end{bmatrix}.$$

The transition intensity for $\nu_t$ is denoted as $\lambda^{(\nu_t,\nu')}$ which only depends on $\mathcal{Q}_g$ and $\mathcal{Q}_c$.

## 2.3 Preferences

Both experts and households have stochastic differential utility of Duffie and Epstein (1992*a*,*b*). This preference is a continuous-time version of the recursive preferences proposed by Kreps and Porteus (1978), Epstein and Zin (1989), and Weil (1990). The Epstein-Zin-Weil recursive preference has become a standard preference in asset pricing and macro literature to capture the reasonable joint behavior of asset prices and macroeconomic quantities. More precisely, the utility is defined recursively as follows:

$$U_0 = \mathbb{E}_0\left[\int_0^\infty \mathbf{f}(c_t, U_t)\mathrm{d}t\right],$$

where

$$\mathbf{f}(c_t, U_t) \equiv \rho\left[\frac{u(c_t)}{((1-\gamma)U_t)^{\theta^{-1}-1}} - \theta U_t\right], \text{ with } \theta \equiv \frac{1-\gamma}{1-\psi^{-1}}$$

and the felicity function $\mathbf{f}(c_t, U_t)$ is an aggregator over current consumption rate $c_t$ and future utility level $U_t$. The coefficient $\rho$ is the rate parameter of time preference, $\gamma$ is the risk-aversion parameter for one-period consumption, and $\psi$ is the parameter of elasticity of intertemporal substitution (EIS) for deterministic consumption paths. The period utility function has the form:

$$u(c_t) = \frac{c_t^{1-\psi^{-1}}}{1-\psi^{-1}}.$$

The preference between consumption and leisure can be viewed as a special case of the KPR preference (King, Plosser and Rebelo, 1988, 2002) and the GHH preference (Greenwood, Hercowitz and Huffman, 1988), where leisure is not appreciated or work is not undervalued. Thus, the labor supply is inelastic.[19]

[19]The inelastic labor supply is adopted for several reasons: (1) this is a useful benchmark that allows a direct comparison to the existing literature on production-based asset pricing and investment in incomplete markets where inelastic labor supply is

To ensure stationarity between experts and households, I assume that agents die independently of each other according to a Poisson process with constant intensity $\mu$. New agents are born at the same rate $\mu$ with a fraction $\frac{\varkappa}{1+\varkappa}$ as experts and $\frac{1}{1+\varkappa}$ as households, so the measure of households and the measure of experts both remain constant. The wealth of agents who die is bestowed on the newly born on a per-capita basis. The subjective discount factor $\rho$ captures the effective time preference because I make it include the adjustment for the likelihood of death for each agent (see Gârleanu and Panageas, 2015).

### 2.4 Labor Markets

The aggregate labor supply is one since each household inelastically supplies their labor-hours endowment. On the demand side, the labor choices are endogenous in both the consumption goods sector and the investment goods sector. Driven by the aggregate shocks in the economy, the share of aggregate labor supply allocated between the two sectors is time-varying. The labor demand in the investment goods sector is straightforward. It is determined by the aggregate investment goods demand. According to (7), it holds that the aggregate labor demand in the investment goods sector is

$$\ell_{\iota,t} = z_{\iota}^{-1} g_t.$$

The optimal labor demand of each firm is a static (i.e., state-by-state) optimization problem. This is because the firm's employment $\ell_{f,c,t}$ affects only the profits rate $y_{f,t} - w_t \ell_{f,c,t}$ at time $t$. As a result, the optimal labor demand maximizes profits state-by-state at time $t$. As a result, given wages, the optimal labor demand can be solved only based on the intratemporal Euler equation, which is independent of the intertemporal optimizations. They are summarized by Proposition 1. All the detailed proofs of propositions and corollaries can be found in the online appendix.

**Proposition 1** (Optimal Labor Demand and Output)**.** *Given $w_t$ and $k_{f,t}$, labor demand and output are linear in $k_{f,t}$ and decreasing in $w_t$: $\ell_{c,f,t} = \ell(w_t) k_{f,t}$ and $y_{f,t} = y(w_t) k_{f,t}$, where $\ell(w_t) \equiv \left[\frac{(1-\varphi)}{w_t}\right]^{1/\varphi}$ and $y(w_t) \equiv \left[\frac{(1-\varphi)}{w_t}\right]^{\frac{1-\varphi}{\varphi}}$.*

From Proposition 1, the aggregate labor demand and the aggregate output, by the Law of Large Numbers, are

$$\ell_{c,t} \equiv \int_{f\in\mathbb{F}} \ell_{c,f,t} \mathrm{d}f = \ell(w_t) k_t \quad \text{and} \quad y_t \equiv \int_{f\in\mathbb{F}} y_{f,t} \mathrm{d}f = y(w_t) k_t, \quad \text{respectively.}$$

the most common assumption (e.g., Danthine and Donaldson, 2002; Angeletos, 2007; Guvenen, 2009; Kogan, Papanikolaou and Stoffman, 2015); (2) this allows us to focus on illustrating our key mechanism that results from the financial friction; (3) in the literature, it is shown that this assumption together with limited risk sharing can provide reasonable asset pricing implications; and (4) this is actually a not-far-off approximation to the reality. Wages have risen in the U.S. over long periods of time, but the proportion of time spent working has not changed very much. This old stylized fact has recently been reconfirmed by Ramey and Francis (2009). I can investigate the extent to which labor supply choice can be endogenized without compromising the performance on the asset pricing side, and study how frictions in labor markets can help improve the quantitative performance of our model. These are definitely important questions to understand but out of the scope of this paper. I leave them as the future research agenda.

The endogenous labor reallocation between the consumption goods sector ($\ell_{c,t}$) and the investment goods sector ($\ell_{\iota,t}$) play a crucial role in understanding the cross-section of stock returns.

### 2.5 Firms' Payouts and Assets' Holding Returns

Because firms face no financing frictions, the irrelevance theorems (see Modigliani and Miller, 1958; Miller and Modigliani, 1961) hold for firms' capital structures and payout policies. Similar to most macroeconomic and asset pricing models, I assume that the firms are all-equity firms and pay out earnings. So, for each firm $f$, its payout is equal to the profits from assets in place $\varphi y(w_t)k_{f,t}\mathrm{d}t$ plus value added by growth options $\pi_{f,t}s_{f,t}p_t\mathrm{d}M_{f,t}$ minus expenditures for assets in place $q_t i_t k_{f,t}\mathrm{d}t$:

$$\mathrm{d}D_{f,t} = \underbrace{\varphi y(w_t)k_{f,t}\mathrm{d}t + \pi_{f,t}s_{f,t}p_t\mathrm{d}M_{f,t}}_{\text{total profits}} \quad - \quad \underbrace{q_t i_t k_{f,t}\mathrm{d}t}_{\text{expenditure for new assets in place}}.$$

Here, $D_{f,t}$ is the cumulative payout of firm $f$ and the incremental payout $\mathrm{d}D_{f,t}$ can theoretically be negative in the model. Because I do not particularly specify the external financing frictions of the firm, the negative payout can be interpreted as issuing new equity by the firm. In fact, numerically, under my baseline calibration, the payout turns out to be negative only in the extreme ranges of the state space which are visited by the economy very rarely in simulations.

The total payout of a firm can be decomposed into two components. One is due to the capital stock of assets in place and the other is due to the capital stock of growth options. They are relevant for the valuation of assets in place and growth options, respectively. More precisely, the decomposition based on the accounting for assets in place and growth options is as follows:

$$\mathrm{d}D_{f,t} = \underbrace{\left[\varphi y(w_t)k_{f,t} - q_t i_t k_{f,t}\right]\mathrm{d}t}_{\text{payout due to assets in place}} + \underbrace{\pi_{f,t}p_t s_{f,t}\mathrm{d}M_{f,t}}_{\text{payout due to growth options}}.$$

Moreover, the instantaneous holding returns of assets in place and growth options for experts are, respectively,

$$\mathrm{d}R^{\mathbf{k}}_{f,t} = \mu^{\mathbf{k}}_{f,t}\mathrm{d}t + (\sigma^q_t + \varphi\sigma)\mathrm{d}Z_t + \sum_{\nu\neq\nu_t} \varsigma^{q,(\nu_t,\nu)}_t \mathrm{d}N^{(\nu_t,\nu)}_t + \mathrm{d}A_{f,t}, \tag{8}$$

and

$$\mathrm{d}R^{\mathbf{s}}_{f,t} = \mu^{\mathbf{s}}_{f,t}\mathrm{d}t + (\sigma^p_t + \varphi\sigma)\mathrm{d}Z_t + \sum_{\nu\neq\nu_t} \varsigma^{p,(\nu_t,\nu)}_t \mathrm{d}N^{(\nu_t,\nu)}_t + \pi_{f,t}\mathrm{d}M_{f,t}, \tag{9}$$

where the drift terms $\mu^{\mathbf{k}}_{f,t}$ and $\mu^{\mathbf{s}}_{f,t}$ can be found in the online appendix and the diffusion terms $\sigma^p_t$ and $\sigma^q_t$ and the jump size terms $\varsigma^{q,(\nu_t,\nu)}_t$ and $\varsigma^{p,(\nu_t,\nu)}_t$ are defined in the beginning of Section 3.

## 2.6 Financial Markets

There is a full set of short-term financial contracts available to all agents. Each financial contract has zero net supply. And their prices are always normalized at one. All agents trade those short-term contracts in a perfectly competitive capital market. The contracts are traded continuously at time $t$ with the payoffs realized at the end of the infinitesimal interval $[t, t + \mathrm{d}t]$. Among the financial contracts, one is the short-term risk-free bond with payoff $1 + r_t \mathrm{d}t$, one is traded on the aggregate $Z_t$ shock with a contingent payoff $1 + r_t^Z \mathrm{d}t + \mathrm{d}Z_t$, one is traded on the growth uncertainty shock with a contingent payoff $1 + r_t^{\nu_g} \mathrm{d}t + \left[\mathrm{d}N_t^{(\nu_{g,t},\nu'_g)} - \lambda^{(\nu_{g,t},\nu'_g)} \mathrm{d}t\right]$, one is traded on the cash-flow uncertainty shock with a contingent payoff $1 + r_t^{\nu_c} \mathrm{d}t + \left[\mathrm{d}N_t^{(\nu_{c,t},\nu'_c)} - \lambda^{(\nu_{c,t},\nu'_c)} \mathrm{d}t\right]$, a continuum of short-term contracts are traded on idiosyncratic cash flow shocks $W_{f,t}$ with payoffs $1 + r_{f,t}^W \mathrm{d}t + \mathrm{d}W_{f,t}$ for all $f \in \mathbb{F}$, and a continuum of short-term contracts are traded on idiosyncratic investment shocks $\varepsilon_{f,t} \mathrm{d}N_{f,t}$ with payoffs $1 + r_{f,t}^N \mathrm{d}t + [\varepsilon_{f,t} \mathrm{d}N_{f,t} - \mathbb{E}(\varepsilon_{f,t}) \lambda \mathrm{d}t]$ for all $f \in \mathbb{F}$. In sum, the financial market is complete.

The expected payoffs $r_t, r_t^Z, r_t^{\nu_g}, r_t^{\nu_c}, r_{f,t}^W$, and $r_{f,t}^N$ are endogenously determined by the market clearing conditions. Importantly, later I shall show that the expected rate of returns are time varying, driven by the cash-flow uncertainty shocks and the growth uncertainty. Moreover, each firm's equity can be freely traded. However, because a full set of contingent claims are already available to all agents, the equities of firms become redundant in terms of spanning the contingent space. Without loss of generality, I assume that a firm's equity on its assets in place and equity on its growth options can be traded separately.

Although a full set of contingent claims are available, the market can be endogenously incomplete due to lack of commitments. Later I show that due to zero commitment in long-term contracts and a moral hazard problem, experts face portfolio constraints including limited access to short-term financial contracts on particular idiosyncratic risks.

## 2.7 Moral Hazard

I now introduce an agency conflict induced by the separation of ownership and control. The diffused investors fund the firm controlled by the expert. In contrast to the neoclassical model in which the firm-specific cash flow process $A_{f,t}$ and the investment opportunity process $M_{f,t}$ are exogenously specified, those processes in my model are affected by expert's unobservable actions. Specifically, the expert is able to secretly divert cash flows and investment opportunities from the firm under her control, which I describe explicitly as follows.

**Hidden actions in cash flows.** The expert $f$'s hidden action $a_{f,t}^A \in \left[0, \overline{a}^A\right]$ determines the expected rate of idiosyncratic cash flow shock $\mathrm{d}A_{f,t}$, so that

$$\mathrm{d}A_{f,t} = -a_{f,t}^A \mathrm{d}t + \nu_{c,t} \mathrm{d}W_{f,t},$$

where $W_{f,t}$ is a Brownian motion capturing the firm $f$'s underlying (short-term) idiosyncratic cash flows. The expert controls the drift, but not the idiosyncratic volatility of the process $A_{f,t}$.[20] When the expert takes the action $a^A_{f,t}$, she enjoys a flow of pecuniary private benefits with intensity $a^A_{f,t}\phi q_t k_{f,t}$ over $[t, t + \mathrm{d}t]$. Here, $0 \le \phi < 1$, which means that the stealing is inefficient. More precisely, the variable $a^A_{f,t}$ can be interpreted as the fraction of cash flows that the expert diverts for her pecuniary private benefits and the parameter $\phi$ captures the expert's net pecuniary benefits per dollar diverted. Given the linearity, this framework of stealing is effectively equivalent to the binary setup in which the expert can steal (i.e., $a^A_{f,t} = \overline{a}^A$) or not steal (i.e., $a^A_{f,t} = 0$).

**Hidden actions in growth options.** Similarly, I assume that the investment opportunity $M_{f,t}$ is affected by the expert's unobserved action in the following way,

$$\mathrm{d}M_{f,t} = (1 - a^M_{f,t})\mathrm{d}N_{f,t}, \tag{10}$$

where $N_{f,t}$ is a Poisson count process that describes the number of investment opportunities of firm $f$ that arrive up to time $t$. The intensity of the underlying Poisson process $N_{f,t}$ is $\lambda$. The action $a^M_{f,t}$ is binary.[21] In particular, the expert does not steal when $a^M_{f,t} = 0$ and steals when $a^M_{f,t} = 1$. When the expert takes the action $a^M_{f,t} = 0$, she obtains zero pecuniary private benefit. By contrast, when the action of $a^M_{f,t} = 1$ is taken by the expert, she steals the investment opportunity from the firm to launch new ventures in her own private account.[22] The lumpy pecuniary benefit is $\phi\pi_{f,t}p_t s_{f,t}$, where the coefficient $\phi$ equals to the expert's net pecuniary benefits per dollar diverted.

**Severity of agency problem.** Here, $1 - \phi$ can be interpreted as the deadweight loss rate of stealing incurred by the expert. Thus, $\phi$ represents the severity of the agency problem and, as I show later, captures the minimum levels of incentives required to prevent the expert from stealing.

**Formulating the optimal contracting problem.** The history paths in

$$\mathcal{H}_t \equiv \sigma\left(\{Z_{t'}, \nu_{g,t'}, \nu_{c,t'}, A_{f,t'}, M_{f,t'} : 0 \le t' \le t, f \in \mathbb{F}\}, \{\varepsilon_{f,t'} : 0 \le t' < t, f \in \mathbb{F}\}\right)$$

[20] A common setting is that there is a menu of projects whose risk characteristics are common knowledge and yet experts can choose which to be undertaken (e.g., Cadenillas, Cvitanic and Zapatero, 2007). My model can be extended to allow the expert to choose among multiple projects and the main mechanism is not altered. Moreover, the expert can also affect the volatility by secretly injecting funds from her own hidden saving accounts. This is not the focus on this paper. To rule out the possibility of altering the idiosyncratic volatility secretly through injecting cash flows from the hidden saving account, I assume that the expert cannot affect the idiosyncratic volatility of (short-term) cash flows and that her net worth is observable, which is without loss of generality due to the Revelation-Principle type of results (e.g., DeMarzo and Fishman, 2007). The similar assumptions are also adopted in DeMarzo et al. (2012), among others. In particular, DeMarzo and Sannikov (2006) restrict the stealing process to be Lipschitz continuous. And, it is well known that all sample paths of a standard Brownian motion have infinite total variation. Thus, idiosyncratic volatility cannot be secretly altered in their model.

[21] As in the free cash flow case, the binary-action setting is equivalent to the continuous-action setting when pecuniary private benefit is linear in actions. However, the binary-action setting has a more natural interpretation for the diversion of investment opportunities.

[22] The investment opportunity is non-replicable; otherwise, the value of growth options is infinity, which is pathological.

are observable and contractable. Denote $H_t$ to be a particular history path in $\mathcal{H}_t$. Similar to He and Krishnamurthy (2011), Brunnermeier and Sannikov (2014) and Di Tella (2014), I take the approach of short-term contracts: the relation only lasts from $t$ to $t + \mathrm{d}t$; at time $t + \mathrm{d}t$, the contract (relation) ends. In fact, the optimal contract can be implemented by a sequence of short-term contracts even when long-term contracts are available in my setting, if experts are assumed to have zero commitment to long-term contracts and to be able to modify the older contracts and offer new contracts at any time in a costless manner. The intuition is that the participation constraint for the diffused investors is always binding in each short period $[t, t + \mathrm{d}t]$, which simply is the capital market non-arbitrage condition, and the incentive compatibility of the contracts in each short period $[t, t + \mathrm{d}t]$ is not affected by the history; hence the current contract is always subject to being replaced by new contracts and hence recontracting continuously is optimal.[23]

Right after the realization of the history $H_t$, the expert and her diffused investors meet up and enter contracts for $[t, t + \mathrm{d}t]$. The expert $f$ offers contracts to her diffused investors (the principals in the contracting relation), which specifies the upfront lumpy payment $P_{f,t}$ collected from the diffused investors and the cash payment $P_{f,t} + \mathrm{d}F_{f,t}$ paid from the expert to her diffused investors over $[t, t + \mathrm{d}t]$. Here, $\mathrm{d}F_{f,t}$ is the net cash payment by the expert over $[t, t + \mathrm{d}t]$. The cumulative net payment process $F_{f,t}$ and the upfront payment $P_{f,t}$ are required to be adapted to the filtration $\mathcal{H}_t$. Thus, a short-term contract consists of a pair of functions $(P_{f,t}, \mathrm{d}F_{f,t})$ specifying the investors' upfront payment to the expert at $t$ and the net cash payment of the expert to the investors over $[t, t + \mathrm{d}t]$. Let $\mathcal{C}_{f,t} \equiv \mathcal{C}_f(\mathcal{H}^t) \equiv (P_{f,t}, \mathrm{d}F_{f,t})$ represent the contract offered by the expert. The participation constraint for the diffused investors is

$$0 = \mathbb{E}_t^a \left[ \mathrm{d}F_{f,t} + (P_{f,t} + \mathrm{d}F_{f,t}) \frac{\mathrm{d}\Lambda_t}{\Lambda_t} \right], \tag{11}$$

where $\Lambda_t$ is the stochastic discount factor of households and is determined in the Walrasian equilibrium with details illustrated in Section 4.3 and $\mathbb{E}^a$ is the expectation operator under the probability measure that is induced by the hidden action processes. The participation constraint for the expert is endogenously mingled with her occupational choice: she endogenously decides whether to become a household by selling off all productive assets (assets in place and growth options). That is, by choosing $k_{f,t} = s_{f,t} = 0$, the expert $f$ endogenously becomes a household. However, in the equilibrium, the expert never converts herself to a household; the expert is always offered a high enough risk premia for holding the productive assets. This is a result of the limited market participation assumption that households cannot choose to become experts due to the lack of the specialized knowledge or skills.[24]

[23] There three important points here. First, it is worth pointing out that this result is very different from the equivalence results of long- and short-term contracts such as Fudenberg, Holmstrom and Milgrom (1990). Those papers investigate sufficient conditions under which a sequence of short-term contracts can achieve the same efficiency level for long-term contracts where commitment is nonzero. Second, if I assume the expert is committed to long-term contracts, like in DeMarzo and Sannikov (2006), Biais et al. (2007), DeMarzo and Fishman (2007), and DeMarzo et al. (2012), the tractability will be worsened with the main mechanism remaining unchanged. Third, the short-term contracting problem I focus on in this paper is analogous to the contracting problem in a one-period principal-agent problem (e.g., Holmstrom and Tirole, 1997).

[24] This is different from the limited market participation of certain financial markets for risky financial securities (e.g., Mankiw and Zeldes, 1991; Allen and Gale, 1994; Basak and Cuoco, 1998; Vissing-Jorgensen, 2002; Guvenen, 2009) in two folds: first, households cannot invest or manage firms' assets and thus the economy stops functioning without experts; second, households can freely trade all financial securities in capital markets.

Given any sequence of contracts characterized by $\mathcal{C}_f \equiv \{\mathcal{C}_{f,t} : t \geq 0\}$, the expert will choose an optimal sequence of strategies $\mathcal{S}_f \equiv \{\mathcal{S}_{f,t} : t \geq 0\}$ that specifies the hidden actions, the consumption, and investment choices $\mathcal{S}_{f,t} \equiv \left(a^A_{f,t}, a^M_{f,t}, c^{\mathrm{e}}_{f,t}, k_{f,t}, s_{f,t}, g_{f,t}\right)$. More precisely, for a sequence of contracts $\mathcal{C}_f$, the expert $f$'s net worth follows the law of motion,

$$\mathrm{d}n^{\mathrm{e}}_{f,t} = -\ c^{\mathrm{e}}_{f,t}\mathrm{d}t\ +\ \overbrace{\underbrace{q_t k_{f,t}\mathrm{d}R^{\mathbf{k}}_{f,t} + p_t k_t s_{f,t}\mathrm{d}R^{\mathbf{s}}_{f,t}}_{\text{gains from assets holdings}}\ -\ \underbrace{\mathrm{d}F_{f,t}}_{\text{contract pay}}\ +\ \underbrace{a^A_{f,t}\phi q_t k_{f,t}\mathrm{d}t + a^M_{f,t}\phi \pi_{f,t} p_t s_{f,t}\mathrm{d}N_{f,t}}_{\text{privite benefits from shirking}}}^{\text{depending on hidden actions } a^A_{f,t} \text{ and } a^M_{f,t}}, \quad (12)$$

where the instantaneous returns from holding the assets can be found in Equation (8). Further, given prices and wages, the expert $f$ chooses the strategies $\mathcal{S}_f$ to solve

$$U\left(H_0, n^{\mathrm{e}}_{f,0}; \mathcal{C}_f\right) = \max_{\mathcal{S}_f} \mathbb{E}^a_0 \left[\int_0^\infty \mathbf{f}\left[c^{\mathrm{e}}_{f,t'}, U(H_{t'}, n^{\mathrm{e}}_{f,t'}; \mathcal{C}_f)\right] \mathrm{d}t'\right],$$

where the net worth process $\left\{n^{\mathrm{e}}_{f,t'} : t' \geq 0\right\}$ includes the potential private benefits from taking a sequence of actions $\left\{a^A_{f,t}, a^M_{f,t} : t' \geq 0\right\}$ and the gain from the holding of firms' assets by taking choosing $\left\{k_{f,t'}, s_{f,t'}, g_{f,t'} : t' \geq 0\right\}$.

The contract-strategy pair $(\mathcal{C}_f, \mathcal{S}_f)$ is *feasible* if it satisfies the solvency constraint $n^{\mathrm{e}}_{f,t} \geq 0$ for all history paths $H_t \in \mathcal{H}_t$. A feasible contract-strategy pair $(\mathcal{C}_f, \mathcal{S}_f)$ is *optimal* if there is no other pair that provides the same payoff to the diffused investors and a higher expected utility to the expert. And, a feasible pair $(\mathcal{C}_f, \mathcal{S}_f)$ is *incentive compatible* if the optimal strategy $\mathcal{S}_f$ implements the efficient actions $a^A_{f,t} = a^M_{f,t} = 0$ all the time given the contracts $\mathcal{C}_f$. To characterize an optimal contract-strategy pair, I start with a Revelation-Principle type result as in the context of mechanism design: given any contract-strategy pair $(\mathcal{C}_f, \mathcal{S}_f)$ for the expert, there exits an incentive-compatible contract-strategy pair $(\mathcal{C}^*_f, \mathcal{S}^*_f)$ with the same payoff to diffused investors and a weakly higher expected utility for the expert. It allows me to focus on the incentive-compatible contract-strategy pairs for finding optimal contracts. The intuition is straightforward (e.g., DeMarzo and Fishman, 2007) and the rigorous proof is in the online appendix. I denote $\mathbb{E}$ to be the expectation operator under the probability measure induced by the efficient actions.

More precisely, an incentive-compatible contract-strategy pair $(\mathcal{C}_f, \mathcal{S}_f)$ is optimal if it maximizes the value function of the expert $f$, given prices and wages,

$$U\left(H_t, n^{\mathrm{e}}_{f,t}\right) = \max_{\mathcal{C}_f} U\left(H_t, n^{\mathrm{e}}_{f,t}; \mathcal{C}_f\right) \quad (13)$$

subject to the participation constraint of diffused investors in (11), where $U\left(H_t, n^{\mathrm{e}}_{f,t}; \mathcal{C}_f\right)$ is the optimal utility achieved by the optimal strategy $\mathcal{S}_f$ given the contracts $\mathcal{C}_f$ with

$$U\left(H_t, n^{\mathrm{e}}_{f,t}; \mathcal{C}_f\right) = \mathbb{E}_t \left[\int_t^\infty \mathbf{f}\left[c^{\mathrm{e}}_{f,t'}, U^{\mathrm{e}}(H_{t'}, n^{\mathrm{e}}_{f,t'}; \mathcal{C}_f)\right] \mathrm{d}t'\right].$$

In summary, I incorporate the optimal contracting problem into a dynamic general equilibrium framework, and thus the optimal contracts are part of the fixed point solution for a (Walrasian) general equilibrium. More precisely, at the decentralized level, optimal contracts are derived as if agents take the aggregate price and wage dynamics as given; in turn, the aggregate level, the demand and supply formed from the aggregation of decentralized optimal contracts need to match so that the markets are cleared. To finally solve the optimal contracts and the general equilibrium, it is useful to first provide a characterization (i.e., a necessary condition for the optimal contracts) and an implementation mechanism for the optimal contracts. After incorporating the characterization and the implementation, the general equilibrium framework with optimal contracting becomes a rather standard model for asset pricing and risk sharing in incomplete markets.

### 2.8 Concentrated Risk: the Optimal Contracts and Implementations

**Characterization of optimal contracts.** Because the cumulative payment process $F_{f,t}$ is adapted to $\mathcal{H}_t$, the net cash payment specified by the contract can be formulated as follows[25]

$$\mathrm{d}F_{f,t} = \mu^F_{f,t}\mathrm{d}t + \underbrace{(1-\beta^A_{f,t})q_t k_{f,t}\mathrm{d}A_{f,t} + \left[\pi_{f,t} - \beta^M_{f,t}(\varepsilon_{f,t})\right] p_t s_{f,t}\mathrm{d}M_{f,t}}_{\text{contingent payments on idiosyncratic shocks}} + \underbrace{\beta^Z_{f,t}\sigma\mathrm{d}Z_t + \sum_{\nu\neq\nu_t}\beta^{V,(\nu_t,\nu)}_{f,t}\mathrm{d}N^{(\nu_t,\nu)}_t}_{\text{contingent payments on aggregate shocks}},$$

where the (functional) processes $\mu^F_{f,t}$, $\beta^A_{f,t}$, $\beta^M_{f,t}(\cdot)$, $\beta^Z_{f,t}$, and $\beta^{V,(\nu_t,\nu)}_{f,t}$ are adaptive to the filtration $\mathcal{H}_t$. Particularly, the function $\beta^M_{f,t}(\cdot)$ can be nonlinear. Plugging the expression of $\mathrm{d}F_{f,t}$ above into (12), the dynamics of the net worth of expert $f$ can be rewritten as follows

$$\mathrm{d}n^{\mathrm{e}}_{f,t} = \underbrace{(\phi-\beta^A_{f,t})q_t k_{f,t}a^A_{f,t}\mathrm{d}t + \left[\phi\pi_{f,t} - \beta^M_{f,t}(\varepsilon_{f,t})\right] p_t s_{f,t}a^M_{f,t}\mathrm{d}N_{f,t}}_{\text{terms altered by } a^A_{f,t} \text{ and } a^M_{f,t}} + \left[\text{terms independent of } a^A_{f,t} \text{ or } a^M_{f,t}\right].$$

Thus, for any incentive-compatible contracts (i.e., satisfying $a^A_{f,t} = a^M_{f,t} = 0$), it must satisfy the following two conditions:

$$\beta^A_{f,t} \geq \phi \quad \text{and} \quad \beta^M_{f,t}(\varepsilon) \geq \phi\pi_{f,t}(\varepsilon) \text{ for all } \varepsilon.$$

It is straightforward that the optimal contracts must satisfy that $\beta^A_{f,t} \equiv \phi$ and $\beta^M_{f,t} \equiv \phi\pi_{f,t}$ for all $f$ and $t$. This is because the expert is risk averse and hence wishes to dump all the idiosyncratic risks $\mathrm{d}W_{f,t}$ and $\mathrm{d}N_{f,t} - \lambda\mathrm{d}t$, while at the same time households can buy it for free due to their capacity to fully diversify any idiosyncratic risks.

[25] The net payment $\mathrm{d}F_{f,t}$ does not depend on the idiosyncratic shocks not associated with the firm $f$, because all agents are risk averse and avoid unnecessary idiosyncratic risk exposures. Another important feature is that jumps with random sizes affect the payoff process (e.g., Sung, 1997; Biais et al., 2010; Hoffmann and Pfeil, 2010). In general, it leads to nonlinear optimal contracts. However, the linearity of optimal contracts in this setting is due to two main reasons: first, it follows the timing convention of taking hidden actions after the realization of shocks (e.g., DeMarzo and Fishman, 2007; Edmans and Gabaix, 2011; Edmans et al., 2012); second, the private pecuniary benefit is contingent and proportional to the payoff.

**Implementation of optimal contracts.** I now characterize the optimal contracts in terms of an optimal mechanism. In particular, I consider the implementation of optimal contracts based on simple financial contracts, including firms' stock shares, options, risk free bond, and indices tracking aggregate states. Specifically, the expert $f$ achieves her optimal incentive-compatible contracting results in the following ways: (1) she buys and manages assets in place $k_{f,t}$ and growth options $s_{f,t}$; (2) she sells $1-\phi$ fraction of the firm's equity to her diffused shareholders; and (3) she trades indices in perfect financial markets. In summary, this implementation features blockholding and active trading on indices.[26] Rather than attempting to describe all possible implementations, I shall focus on this simple yet empirically relevant mechanism.

The following proposition describes the detailed specifications of the implementations and establishes their optimality.

**Proposition 2** (Blockholding and Indexation). *For each $f \in \mathbb{F}$, suppose the expert $f$ has initial net worth $n_{f,0}$. She gets infinite penalty unless the solvency condition $n_{j,t} \geq 0$ holds. The expert $f$ is required to hold $\phi$ share of the firm $f$'s equity. The expert $f$ is not allowed to diversify or hedge away the idiosyncratic risks of firm $f$ as a blockholder. She can trade a risk-free bond and financial indices tracking aggregate shocks. Under the capital market configuration, it is optimal for each expert $f$ to choose actions $a^A_{f,t} = a^M_{f,t} = 0$.*

### 2.9 Aggregation: Investments and Productions

In this section, I discuss the aggregation results on the production and investment side of the economy.[27] An important feature of our model is that, the evolution of the aggregate assets in place follows the standard process as in the neoclassical growth model, though heterogeneous firms make decentralized investment decisions in my economy.

More precisely, under incentive-compatible optimal contracts, the law of motion for the aggregate capital stock of assets in place $k_t = \int_{f\in\mathbb{F}} k_{f,t}\mathrm{d}f$ is not affected by any particular idiosyncratic shocks; it can be characterized as follows:

$$\mathrm{d}k_t = (i_t - \delta)k_t\mathrm{d}t + \sigma k_t \mathrm{d}Z_t.$$

Here $i_t \equiv \int_{f\in\mathbb{F}} i_{f,t}\mathbf{1}_{\{\varepsilon_{f,t}>\xi_t\}}\mathrm{d}N_{f,t}$ is the aggregate investment rate with the analytical formula:

$$i_t = \underbrace{\overbrace{\mathcal{G}_\alpha\left(\nu_{g,t};\xi_t\right)}^{\uparrow \text{ in } \nu_{g,t}}}_{\text{marginal efficiency of investment}} \times \underbrace{\overbrace{o_\iota\left(\frac{q_t k_t}{\tau_t}\right)^{\frac{\alpha}{1-\alpha}}}^{\uparrow \text{ in } q_t/\tau_t}}_{\text{conventional q theory}}. \tag{14}$$

[26]An alternative theory that generates the same results is that the experts bargain with diffused shareholders' for the rents, subject to some capital market constraints. Rents can be efficient. For example, Myers (2000) and Lambrecht and Myers (2007, 2008, 2012) show how rents can align managers' and shareholders' interests if the managers maximize the present value of rents subject to a capital market constraint. Also, Eisfeldt and Papanikolaou (2013) develop a model in which the outside option of the key talent determines the share of firm cash flows that accrue to shareholders. This outside option varies systematically and renders firms depending more on the key talents riskier from shareholders' perspective.

[27]The detailed derivations in this section can be found in the online appendix.

The term $\mathcal{G}_\alpha(\nu_{g,t};\xi_t)$ acts as the endogenous marginal efficiency of investment and the shocks that drive its fluctuations are endogenous aggregate investment shocks. As shown in Proposition 3, the endogenous investment shock $\mathcal{G}_\alpha(\nu_{g,t};\xi_t)$ is increasing in $\nu_{g,t}$ and decreasing in $\xi_t$. In fact, it has the following analytical expression

$$\mathcal{G}_\alpha(\nu_{g,t};\xi_t) \equiv \lambda \times \nu_{g,t}^{\frac{1}{1-\alpha}} \times \bar{\Gamma}_\alpha\left(\xi_t/\nu_{g,t}\right), \tag{15}$$

and the function $\bar{\Gamma}_\alpha(\cdot)$ is defined as

$$\bar{\Gamma}_\alpha(\xi_t/\nu_{g,t}) \equiv o_\alpha \times \bar{\Gamma}\left(\frac{1}{2}\left(\xi_t/\nu_{g,t}\right)^2, \frac{2-\alpha}{2-2\alpha}\right) \tag{16}$$

where $\bar{\Gamma}(\cdot,\cdot)$ is the standard upper incomplete gamma function and $o_\alpha$ is a universal constant.[28]

The function $\mathcal{G}_\alpha(\nu_{g,t};\xi_t)$ is the key to understand how growth uncertainty can increase aggregate investment. More precisely, I decompose the function into a multiplication of two terms that capture the complementary effect and the option effect of growth uncertainty on the aggregate investment

$$\underbrace{\mathcal{G}_\alpha(\nu_{g,t};\xi_t)}_{\text{marginal efficiency of investment}} = \underbrace{\nu_{g,t}^{\frac{\alpha}{1-\alpha}}}_{\text{complementary effect}} \times \underbrace{\left[\overbrace{\nu_{g,t}}^{\text{intensive margin}} \times \overbrace{\bar{\Gamma}_\alpha(\xi_t/\nu_{g,t}) \times \lambda}^{\text{extensive margin}}\right]}_{\text{option effect}}. \tag{17}$$

The first term $\nu_{g,t}^{\frac{\alpha}{1-\alpha}}$ captures the exogenous positive effect of growth uncertainty on aggregate investment. It is similar to the Oi-Hartman-Abel-Caballero effect: the flexible inputs, which can be adjusted after productivity shocks are realized and are complementary to the productivity of the capital, create optionality in the capital. In my case, when $\alpha = 0$, investment goods are not needed in creating new assets in place (i.e. zero complementarity). As a result, the Oi-Hartman-Abel-Caballero effect disappears. The second term $\nu_{g,t} \times \bar{\Gamma}_\alpha(\xi_t/\nu_{g,t}) \times \lambda$ captures the option effect of exercising investment opportunities. The variable $\nu_{g,t}$ captures the intensive margin effect caused by growth uncertainty shocks: the high-quality investment opportunities are likely to be more profitable when growth uncertainty increases. Moreover, the function $\bar{\Gamma}_\alpha(\xi_t/\nu_{g,t})$ captures the extensive margin effect caused by growth uncertainty shocks: more experts endogenously choose to make investment for fixed exercising boundary $\xi_t$. However, the exercising boundary is endogenously adjusted in the economy, which can partly offset the exogenous effect of increasing growth uncertainty; this is called wait-and-see effect (e.g., Miao and Wang, 2007; Bloom, 2009).

Another important feature of our model is that, the aggregate output is Cobb-Douglas with diminishing return to scale in the aggregate assets in place as in a standard neoclassical growth model, though each firm's optimal output is linear in terms of its own assets in place. More precisely, the aggregate output of the consumption goods sector is

$$y_t = k_t^{\varphi} \ell_{c,t}^{1-\varphi},$$

[28]The upper incomplete gamma function is defined as $\bar{\Gamma}(x_1, a_1) = \int_{x_1}^{\infty} x^{a_1-1}e^{-x}\mathrm{d}x$ and $o_\alpha \equiv 2^{(2\alpha-1)/(2-2\alpha)}\boldsymbol{\pi}^{-1/2}$ where $\boldsymbol{\pi}$ is the mathematical constant but not the profit rate of growth options $\pi$.

and under incentive-compatible optimal contracts, the aggregate output of the investment goods sector is

$$g_t = \mathcal{G}_\alpha(\nu_{g,t};\xi_t) \times o_g \left(\frac{q_t k_t}{\tau_t}\right)^{\frac{1}{1-\alpha}}.$$

Intuitively, the aggregate investment goods demand $g_t$ is affected by the growth uncertainty $\nu_{g,t}$ similarly to the aggregate investment rate $i_t$ through the function $\mathcal{G}_\alpha(\nu_{g,t};\xi_t)$.

The aggregate payout from assets in place and the aggregate profit from growth options are summarized as follows. Particularly, the analytical formula of aggregate profit from growth options provides intuitions that help understand how growth uncertainty shocks affect the value of growth options. More precisely, under incentive-compatible optimal contracts, the aggregate net payout due to assets in place is

$$d_t = \varphi y_t - q_t i_t k_t$$

and the aggregate profit of growth options is $\Pi_t \equiv \pi_t p_t \overline{s}$ where

$$\frac{\pi_t}{\lambda} = \underbrace{\varpi\left(\frac{\nu_{g,t}}{\xi_t}\right)^{\frac{1}{1-\alpha}}}_{\text{effective payoff}} \times \underbrace{\bar{\Gamma}_\alpha(\xi_t/\nu_{g,t})}_{\text{adj. prob. of exercising}} - \underbrace{\varpi}_{\text{strike price}} \times \underbrace{\bar{\Phi}(\xi_t/\nu_{g,t})}_{\text{prob. of exercising}}, \tag{18}$$

where the function $\bar{\Gamma}_\alpha(\cdot)$ is defined in (16), and the function $\bar{\Phi}(\cdot)$ is the complementary cumulative distribution function (CCDF) of a standard normal variable. The net profit rate of growth options derived in (18) resembles the well-known Black-Scholes-Merton option pricing formula (Black and Scholes, 1973; Merton, 1973*b*). In the following decomposition, the term $\varpi\left(\frac{\nu_{g,t}}{\xi_t}\right)^{\frac{1}{1-\alpha}}$ can be viewed as the effective payoff when the option is exercised, the term $\bar{\Gamma}_\alpha(\xi_t/\nu_{g,t})$ can be interpreted as the adjusted likelihood of exercising the option ($\varepsilon_{f,t} > \xi_t$), the term $\varpi$ is strike price, and the term $\bar{\Phi}(\xi_t/\nu_{g,t})$ is the actual probability of exercising the growth option ($\varepsilon_{f,t} > \xi_t$).

Thus, keeping the exercising boundary $\xi_t$ fixed, the profit rate of growth options is monotonically increasing in growth uncertainty. This is summarized in the following proposition.

**Proposition 3** (Optionality)**.** *Under incentive-compatible optimal contracts, the aggregate profit rate of growth options ($\pi_t$) is strictly increasing in growth uncertainty ($\nu_{g,t}$) and strictly decreasing in the exercising boundary $\xi_t$ fixed. At the same time, the endogenous investment efficiency That is, the partial derivatives always hold the following signs: $\partial\pi_t/\partial\nu_{g,t} > 0$, $\partial\pi_t/\partial\xi_t < 0$, $\partial\mathcal{G}_\alpha/\partial\nu_{g,t} > 0$, and $\partial\mathcal{G}_\alpha/\partial\xi_t < 0$.*

## 3 Equilibrium

I denote $\eta_t$ to be the market price of risk for the aggregate shock $z_t$, and denote $\kappa_t^{(\nu_t,\nu)}$ to be the market price of risk for the uncertainty shock $N_t^{(\nu_t,\nu)}$. The market prices of the aggregate shocks depend only upon the aggregate state variables, though the economy is full of idiosyncratic shocks. I define the de-trended asset prices and human capital after taking out the economy's balanced growth path as follows:

$\tilde{p}_t \equiv p_t/k_t^{\varphi}$, $\tilde{q}_t \equiv q_t/k_t^{\varphi-1}$, and $\tilde{\hbar}_t \equiv \hbar_t/k_t^{\varphi}$. I conjecture that the prices $\tilde{q}_t$, $\tilde{p}_t$, and the human capital $\tilde{\hbar}_t$ follow the Ito processes with jumps

$$\frac{\mathrm{d}\tilde{q}_t}{\tilde{q}_t} = \mu_t^q \mathrm{d}t + \sigma_t^q \mathrm{d}Z_t + \sum\nolimits_{\nu \neq \nu_t} \varsigma^{q,(\nu_t,\nu)} \mathrm{d}N_t^{(\nu_t,\nu)},$$

and

$$\frac{\mathrm{d}\tilde{p}_t}{\tilde{p}_t} = \mu_t^p \mathrm{d}t + \sigma_t^p \mathrm{d}Z_t + \sum\nolimits_{\nu \neq \nu_t} \varsigma^{p,(\nu_t,\nu)} \mathrm{d}N_t^{(\nu_t,\nu)},$$

and

$$\frac{\mathrm{d}\tilde{\hbar}_t}{\tilde{\hbar}_t} = \mu_t^{\hbar} \mathrm{d}t + \sigma_t^{\hbar} \mathrm{d}Z_t + \sum\nolimits_{\nu \neq \nu_t} \varsigma^{\hbar,(\nu_t,\nu)} \mathrm{d}N_t^{(\nu_t,\nu)}.$$

Here, the coefficient functions $\mu_t^p$, $\mu_t^q$, $\mu_t^{\hbar}$, $\sigma_t^p$, $\sigma_t^q$, $\sigma_t^{\hbar}$, $\varsigma^{p,(\nu_t,\nu)}$, $\varsigma^{q,(\nu_t,\nu)}$, and $\varsigma^{\hbar,(\nu_t,\nu)}$ are endogenously determined in equilibrium. In equilibrium, the prices and human capital are driven by the aggregate shocks $Z_t$ and $N_t^{(\nu_t,\nu)}$, but not by the idiosyncratic shocks $\{W_{f,t}\}_{f\in\mathbb{F}}$, $\{N_{f,t}\}_{f\in\mathbb{F}}$, or $\{\varepsilon_{f,t}\}_{f\in\mathbb{F}}$. Later, I shall show that the productivity shock $\mathrm{d}Z_t$ does not affect the fluctuations of the de-trended prices (see Corollary 2). So, it holds that $\sigma_t^q \equiv \sigma_t^p \equiv \sigma_t^{\hbar} \equiv 0$.

### 3.1 Households' Optimization Problem

Given prices and wages, households face a standard portfolio problem with labor income. Although they cannot manage or trade firm assets, they can freely access to a complete financial market. Taking the processes of market price of risk $\eta_t$ and $\left\{\kappa^{(\nu_t,\nu)} \ : \ \nu_t, \nu \in \mathcal{V}\right\}$ and the prices $p_t$, $q_t$, $\tau_t$ and the wages $w_t$ as given, they solve the following utility maximization problem

$$U_{h,0}^{\mathrm{h}} = \max_{\left\{c_{h,t}^{\mathrm{h}}, \hat{\vartheta}_{h,t}^{\mathrm{h}}, \hat{\vartheta}_{h,t}^{\mathrm{h},(\nu_t,\nu)}\right\}_{t\geq 0}} \mathbb{E}_0 \left[\int_0^{\infty} \mathbf{f}(c_{h,t}^{\mathrm{h}}, U_{h,t}^{\mathrm{h}}) \mathrm{d}t\right] \tag{19}$$

subject to the solvency constraint $n_{h,t}^{\mathrm{h}} \geq 0$ the dynamic budget constraint

$$\frac{\mathrm{d}n_{h,t}^{\mathrm{h}}}{n_{h,t}^{\mathrm{h}}} = \left[\mu_{h,n,t}^{\mathrm{h}} - \hat{c}_{h,t}^{\mathrm{h}}\right] \mathrm{d}t + \underbrace{\sigma_{h,n,t}^{\mathrm{h}} \mathrm{d}Z_t + \sum_{\nu \neq \nu_t} \varsigma_{h,n,t}^{\mathrm{h},(\nu_t,\nu)} \left[\mathrm{d}N_t^{(\nu_t,\nu)} - \lambda^{(\nu_t,\nu)} \mathrm{d}t\right]}_{\text{only aggregate risk expsoures}}, \tag{20}$$

where the expected growth rate on net worth (pre consumption) is $\mu_{h,n,t}^{\mathrm{h}}$ and the aggregate risk exposures are

$$\sigma_{h,n,t}^{\mathrm{h}} = \underbrace{\hat{\vartheta}_{h,t}^{\mathrm{h}}}_{\text{indices}} + \underbrace{(1-\phi)\frac{q_t k_t}{n_t^{\mathrm{h}}}(\sigma_t^q + \varphi\sigma) + (1-\phi)\frac{p_t \overline{s}}{n_t^{\mathrm{h}}}(\sigma_t^p + \varphi\sigma)}_{\text{diversified equity holdings}} + \underbrace{\varrho\frac{\hbar_t}{n_t^{\mathrm{h}}}(\sigma_t^{\hbar} + \varphi\sigma)}_{\text{pledgeable human capital}},$$

and

$$\varsigma_{h,n,t}^{\mathrm{h},(\nu_t,\nu)} = \underbrace{\hat{\vartheta}_{h,t}^{\mathrm{h},(\nu_t,\nu)}}_{\text{indices}} \;+\; \underbrace{(1-\phi)\frac{q_t k_t}{n_t^{\mathrm{h}}}\varsigma_t^{q,(\nu_t,\nu)} + (1-\phi)\frac{p_t\overline{s}}{n_t^{\mathrm{h}}}\varsigma_t^{p,(\nu_t,\nu)}}_{\text{diversified equity holdings}} \;+\; \underbrace{\varrho\frac{\hbar_t}{n_t^{\mathrm{h}}}\varsigma_t^{\hbar,(\nu_t,\nu)}}_{\text{pledgeable human capital}}, \qquad \text{with } \nu \in \mathcal{V}.$$

Here, the shares $\hat{\vartheta}_{h,t}^{\mathrm{h}} \equiv \vartheta_{h,t}^{\mathrm{h}}/n_{h,t}^{\mathrm{h}}$ and $\hat{\vartheta}_{h,t}^{\mathrm{h},(\nu_t,\nu)} \equiv \vartheta_{h,t}^{\mathrm{h},(\nu_t,\nu)}/n_{h,t}^{\mathrm{h}}$ characterize the household's positions in risky assets. Here, the hatted consumption rate $\hat{c}_{h,t}^{\mathrm{h}}$ denotes the consumption rate normalized by the household $h$'s net worth, i.e. $\hat{c}_{h,t}^{\mathrm{h}} \equiv c_{h,t}^{\mathrm{h}}/n_{h,t}^{\mathrm{h}}$. Because all households are homogenous up to their net worth levels, they choose homogeneous risk exposures in equity holdings and pledgeable human capital holdings. In other words, they hold the diversified equity portfolios and the pledgeable human capital proportional to their net worth. The total net worth of all households is $n_t^{\mathrm{h}} \equiv \int_{h\in\mathbb{H}} n_{h,t}^{\mathrm{h}}\mathrm{d}h$. Because the firm-level idiosyncratic risks $\{W_{f,t}, N_{f,t}\}_{t\geq 0}$ are priced at zero by households in equilibrium, the risk averse household will never have any exposure to them in equilibrium. The expected growth rate $\mu_{h,n,t}^{\mathrm{h}}$ includes three components: (i) the expected returns from the index holdings, (ii) the expected returns from the diversified equity holdings, and (iii) the identical labor income rate $\hat{w}_t \equiv w_t/n_t^{\mathrm{h}}$, which is guaranteed by perfect labor insurances among all households. More detailed explanations are in the online appendix.

### 3.2 Experts' Optimization Problem

Given prices and wages, experts face a joint problem of optimal portfolio allocation and optimal real investment, subject to portfolio constraints. The portfolio constraints arise endogenously as a result of incentive compatibility constraints in a moral hazard problem (see Section 2.7). Experts can continuously trade firm's assets in spot markets. Meanwhile, they can also access to the short-term financial contracts in the capital markets. Taking the processes of market price of risk $\eta_t$ and $\left\{\kappa^{(\nu_t,\nu)} \;:\; \nu_t, \nu \in \mathcal{V}\right\}$ and the prices $p_t$, $q_t$, $\tau_t$ and the wages $w_t$ as given, the expert $f$ maximizes the utility

$$U_{f,0}^{\mathrm{e}} = \max_{\left\{\hat{c}_{f,t}^{\mathrm{e}}, g_{f,t}, k_{f,t}, s_{f,t}, \hat{\vartheta}_{f,t}^{\mathrm{e}}, \hat{\vartheta}_{f,t}^{\mathrm{e},(\nu_t,\nu)}\right\}_{t\geq 0}} \mathbb{E}_0\left[\int_0^\infty \mathbf{f}(c_{f,t}^{\mathrm{e}}, U_{f,t}^{\mathrm{e}})\mathrm{d}t\right] \tag{21}$$

subject to the solvency constraint $n_{f,t}^{\mathrm{e}} \geq 0$ and the dynamic budget constraint

$$\begin{aligned}\frac{\mathrm{d}n_{f,t}^{\mathrm{e}}}{n_{f,t}^{\mathrm{e}}} = \left(\mu_{f,n,t}^{\mathrm{e}} - \hat{c}_{f,t}^{\mathrm{e}}\right)\mathrm{d}t + &\underbrace{\sigma_{f,n,t}^{\mathrm{e}}\mathrm{d}Z_t + \sum\nolimits_{\nu\neq\nu_t}\varsigma_{f,n}^{\mathrm{e},(\nu_t,\nu)}\left[\mathrm{d}N_t^{(\nu_t,\nu)} - \lambda^{(\nu_t,\nu)}\mathrm{d}t\right]}_{\text{aggregate risk exposures}} \\ &+ \underbrace{\sigma_{f,n,W,t}^{\mathrm{e}}\mathrm{d}W_{f,t} + \left[\varsigma_{f,n,N,t}^{\mathrm{e}}\mathrm{d}N_{f,t} - \mathbb{E}^{\varepsilon}\left(\varsigma_{f,n,N,t}^{\mathrm{e}}\right)\lambda\mathrm{d}t\right]}_{\text{idiosyncratic risk exposures}}\end{aligned} \tag{22}$$

where the consumption rate is $\hat{c}^{\mathrm{e}}_{f,t} \equiv c^{\mathrm{e}}_{f,t}/n^{\mathrm{e}}_{f,t}$ and the expected growth rate on net worth (pre consumption) is $\mu^{\mathrm{e}}_{f,n,t}$. Furthermore, the exposure to the aggregate shock $\mathrm{d}Z_t$ is

$$\sigma^{\mathrm{e}}_{f,n,t} = \underbrace{\hat{\vartheta}^{\mathrm{e}}_{f,t}}_{\text{indices}} + \underbrace{\phi\frac{q_t k_{f,t}}{n^{\mathrm{e}}_{f,t}}(\sigma^q_t + \varphi\sigma) + \phi\frac{p_t s_{f,t}}{n^{\mathrm{e}}_{f,t}}(\sigma^p_t + \varphi\sigma)}_{\text{concentrated equity holdings}}, \tag{23}$$

and the exposure to the aggregate uncertainty risk $\mathrm{d}N^{(\nu_t,\nu)}_t$ is

$$\varsigma^{\mathrm{e},(\nu_t,\nu)}_{f,n} = \underbrace{\hat{\vartheta}^{\mathrm{e},(\nu_t,\nu)}_{f,t}}_{\text{indices}} + \underbrace{\phi\frac{q_t k_{f,t}}{n^{\mathrm{e}}_{f,t}}\varsigma^{q,(\nu_t,\nu)}_t + \phi\frac{p_t s_{f,t}}{n^{\mathrm{e}}_{f,t}}\varsigma^{p,(\nu_t,\nu)}_t}_{\text{concentrated equity holdings}}, \tag{24}$$

The exposures to the idiosyncratic risks are

$$\underbrace{\sigma^{\mathrm{e}}_{f,n,W,t} = \phi\frac{q_t k_{f,t}}{n^{\mathrm{e}}_{f,t}}\nu_{c,t} \quad \text{and} \quad \varsigma^{\mathrm{e}}_{f,n,N,t} = \phi\frac{p_t s_{f,t}}{n^{\mathrm{e}}_{f,t}}\pi_{f,t}.}_{\text{concentrated equity holdings}}$$

Here, the shares $\hat{\vartheta}^{\mathrm{e}}_{f,t} \equiv \vartheta^{\mathrm{e}}_{f,t}/n^{\mathrm{e}}_{f,t}$ and $\hat{\vartheta}^{\mathrm{e},(\nu_t,\nu)}_{f,t} \equiv \vartheta^{\mathrm{e},(\nu_t,\nu)}_{f,t}/n^{\mathrm{e}}_{f,t}$ characterize the expert's positions in risky short-term financial contracts. The portfolio constraints forced experts to bear uninsured idiosyncratic risks $\sigma^{\mathrm{e}}_{f,n,W,t}$ and $\varsigma^{\mathrm{e}}_{f,n,N,t}$. The implementation described in Proposition 2 requires expert $f$ to retain $\phi$ fraction of firm $f$'s equity stake. The concentrated holdings of the aggregate risks in firm $f$'s equity can be offset by the holdings of aggregate indices as in (23) and (24). Thus, the true effect of the financial restriction is to force each expert to bear the background risks, which are the uninsurable idiosyncratic investment risks. The expected growth rate $\mu^{\mathrm{e}}_{f,n,t}$ includes three components: (i) the expected returns from financial index holdings, (ii) the expected returns from firm's assets holdings, and (iii) minus the expected returns of firm's equity paid out to diffused shareholders.[29]

### 3.3 Competitive Equilibrium

Now, I provide the formal definition of the competitive equilibrium with incomplete markets.

**Definition 1.** *Given the initial aggregate assets in place $k_0 > 0$ and growth options $s_0 > 0$ and the distributions among agents which satisfy $\int_{f\in\mathbb{F}} k_{f,0}df + \int_{h\in\mathbb{H}} k_{h,0}dh = k_0$ and $\int_{f\in\mathbb{F}} s_{f,0}df + \int_{h\in\mathbb{H}} s_{h,0}dh = s_0$. Each agent starts with strictly positive and identical net worth $k_{j,0} > 0$ and $s_{j,0} > 0$ for all $j \in \mathbb{F}\cup\mathbb{H}$. Households sell their capital to experts immediately at time 0. A* **competitive equilibrium** *is a set of aggregate and idiosyncratic stochastic processes adapted to the filtration generated by aggregate and idiosyncratic stochastic processes $\mathcal{F}_t \equiv \sigma\{Z_{t'}, N^{(\nu_{t'},\nu)}_{t'}, W_{f,t'}, N_{f,t'}, \varepsilon_{f,t'} : 0 \le t' \le t, f \in \mathbb{F}, \nu_{t'}, \nu \in \mathcal{V}\}$. The set of aggregate stochastic processes include the prices of productive capitals $\{q_t, p_t\}$, the market prices of aggregate risks $\{\eta_t, \kappa^{(\nu_t,\nu)}_t : \nu_t, \nu \in \mathcal{V}\}$, the aggregate productive capital stocks $\{k_t, s_t\}$, the wage process*

[29] More details on the budget constraint of the expert and the household can be found in the online appendix.

*$\{w_t\}$, the price of investment goods $\{\tau_t\}$, and the human capital $\{\hbar_t\}$. The set of agent-level stochastic processes include the net worth processes $\{n^e_{f,t}, n^h_{h,t}\}$, the consumptions $\{c^e_{f,t}, c^h_{h,t}\}$, the holdings of firm assets $\{k_{f,t}, s_{f,t}\}$, the investment rates $\{i_{f,t}\}$, the demands for the investment goods $\{g_{f,t}\}$, the labor demands $\{\ell_{c,f,t}, \ell_{\iota,t}\}$, and risk exposures $\{\sigma^e_{f,n,t}, \sigma^h_{h,n,t}, \varsigma^{e,(\nu_t,\nu)}_{f,n,t}, \varsigma^{h,(\nu_t,\nu)}_{h,n,t}\}$, for all $f \in \mathbb{F}$ and $h \in \mathbb{H}$, such that*

*(i) Initial expert net worth satisfies $n^e_{f,0} = q_0 k^e_{f,0} + p_0 s^e_{f,0}$ and initial household net worth satisfies $n^h_{h,0} = q_0 k^h_{h,0} + p_0 s^h_{h,0}$.*

*(ii) Given the aggregate dynamics, each household solves her utility optimization problem (19) and each expert solves her utility optimization problem (21).*

*(iii) Market clearing conditions:*

*(a) Assets in place market and growth options market:*

$$\int_{f\in\mathbb{F}} k_{f,t}\,df = k_t \quad \text{and} \quad \int_{f\in\mathbb{F}} s_{f,t}\,df = \bar{s}.$$

*(b) Consumption goods market:*

$$\int_{f\in\mathbb{F}} c^e_{f,t}\,df + \int_{h\in\mathbb{H}} c^h_{h,t}\,dh = \int_{f\in\mathbb{F}} k^{\varphi}_{f,t}\ell^{1-\varphi}_{c,f,t}\,df - \varpi p_t \bar{s}\lambda\bar{\Phi}(\xi_t/\nu_{g,t}).$$

*(c) Investment goods market:*

$$\int_{f\in\mathbb{F}} g_{f,t}\,dN_{f,t} = z_\iota \ell_{\iota,t}.$$

*(d) Labor markets:*

$$\int_{f\in\mathbb{F}} \ell_{c,f,t}\,df + \ell_{\iota,t} = 1.$$

*(e) Financial market for insurance $Z_t$ risk:*

$$\int_{f\in\mathbb{F}} \sigma^e_{f,n,t} n^e_{f,t}\,df + \int_{h\in\mathbb{H}} \sigma^h_{h,n,t} n^h_{h,t}\,dh = q_t k_t(\sigma^q_t + \varphi\sigma) + p_t\bar{s}(\sigma^p_t + \varphi\sigma) + \varrho\hbar_t(\sigma^{\hbar}_t + \varphi\sigma).$$

*(f) Financial market for insurance $N^{(\nu_t,\nu)}_t$ risk:*

$$\int_{f\in\mathbb{F}} \varsigma^{e,(\nu_t,\nu)}_{f,n,t} n^e_{f,t}\,df + \int_{h\in\mathbb{H}} \varsigma^{h,(\nu_t,\nu)}_{h,n,t} n^h_{h,t}\,dh = q_t k_t \varsigma^{q,(\nu_t,\nu)}_t + p_t\bar{s}\varsigma^{p,(\nu_t,\nu)}_t + \varrho\hbar_t\varsigma^{\hbar,(\nu_t,\nu)}_t.$$

*(iv) Law of motion of aggregate capital*

$$dk_t = \left(\int_{f\in\mathbb{F}} i_{f,t}\,dN_{f,t} - \delta\right) k_t\,dt + \sigma k_t\,dZ_t \quad \text{and} \quad ds_t = 0.$$

By Walras' law, the market for risk-free debt clears automatically.

### 3.4 Solving for the Equilibrium Recursively

In order to solve the competitive equilibrium, I have to determine how the prices, investments, and consumptions of all agents depend on the historical paths of the aggregate shock $Z_t$, the aggregate uncertainty shocks $N_t^{(\nu_t,\nu)}$ and idiosyncratic shocks $W_{f,t}, N_{f,t}, \varepsilon_{f,t}$. In fact, I show that the equilibrium can be characterized, in a recursive formulation, by policy functions of three exogenous state variables $(z_t, \nu_{g,t}, \nu_{c,t})$ and two endogenous state variables. One endogenous state variable is the cross-sectional distribution of net worth among experts and households. Because Epstein-Zin-Weil preference is homothetic, the optimal control variables are all linear in the agent's net worth. The linear property allows me to simplify the endogenous state space, from an infinite-dimensional state space to a one-dimensional space. More precisely, I only need to track the evolution of experts' net worth relative to the total net worth held by all agents in equilibrium $x_t = \frac{n_t^{\mathrm{e}}}{Q_t}$, where $n_t^{\mathrm{e}} = \int_{f\in\mathbb{F}} n_{f,t}^{\mathrm{e}}\mathrm{d}f$ and $Q_t \equiv q_t k_t + \overline{s}p_t + \varrho\hbar_t$. The other endogenous state variable is the aggregate assets in place $k_t$, which captures the stochastic trend of the economy. Thus, the equilibrium can be characterized by state variables $(z_t, \nu_t, k_t, x_t)$ where $\nu_t \equiv (\nu_{g,t}, \nu_{c,t})$. Moreover, the Brownian motion $Z_t$ only affects the economy through the i.i.d. shocks $(\mathrm{d}Z_t)$ driving the stochastic trend of the economy and it is independent of the state variable $\nu_t$. So, the variable $Z_t$ does not really serve as a state variable characterizing the equilibrium.[30] As a result, the equilibrium is characterized by $(\nu_t, k_t, x_t)$.

**Dynamic evolution of the economy** In equilibrium, all variables evolve around the stochastic trend $k_t^{\varphi}$. Moreover, the transitory fluctuations along the stochastic trend can be characterized by the state variables $(\nu_t, x_t)$. The uncertainty state variable $\nu_t$ is stationary by assumption. The endogenous state variable $x_t$ is also mean-reverting in equilibrium. The dynamics of the variables in equilibrium can be summarized in Proposition 4.

**Proposition 4** (Growth-trending Variables)**.** *The price variables, the firm-level output and payout variables, and the agent-level net worth variables in equilibrium have the following forms:*

$$\begin{aligned} p_t =& \tilde{p}_t k_t^{\varphi},\ q_t = \tilde{q}_t k_t^{\varphi-1},\ w_t = \tilde{w}_t k_t^{\varphi},\ \tau_t = \tilde{\tau}_t k_t^{\varphi},\ \hbar_t = \tilde{\hbar}_t k_t^{\varphi},\ \textit{and} \\ y_{f,t} =& \tilde{y}_t k_t^{\varphi},\ d_{f,t} = \tilde{d}_t k_t^{\varphi},\ n_{f,t}^{e} = \tilde{n}_{f,t}^{e} k_t^{\varphi},\ n_{h,t}^{h} = \tilde{n}_{h,t}^{h} k_t^{\varphi}, \quad \textit{for all } f \in \mathbb{F} \textit{ and } h \in \mathbb{H}, \end{aligned}$$

*where $\tilde{p}_t$, $\tilde{q}_t$, $\tilde{w}_t$, $\tilde{\tau}_t$, $\tilde{\hbar}_t$, $\tilde{y}_t$, $\tilde{d}_t$, $\tilde{n}_{f,t}^{e}$, and $\tilde{n}_{h,t}^{h}$ are independent of the state variables $z_t$ and $k_t$ and are only driven by the state variables $\nu_t$ and $x_t$.*

**Corollary 1** (Stationary Variables)**.** *The firm-level profit rate of growth options $\pi_{f,t}$, labor demand for production $\ell_{c,f,t}$, investment goods demand $g_{f,t}$, and investment rate $i_{f,t}$ do not depend on the growth-trend state variable $k_t$. They depend only on the stationary state variables $\nu_t$ and $x_t$.*

I now consider the agent-level consumption, real investment, and portfolio holdings. In equilibrium, as in classic consumption-portfolio problems studied by Samuelson (1969) and Merton (1969), the individual

[30] The same feature of i.i.d. cash flow shocks is also adopted in Bolton, Chen and Wang (2011, 2013) and Dou and Ji (2015).

consumption, real investment, and portfolio holdings are linear in terms of the individual net worth. This is because Epstein-Zin-Weil preferences are homothetic. Moreover, the linearity and symmetry of an individual's decision makes it unnecessary to track either the cross-sectional distribution of experts' net worth or the cross-sectional distribution of households' net worth to characterize the equilibrium. It facilitates the aggregation by making the two infinite-dimensional cross-sectional distributions irrelevant in equilibrium.

**Proposition 5** (Linearity and Symmetry)**.** *In equilibrium, the agent-level consumptions* $c^e_{f,t}$ *and* $c^h_{h,t}$*, the firm assets held by individual experts* $k_{f,t}$ *and* $s_{f,t}$*, and the positions of financial short-term contracts shosen by individual agents* $\vartheta^e_{f,t}$*,* $\vartheta^{e,(\nu_t,\nu)}_{f,t}$*,* $\vartheta^h_{h,t}$*, and* $\vartheta^{h,(\nu_t,\nu)}_{h,t}$ *for any* $\nu_t, \nu \in \mathcal{V}$*,* $f \in \mathbb{E}$ *and* $h \in \mathbb{H}$*, have the following forms:*

$$
\begin{aligned}
&c^e_{f,t} = \hat{c}^e_t n^e_{f,t},\ c^h_{h,t} = \hat{c}^h_t n^h_{h,t},\ k_{f,t}/k_t = \hat{k}_t \tilde{n}^e_{f,t},\ s_{f,t} = \hat{s}_t \tilde{n}^e_{f,t},\ \textit{and}\\
&\vartheta^e_{f,t} = \hat{\vartheta}^e_t n^e_{f,t},\ \vartheta^{e,(\nu_t,\nu)}_{f,t} = \hat{\vartheta}^{e,(\nu_t,\nu)}_t n^e_{f,t},\ \vartheta^h_{f,t} = \hat{\vartheta}^h_t n^h_{f,t},\ \vartheta^{h,(\nu_t,\nu)}_{f,t} = \hat{\vartheta}^{h,(\nu_t,\nu)}_t n^h_{f,t},
\end{aligned}
$$

*for all* $\nu_t, \nu \in \mathcal{V}$*,* $f \in \mathbb{F}$*,* $h \in \mathbb{H}$*. Importantly, the hatted variables* $\hat{c}^e_t$*,* $\hat{c}^h_t$*,* $\hat{k}_t$*,* $\hat{s}_t$*,* $\hat{\vartheta}^e_t$*,* $\hat{\vartheta}^h_t$*,* $\hat{\vartheta}^{e,(\nu_t,\nu)}_t$*, and* $\hat{\vartheta}^{h,(\nu_t,\nu)}_t$ *are only dependent on the aggregate stationary state variables* $\nu_t$ *and* $x_t$*. The detrended net worth* $\tilde{n}^e_{f,t}$ *and* $\tilde{n}^h_{h,t}$ *are defined in Proposition 4.*

**Value functions.** Due to homotheticity of EZW preferences, I know that the value function for an expert with net worth $n^{\mathrm{j}}_t$ takes the following power form:

$$
U^{\mathrm{j}}(\zeta^{\mathrm{j}}_t, n^{\mathrm{j}}_t) = \frac{\left(\zeta^{\mathrm{j}}_t n^{\mathrm{j}}_t\right)^{1-\gamma}}{1-\gamma},
$$

where $\zeta^{\mathrm{j}}_t$ is the marginal value of net worth for the agent $\mathrm{j} \in \{\mathrm{e}, \mathrm{h}\}$. The marginal value $\zeta^{\mathrm{j}}_t$ captures the general equilibrium investment environment the agent faces. In particular, a higher marginal value of net worth $\zeta^{\mathrm{j}}_t$ means a better investment environment for the agent. I conjecture that $\zeta^{\mathrm{j}}_t$ follows the dynamic

$$
\frac{\mathrm{d}\zeta^{\mathrm{j}}_t}{\zeta^{\mathrm{j}}_t} = \mu^{\mathrm{j}}_{\zeta,t}\mathrm{d}t + \sigma^{\mathrm{j}}_{\zeta,t}\mathrm{d}Z_t + \sum_{\nu \neq \nu_t} \varsigma^{\mathrm{j},(\nu_t,\nu)}_{\zeta}\mathrm{d}N^{(\nu_t,\nu)}_t, \tag{25}
$$

where all the coefficients $\mu^{\mathrm{j}}_{\zeta,t}$, $\sigma^{\mathrm{j}}_{\zeta,t}$, and $\varsigma^{\mathrm{j},(\nu_t,\nu)}_{\zeta}$ for $\mathrm{j} \in \{\mathrm{e}, \mathrm{h}\}$ are determined in equilibrium.[31]

**Wealth distribution dynamics.** Due to the homogeneity of experts and the homogeneity of households up to their own individual net worth levels, I only need to track the distribution between the aggregate experts' net worth $n^{\mathrm{e}}_t$ and the aggregate households' net worth $n^{\mathrm{h}}_t$. I define $\tilde{Q}_t \equiv Q_t/k^{\varphi}_t = \tilde{n}^{\mathrm{e}}_t + \tilde{n}^{\mathrm{h}}_t$ and

[31] The HJB equations for experts and households can be found in the online appendix. The expressions of the Ito coefficients in (25), (26), and (27) are also in the online appendix.

conjecture that

$$\mathrm{d}\tilde{Q}_t/\tilde{Q}_t = \mu_t^Q \mathrm{d}t + \sigma_t^Q \mathrm{d}Z_t + \sum_{\nu \neq \nu_t} \varsigma^{Q,(\nu_t,\nu)} \mathrm{d}N_t^{(\nu_t,\nu)}, \tag{26}$$

where the coefficients depends on those of the prices $q_t$, $p_t$ and human capital $\hbar_t$. Thus, in equilibrium, the law of motion of $x_t$ can be characterized as follows:

$$\frac{\mathrm{d}x_t}{x_t} = \mu_{x,t}\mathrm{d}t + \sigma_{x,t}\mathrm{d}Z_t + \sum_{\nu \neq \nu_t} \varsigma_{x,t}^{(\nu_t,\nu)} \mathrm{d}N_t^{(\nu_t,\nu)}, \tag{27}$$

where $\mu_{x,t}$ is the expected growth rate and the volatility of wealth share $\sigma_{x,t}$ and the jump size $\varsigma_{x,t}^{(\nu_t,\nu)}$ are,

$$\sigma_{x,t} = \sigma_{f,n,t}^{\mathrm{e}} - \sigma_t^Q - \varphi\sigma, \quad \text{and} \quad \varsigma_{x,t}^{(\nu_t,\nu)} = \frac{\varsigma_{f,n,t}^{\mathrm{e},(\nu_t,\nu)} + 1}{\varsigma_t^{Q,(\nu_t,\nu)} + 1} - 1, \text{ respectively.}$$

Because the aggregate $Z_t$ process characterizes i.i.d. shocks in the economy which are independent with all other aggregate shocks and it is not a state variable, it only affects agents' myopic portfolio decisions and hence is perfectly shared by agents using contract term contracts on the shock. Thus, in equilibrium, the aggregate shock $\mathrm{d}Z_t$ should have zero impact on the endogenous state variable $x_t$. In fact, it is not hard to show the following results.

**Proposition 6.** *In the equilibrium, the aggregate risk $Z_t$ is perfectly shared. Thus, each agent's exposure $\sigma_{f,n,t}^e$ to the productivity shock $dZ_t$ is simply the constant myopic component:*

$$\sigma_{x,t} = \sigma_{f,n,t}^e - \sigma_t^Q - \varphi\sigma = 0.$$

**Corollary 2.** *In the equilibrium, the loadings of de-trended variables on the productivity shock $dZ_t$ are all zero, since the risk $Z_t$ is perfectly shared among heterogeneous agents. In particular,*

$$\sigma_{\xi,t}^e \equiv \sigma_{\xi,t}^h \equiv \sigma_t^Q \equiv \sigma_t^p \equiv \sigma_t^q \equiv \sigma_t^{\hbar} \equiv 0.$$

**Recursive Markov equilibria.**

**Definition 2.** *A Recursive Markov Equilibrium characterized by state variables $(x_t, \nu_t)$ is a set of aggregate functions: marginal values of net worth in value functions $\zeta^e, \zeta^h$, price functions $p, q, w, \hbar, \eta, r$, and $\kappa^{(\nu,\nu')}$ and policy functions $\hat{c}^e, \hat{c}^h, g, \hat{k}, \hat{s}, \hat{\theta}^e, \hat{\theta}^h, \hat{\vartheta}^{e,(\nu,\nu')}$, and $\hat{\vartheta}^{h,(\nu,\nu')}$, and law of motions for the endogenous state variable $x_t$ such that*

*(i) the marginal value of net worths $\zeta^e$ and $\zeta^h$ solve the experts' and households' HJB equations, and $\hat{c}^e, \hat{c}^h, g, \hat{k}, \hat{s}, \hat{\theta}^e, \hat{\theta}^h, \hat{\vartheta}^{e,(\nu,\nu')}, \hat{\vartheta}^{h,(\nu,\nu')}$ are the optimal control variables, taking prices $\tilde{q}, \tilde{p}, \tilde{w}, \tilde{\hbar}, r, \eta$, and $\kappa^{(\nu,\nu')}$ and the law of motion of state variables $x_t$ and $\nu_t$ as given;*

*(ii) the market clearing conditions are satisfied:*

$$\hat{c}^e\tilde{Q}x + \hat{c}^h\tilde{Q}(1-x) = \tilde{y} - \varpi\bar{s}\tilde{p}\lambda\bar{\Phi}(\xi/\nu_g) \qquad \textit{(Consumption Goods)}$$

$$g = z_\iota \ell_\iota \quad \textit{(Investment Goods)}$$

$$\tilde{Q}\hat{k}x = 1 \textit{ and } \tilde{Q}x\hat{s} = \bar{s} \quad \textit{(Tangible and Intangible Capitals)}$$

$$\ell_c + \ell_\iota = 1 \quad \textit{(Labor Hours)}$$

$$\sigma_n^e x + \sigma_n^h (1-x) = \varphi\sigma \quad \textit{(Financial Securities for } Z_t \textit{ and } \sigma_t^Q \equiv 0\textit{)}$$

$$\varsigma_n^{e,(\nu,\nu')} x + \varsigma_n^{h,(\nu,\nu')} (1-x) = \varsigma^{Q,(\nu,\nu')} \quad \textit{(Financial Securities for } N_t^{(\nu,\nu')}\textit{)}$$

*(iii) the law of motion of endogenous state variable $x_t$ is characterized as in (27).*

The fixed-point conditions that characterize the Recursive Markov equilibrium can be summarized by a set of coupled highly-nonlinear ordinary differential equations, whose details can be found in the online appendix.

# 4 Quantitative Results

In this section, I first explore whether a real business cycle model with two sources of uncertainty shocks and imperfect risk sharing can simultaneously match the key moments of macroeconomic variables and asset returns. This exercise reveals the quantitative importance of two uncertainty shocks, interacting with endogenous imperfect risk sharing, as drivers of macroeconomic fluctuations and determinants of risk premia. Then, the calibrated general equilibrium model provides a laboratory allowing me to examine the quantitative relevance of the key mechanism discovered in this paper. I show that the implications of the key mechanism are quantitatively significant and coherent within such an empirically-validated framework. Furthermore, in Section 5, I explore whether the implications of the key mechanism is observed in the data.

## 4.1 Calibration and Parameter Choices

Table 1 summarizes the parameter choices used in my calibration. The key parameter that characterizes the risk sharing imperfection is the severity of agency problem, denoted by $\phi$. In the model, the experts effectively constitute blockholders.[32] The blockholders (including the inside blockholders) control the firm: they can either directly or indirectly intervene in firm's operations (e.g., Edmans and Manso, 2011; Holderness, 2003, 2009). Holderness (2009) reports 96% of randomly selected U.S. firms in 1995 have blockholders, and the average percentage of the voting rights in common stocks held by all blockholders is 43%. Khan, Dharwadkar and Brandes (2005) show that from 1992 to 1999, the total institutional ownership increases from 52.6% to 58.8%, and the CEO ownership is ranged from 2.17% and 2.94%, based on a complete 8-year sample with 224 U.S. public firms. More recent data show that the institutional

[32]Each expert is a representative agent of the managers and insiders who actively intervene in the management and hold significant stake of a firm. The degree of blockholding tends to underestimate the concentrated ownership of the experts, because experts do not only hold stake in the firm through common stocks but also through compensations and rents. On the other hand, not all blockholders are forced to bear uninsurable idiosyncratic risks of the firm's equity. Hoping the two forces cancel out each other, I take the blockholding level as an approximation for the parameter $\phi$.

blockholders in U.S. equity markets holding over 66% of the total equity.[33] I choose $\phi = 0.5$ to provide a reasonable blockholding in my model.

To illustrate the role of the two uncertainty shocks, I need to choose the evolution rules governing how the uncertainty fluctuates over time and to choose the uncertainty levels that characterizing the scale of fluctuations in uncertainty. The transition intensities $\lambda^{(\nu_g,\nu_g')}$ and $\lambda^{(\nu_c,\nu_c')}$ are estimated based on the regime-switching dynamics of estimated growth uncertainty and cash-flow uncertainty, respectively. Specifically, they are estimated based on Table 6 for which the details are in the online appendix. The uncertainty levels are calibrated such that the interdecile range (IDR) of sales growth rates and the cross-sectional standard deviation (CSD) of investment rates have means and standard deviations that reasonably match the moments in the data summarized in Table 4. More precisely, the means of the IDR of sales growth rates are 53.02% in the model and 49.02% in the data; moreover, its standard deviations are 16.03% in the model and 12.32% in the data. The means of the CSD of investment rates are 45.12% in the model versus 40.85% in the data; and, its standard deviations are 13.50% in the model versus 7.25% in the data.

To calibrate the specification of preferences, I choose a value for EIS $\psi = 2$ consistent with Bansal and Yaron (2004); Bansal, Kiku and Yaron (2012), who emphasize that the preference of early resolution of uncertainty is important to understand uncertainty shock's impact on asset prices. Consistent with macroeconomic models of asset prices such as Guvenen (2009), I choose a value for risk aversion no bigger than 10. Here, I use $\gamma = 6$ to provide a comparable capital-to-output ratio to the data as summarized in Table 3 (196.20% in the model versus 169.24% in the data). The subjective discount factor is chosen to be $\delta = 0.0111$ to help the model match the average level of risk-free rates as in Table 5 (1.53% in the model versus 1.31% in the data).

The average lifespan parameter is chosen to be $\mu = 1/40$, which is a standard choice since the average number of working years in U.S. is about 40. The pledgeability of human capital chosen to be $\varrho = 5\%$ which is consistent with Lustig and Nieuwerburgh (2010). The population of experts is estimated based on the U.S. income distribution observations provided by U.S. census. A simple linear extrapolation estimates that, on average, about 2% of U.S. households earn annual salary $30,000$ dollars. Presumably, the experts make at least $30,000$ dollars a year, so 2% is a reasonable approximation for the population share of experts in the economy.

Lastly, as for the production and investment of consumption goods firms, the productivity volatility $\sigma$ is calibrated in a standard way. I choose $\sigma = 10\%$ to match the standard deviation of output log growth as summarized in Table 2 (1.92% in the model versus 1.67% in the data). The shares of capital are chosen to be $\varphi = 0.3$ and $1 - \alpha = 0.1$, which help match the relative size of the consumption goods sector and the investment goods sector (approximately 23% for the investment goods sector in terms of sectoral outputs in the data), while generating a labor share of output of approximately 75% as in Table 3 (75.25% in the model versus 75.26% in the data). This is also in line with Papanikolaou (2011). The constant arrival rate of investment opportunities is chosen at $\lambda = 3.33$ and the fixed adjustment cost rate is chosen at

[33] See, e.g., Carolyn K. Brancato and Stephan Rabimov, "The 2008 Institutonal Investment Report: Trends in Institutional Investor Assets and Equity Ownership of U.S. Corporations" (The Conference Board, 2008).

$\varpi = 0.83\%$ to match the average annual positive investment rate (approximately 79% in the data) and the standard deviation of aggregate investment log growth (Table 2) simultaneously. The standard deviations of log growth rates of aggregate investment are 55.38% in the model and 36.00% in the data. The average productivity in the investment goods sector $z_\iota = 1.03$ and the depreciation rate $\delta = 15\%$ help match the average investment-to-output ratio (Table 3: 16.60% in the model versus 16.47% in the data) and the average payout-to-consumption ratio (Table 3: 6.30% in the model versus 5.46% in the data).

### 4.2 Model Implications

**Macroeconomic moments.** I report the model-implied moments of the growth rates of log consumption, log investment, and log output in Panel B of Table 2; for comparison, I also report their empirical counterparts in Panel A of the same table. Columns (1) – (3) of the table report means, standard deviations, and autocorrelations for each growth variable; Columns (4) – (6) report the correlations among the three growth variables. For the simulated data in panel B, the table shows the average values across independent simulations, along with the 5th and 95th percentiles reported in brackets. For the moments in the data, the table reports the point estimates and the corresponding confidence intervals in brackets estimated by stationary block bootstrap methods.

The moments in blue and bold are those used for calibration. For most of the moments of interest in Table 2, the data and the model are close statistically. However, the model fails to produce the right pattern of comovement between investment and consumption growth ($-0.44$ in the model versus 0.83 in the data). The main reason is that the growth uncertainty resembles the investment shock, especially when the risk sharing condition is good. This can be seen from (14) and (17). The implied investment shock generates opposite responses in investment and consumption. The negative correlation arises for two reasons. First, the high value of EIS implies that consumption does fall heavily in response to an implied positive investment shock. It generates exceedingly negative correlation between consumption and investment. Second, the aggregate productivity shock $\mathrm{d}Z_t$ moves investment and consumption in the same direction. However, when EIS is large, the effect of implied investment shocks dominates, generating a negative correlation between investment and consumption. This is a well-known issue for real business cycle models with investment shocks. In general, labor market frictions can help restore the positive correlation.

For a macroeconomic growth model, the valid quantitative analysis requires the key macroeconomic ratios characterizing the steady state along the balanced growth path to be replicated by the model with reasonably small errors. Basically, the calibration of the model should be able to generate the steady-state ratios consistent with the data. Table 3 compares the empirical moments of investment-to-output ratios, net-payout-to-consumption ratios, wage-income-to-output ratios, and capital-to-output ratios with their correspondences in the simulated data generated from the model. The moments in blue and bold are used for calibration. It shows that the data and the model are statistically close for most of the moments of interest. However, the model fails to generate a high enough standard deviation of wage-income-to-output ratio (2.04% in the model versus 4.02% in the data); moreover, their confidence intervals are not even overlapped. The main reason is that the asymmetric and volatile fluctuations in unemployment is hard to be captured by a model without frictions in labor markets (e.g., Petrosky-Nadeau and Zhang, 2013). In the model with frictionless labor markets, agents can efficiently smooth out the labor income shocks over time.

Table 4 compares the empirical moments of cross-sectional dispersions to the simulated ones from the model. Panel A shows that the sales growth dispersion is countercyclical (the correlation with log output growth is $-17.32\%$), while the investment rate dispersion is procyclical in the data (the correlation with log output growth is 43.28%). It is consistent with the findings in Bachmann and Bayer (2014) who emphasize that the uncertainty-driving real business cycle models need to reconcile the two prominent patterns. Column (4) of Panel B shows that the model generates countercyclical sales growth dispersions (the correlation with log output growth is $-27.66\%$) and procyclical investment rate dispersion (the correlation with log output growth is 23.82%) simultaneously. In the model, the sales growth dispersion is mainly driven by cash-flow uncertainty shocks (as shown in (42)), which decrease the output and investment due to the imperfect risk sharing; the investment rate dispersion is mainly driven by growth uncertainty shocks, which has asymmetrically stronger effect when risk sharing is less limited. The asymmetric effect of growth uncertainty shocks on the investment rate dispersion implies procyclical dispersion in equilibrium.

Table 5 compares the asset pricing moments in the data to the simulated moments from the model. In particular, the sizable equity premium (4.95% in the model versus 4.47% in the data) is main a result of the market incompleteness and the amplification effect of financial frictions on the uncertainty shocks. The model also reproduces the sizable value premium (7.57% in the model versus 5.05% in the data). The large average value spread is mainly due to the cash-flow uncertainty shock which carries a negative market price of risk and decreases the value of assets in place relative to growth options.

Figure 2 and Figure 3 show the key results of this paper. In Figure 2, while the market price of risk for the cash-flow uncertainty shock is always negative, the market price of risk for the growth uncertainty shock changes from negative to positive as the risk sharing condition gets better. In Figure 3, while the exposure of value spreads to cash-flow uncertainty shocks is always negative, their exposure to the growth uncertainty shock changes from positive to negative as the risk sharing condition improves.

### 4.3 Basic Mechanisms

**Stochastic discount factors and idiosyncratic risk premia** There is a full menu of short-term contingent claims on both aggregate shocks and idiosyncratic shocks available to the agents. However, the moral hazard makes the enforcement of some contingent claim contracts on idiosyncratic shocks imperfect.

I denote by $\mathcal{M}^{\mathrm{e}}_{f,t}$ the utility gradients of the expert $f$ at her optimal consumption policy. According to Duffie and Skiadas (1994, Theorem 2), the utility gradient of expert $f$ has the following expression:

$$\mathcal{M}^{\mathrm{e}}_{f,t} = \exp\left[\int_0^t \mathbf{f}_U(c^{\mathrm{e}}_{f,t'}, U^{\mathrm{e}}_{f,t'})\mathrm{d}t'\right] \mathbf{f}_c(c^{\mathrm{e}}_{f,t}, U^{\mathrm{e}}_{f,t}).$$

Thus, the instantaneous intertemporal marginal rate of substitution (IMRS) of expert $f$ is

$$\frac{\mathrm{d}\mathcal{M}^{\mathrm{e}}_{f,t}}{\mathcal{M}^{\mathrm{e}}_{f,t}} = -\mu^{\mathrm{e}}_t - \eta^{\mathrm{e}}_t \mathrm{d}Z_t - \sum_{\nu'_g \neq \nu_g} \kappa^{\mathrm{e},(\nu_g,\nu'_g)} \left[\mathrm{d}N^{(\nu_g,\nu'_g)}_t - \lambda^{(\nu_g,\nu'_g)}\mathrm{d}t\right] - \gamma\sigma^{\mathrm{e}}_{f,n,W,t}\mathrm{d}W_{f,t} - \gamma\sigma^{\mathrm{e}}_{f,n,N,t}\mathrm{d}N_{f,t}, \tag{28}$$

where the drift $\mu^{\mathrm{e}}_t$ and the coefficients of aggregate shocks $\eta^{\mathrm{e}}_t$ and $\kappa^{\mathrm{e},(\nu_g,\nu'_g)}_t$ only depend on aggregate state variables in equilibrium. The coefficients of idiosyncratic shocks $\sigma^{\mathrm{e}}_{f,n,W,t}$ and $\sigma^{\mathrm{e}}_{f,n,N,t}$ also only depend on aggregate state with the following expressions:

$$\sigma^{\mathrm{e}}_{f,n,W,t} \equiv \nu_{c,t}\frac{\phi}{x_t}\digamma_{\mathbf{k}}(x_t,\nu_t) \quad \text{and} \quad \sigma^{\mathrm{e}}_{f,n,N,t} = \frac{1}{\gamma}\left[1-(1+\varsigma^{\mathrm{e}}_{f,n,N,t})^{-\gamma}\right].$$

Effectively, the term $\gamma\sigma_{f,n,W,t}$ is the market price of the idiosyncratic cash flow risk $\mathrm{d}W_{f,t}$ required by the expert $f$, while the term $\gamma\sigma^{\mathrm{e}}_{f,n,N,t}$ is the market price of the idiosyncratic growth risk $\mathrm{d}N_{f,t}$ required by the expert $f$. The term $\sigma_{f,n,W,t}$ is simply the loading of idiosyncratic cash flow risks. The term $\sigma^{\mathrm{e}}_{f,n,N,t}$ is approximately equal to $\varsigma^{\mathrm{e}}_{f,n,N,t}$ when the latter is small according to the Taylor expansion.

In an economy with complete and frictionless financial market, there is a unique stochastic discount factor which is equal to every agent's utility gradient. In an incomplete market, for any particular set of assets, according to the intertemporal Euler equations, the non-arbitrage condition implies that the stochastic discount factor is equal to the highest utility gradient across all agents who have access to the particular set of assets. In fact, for the unconstrained agent in some state, her utility gradient must equal to the stochastic discount factor in that state. This is the similar idea in Chien and Lustig (2010) and Alvarez and Jermann (2001) for asset pricing in an incomplete market.

Because all experts can freely access all financial assets whose payoffs are contingent on the aggregate shocks, the cross-sectional average of these individual experts' IMRS is a valid SPD for those financial in all states. Thus, the following results can be derived readily. For those financial assets whose payoffs depend only on aggregate states, one SPD that prices their returns is provided by the average IMRS of experts. More precisely, it is the SPD $\Lambda_t$ such that

$$\frac{\mathrm{d}\Lambda_t}{\Lambda_t} \equiv \frac{1}{\varkappa}\int_{f\in\mathbb{F}}\left[\frac{\mathrm{d}\mathcal{M}^{\mathrm{e}}_{f,t}}{\mathcal{M}^{\mathrm{e}}_{f,t}}\right]\mathrm{d}f = -r_t\mathrm{d}t - \eta_t\mathrm{d}Z_t - \sum_{\nu\neq\nu_t}\kappa^{(\nu_t,\nu)}_t\left[\mathrm{d}N^{(\nu_t,\nu)}_t - \lambda^{(\nu_t,\nu)}\mathrm{d}t\right], \tag{29}$$

Here $r_t \equiv \mu^{\mathrm{e}}_t$ is the risk-free interest rate, $\eta_t \equiv \eta^{\mathrm{e}}_t$ is the market price of aggregate cash flow risk $Z_t$, and $\kappa^{(\nu_t,\nu)}_t \equiv \kappa^{\mathrm{e},(\nu_t,\nu)}_t$ is the market price of uncertainty risk $N^{(\nu_t,\nu)}_t - \lambda^{(\nu_t,\nu)}t$ for each $\nu\in\mathcal{V}$. All market prices $\eta_t$, $\kappa^{(\nu_t,\nu)}$ for all $\nu\in\mathcal{V}$ and interest rate $r_t$ are determined endogenously in equilibrium. Households agree on the market prices of risk. It is straightforward to derive that the market price of risk $\eta_t$ is constant $\eta\equiv\gamma\varphi\sigma$, because all agents (experts and households) perfectly share the aggregate risk of productivity shock $\mathrm{d}Z_t$ by holding constant risk exposure $\varphi\sigma$.

However, experts are the only agents who can trade firm's assets freely. For each firm-specific asset, the following Euler equations hold. More precisely, for each $f \in \mathbb{F}$, it holds that

$$\mathbb{E}_t\left[\mathrm{d}R_t^{\mathbf{k}}\right]/\mathrm{d}t - r_t = -\mathbb{E}_t\left[\frac{\mathrm{d}\mathcal{M}_{f,t}^{\mathrm{e}}}{\mathcal{M}_{f,t}^{\mathrm{e}}} \times \frac{d_{f,t}\mathrm{d}t - (1-\phi)q_t k_{f,t}\nu_{c,t}\mathrm{d}W_{f,t} + \mathrm{d}\left(q_t k_{f,t}\right)}{q_t k_{f,t}}\right] \quad \text{for all } f \in \mathbb{F},$$

and

$$\mathbb{E}_t\left[\mathrm{d}R_t^{\mathbf{s}}\right]/\mathrm{d}t - r_t = -\mathbb{E}_t\left[\frac{\mathrm{d}\mathcal{M}_{f,t}^{\mathrm{e}}}{\mathcal{M}_{f,t}^{\mathrm{e}}} \times \frac{\phi p_t s_{f,t}\mathrm{d}N_{f,t} + \mathrm{d}\left(p_t s_{f,t}\right)}{p_t s_{f,t}}\right] \quad \text{for all } f \in \mathbb{F}.$$

Here, $d_{f,t}\mathrm{d}t - (1-\phi)q_t k_{f,t}\nu_{c,t}\mathrm{d}W_{f,t}$ is the effective consumption goods net payout of assets in place to expert $f$ since she can dump the amount $(1-\phi)q_t k_{f,t}\nu_{c,t}\mathrm{d}W_{f,t}$ of the idiosyncratic cash flow exposure for free. And, $\phi\pi_{f,t}p_t\bar{s}\mathrm{d}N_{f,t}$ is the effective pecuniary net payout of growth options to expert $f$ since she can dump the amount $(1-\phi)\pi_{f,t}p_t\bar{s}\mathrm{d}N_{f,t}$ of idiosyncratic growth exposure for free.

The relations of (28) – (31) leads to the following beta pricing rules for assets in place and growth options. The expected return from holding assets in place (i.e. assets in place) in excess of the risk-free rate equals

$$\underbrace{\mathbb{E}_t\left[\mathrm{d}R_t^{\mathbf{k}}\right]/\mathrm{d}t - r_t}_{\text{risk premium of holding } k_t} = \underbrace{\overbrace{\varphi\sigma\eta + \sum_{\nu\neq\nu_t} \varsigma_t^{q,(\nu_t,\nu)}\kappa_t^{(\nu_t,\nu)}\lambda_t^{(\nu_t,\nu)}}^{\mathbb{E}_t\left[\mathrm{d}\tilde{R}_t^{\mathbf{k}}\right]/\mathrm{d}t - r_t}}_{\text{aggregate premium for all agents}} + \underbrace{\gamma(\phi\nu_{c,t})^2\frac{\digamma_{\mathbf{k}}(x_t,\nu_t)}{x_t}}_{\text{idiosyncratic premium for experts}}, \tag{30}$$

where $\tilde{R}_t^{\mathbf{k}}$ is the equity return on assets in place and $\digamma_{\mathbf{k}}(x_t,\nu_t) \equiv q_t k_t/Q_t$ is the assets-in-place share in the total net worth $Q_t$.

And, the expected return from holding growth options (i.e. growth options) in excess of the risk-free rate equals

$$\underbrace{\mathbb{E}_t\left[\mathrm{d}R_t^{\mathbf{s}}\right]/\mathrm{d}t - r_t}_{\text{risk premium of holding } s_t} = \underbrace{\overbrace{\varphi\sigma\eta + \sum_{\nu\neq\nu_t} \varsigma_t^{p,(\nu_t,\nu)}\kappa_t^{(\nu_t,\nu)}\lambda_t^{(\nu_t,\nu)}}^{\mathbb{E}_t\left[\mathrm{d}\tilde{R}_t^{\mathbf{s}}\right]/\mathrm{d}t - r_t}}_{\text{aggregate premium for all agents}} + \underbrace{\lambda\phi\mathbb{E}_t^{\varepsilon}\left\{\left[1-\left(1+\varsigma_{f,n,N,t}^{\mathrm{e}}\right)^{-\gamma}\right]\pi_{f,t}\right\}}_{\text{idiosyncratic premium for experts}}, \tag{31}$$

where $\tilde{R}_t^{\mathbf{s}}$ is the equity return on growth options and $\digamma_{\mathbf{s}}(x_t,\nu_t) \equiv p_t s_t/Q_t$ is the growth-options share in the total net worth $Q_t$.

Alternatively, the beta pricing rules (30) – (31) can also be derived using the first-order conditions of experts' Hamilton-Jacobi-Bellman (HJB) equations, together with their dynamic budget constraints.

Using the Taylor-expansion approximation, the idiosyncratic risk premium on growth options can be approximated by

$$\lambda\phi\mathbb{E}_t^{\varepsilon}\left\{\left[1-\left(1+\varsigma_{f,n,N,t}^{\mathrm{e}}\right)^{-\gamma}\right]\pi_{f,t}\right\} \approx \lambda\phi\gamma\mathbb{E}_t^{\varepsilon}\left[\pi_{f,t}\varsigma_{f,n,N,t}^{\mathrm{e}}\right] = \gamma\left[\mathcal{I}_\alpha(\nu_{g,t}/\xi_t)\right]^2\frac{\digamma_{\mathbf{s}}(x_t,\nu_t)}{x_t} \times \frac{\phi^2}{\lambda}.$$

Figure 4 illustrates the idiosyncratic risk premia under the calibration summarized in Table 1. The uncertainty shocks increase the risk premia on the idiosyncratic risks. There are several additional observations that worth mentioning. First, the effect of uncertainty shocks on the idiosyncratic risk premia increases nonlinearly as the risk sharing becomes more limited (i.e., $x_t$ decreases). The reason is that the expert's net worth has larger exposure to the idiosyncratic shocks when $x_t$ is lower. Second, the risk premium on the idiosyncratic cash-flow shock is mainly affected by the cash-flow uncertainty shock, while the risk premium on the idiosyncratic investment shock is mainly affected by the growth uncertainty shock. These heterogeneous impacts are due to the distinct nature of the two uncertainty shocks. Third, while the cash-flow uncertainty always has significant positive impact on the idiosyncratic cash flow risk premium, the growth uncertainty has almost no effect on idiosyncratic risk premia when the risk sharing condition is good. The reason is that the investment shock effect dominates when the risk sharing condition is good.

**Amplification: uncertainty shocks compromise risk sharing conditions.** In the model, the risk sharing condition is endogenously affected by the two uncertainty shocks. To establish the link between uncertainty shocks and the risk sharing condition, I consider two different types of measures of how much risk sharing is limited. The first type of measure is based on the idea that the covariance between an agent's net worth and idiosyncratic risks is always zero when the market is complete (i.e., risk sharing is perfect). When experts have a larger exposure to idiosyncratic shocks in their net worth, there is a larger cross-sectional dispersion in growth rates of individual consumption shares. So, the cross-sectional dispersion in growth rates of individual consumption shares provides a reasonable measure for the risk sharing imperfectness. The second type of measure is based on the idea that the marginal value of net worth should be identical across all agents when the market is complete. Therefore, the discrepancy between agents' marginal value of net worth serves as another natural measure of risk sharing imperfectness.[34]

*Consumption dispersion.* In equilibrium, the household's net worth is independent of all idiosyncratic risks, while the incentive constraints force each expert $f$ to expose his own net worth to the particular idiosyncratic risk $\mathrm{d}W_{f,t}$. Importantly, the expert's idiosyncratic risk exposure is endogenous and hence time varying.

But, it only depends on the aggregate states in the economy. For each expert $f$, the conditional instantaneous covariance of net worth growth with the idiosyncratic shock $\mathrm{d}W_{f,t}$ is

$$\mathrm{Cov}_t\left(\frac{\mathrm{d}n^{\mathrm{e}}_{f,t}}{n^{\mathrm{e}}_{f,t}},\ \mathrm{d}W_{f,t}\right)/\mathrm{d}t = \nu_{c,t} \times \phi \times \frac{1}{x_t} \times \digamma_{\mathbf{k}}(x_t, \nu_t),$$

where $\digamma_{\mathbf{k}}(x_t, \nu_t) \equiv q_t/Q_t$ is the value share of asset in place in total financial wealth. Moreover, the conditional instantaneous covariance of net worth growth with the idiosyncratic standardized shock $\mathrm{d}\tilde{N}_{f,t}$,

[34] The detailed derivations in this section can be found in the online appendix.

which is normalized by aggregate profit rate of growth options $\pi_t$, is

$$\mathrm{Cov}_t\left(\frac{\mathrm{d}n^{\mathrm{e}}_{f,t}}{n^{\mathrm{e}}_{f,t}},\ \mathrm{d}\tilde{N}_{f,t}\right)/\mathrm{d}t = \mathcal{I}_{\alpha}(\nu_{g,t}/\xi_t) \times \phi \times \frac{1}{x_t} \times \digamma_{\mathbf{s}}(x_t, \nu_t),$$

where $\digamma_{\mathbf{s}}(x_t, \nu_t) \equiv p_t\bar{s}/Q_t$ is the value share of growth options in total financial wealth, and $\mathcal{I}_{\alpha}(\cdot)$ is a deterministic function which is strictly increasing.

It can be seen that the severity of agency problems characterized by $\phi$, the expert's wealth share $x_t$, and the uncertainties $\nu_{c,t}$ and $\nu_{g,t}$ have direct monotonic impact on the risk sharing capacity measures, up to some general equilibrium valuation effects $\digamma_{\mathbf{k}}(x_t, \nu_t)$ and $\digamma_{\mathbf{s}}(x_t, \nu_t)$. The severity of agency problem, the wealth share and the uncertainties can also affect the risk sharing capacity indirectly, which can be summarized by the general equilibrium effects $\digamma_{\mathbf{k}}(x_t, \nu_t)$ and $\digamma_{\mathbf{s}}(x_t, \nu_t)$.

For each expert $f$, the consumption share is defined as $S^{\mathrm{e}}_{f,t} \equiv c^{\mathrm{e}}_{f,t}/c^{\mathrm{e}}_t$. I use the cross-sectional standard deviation (CSD) of consumption share growth rates to capture the dispersion. The cross-sectional dispersion of consumption growth depends on the idiosyncratic risk exposures. The instantaneous cross-sectional variance of consumption share growth rates has the following analytical expression

$$\mathrm{var}_t\left(\frac{\mathrm{d}S^{\mathrm{e}}_{f,t}}{S^{\mathrm{e}}_{f,t}}\right)/\mathrm{d}t = \left[\nu_{c,t}\frac{\phi}{x_t}\digamma_{\mathbf{k}}(x_t, \nu_t)\right]^2 + \left[\mathcal{I}_{\alpha}(\nu_{g,t}/\xi_t)\frac{\phi}{x_t}\digamma_{\mathbf{s}}(x_t, \nu_t)\right]^2.$$

The basic idea of the proof is that each individual expert's consumption share is equal to his net worth share in the equilibrium. That is, $c^{\mathrm{e}}_{f,t}/c^{\mathrm{e}}_t = n^{\mathrm{e}}_{f,t}/n^{\mathrm{e}}_t$. Therefore, the cross-sectional instantaneous variance of the growth rates of consumption shares is equal to the instantaneous idiosyncratic variance of individual consumption growth.

Therefore, the instantaneous cross-sectional variance of the experts' consumption share growth rates is linked to their exposures of idiosyncratic risks in the following way:

$$\mathrm{var}_t\left(\frac{\mathrm{d}S^{\mathrm{e}}_{f,t}}{S^{\mathrm{e}}_{f,t}}\right)/\mathrm{d}t = \left[\mathrm{Cov}_t\left(\frac{\mathrm{d}n^{\mathrm{e}}_{f,t}}{n^{\mathrm{e}}_{f,t}}, \mathrm{d}W_{f,t}\right)/\mathrm{d}t\right]^2 + \left[\mathrm{Cov}_t\left(\frac{\mathrm{d}n^{\mathrm{e}}_{f,t}}{n^{\mathrm{e}}_{f,t}}, \mathrm{d}\tilde{N}_{f,t}\right)/\mathrm{d}t\right]^2.$$

The instantaneous cross-sectional standard deviation of $\frac{\mathrm{d}S^{\mathrm{e}}_{f,t}}{S^{\mathrm{e}}_{f,t}}$ across all experts is defined as a measure for the risk sharing imperfection (i.e. the inverse of risk sharing condition). More precisely, I define

$$\Xi_t \equiv \sqrt{\mathrm{var}_t\left(\frac{\mathrm{d}S^{\mathrm{e}}_{f,t}}{S^{\mathrm{e}}_{f,t}}\right)} = \sqrt{\left[\nu_{c,t}\frac{\phi}{x_t}\digamma_{\mathbf{k}}(x_t, \nu_t)\right]^2 + \left[\mathcal{I}_{\alpha}(\nu_{g,t}/\xi_t)\frac{\phi}{x_t}\digamma_{\mathbf{s}}(x_t, \nu_t)\right]^2}.$$

*Marginal value gap.* In complete market, the marginal value of wealth for agents should be identical. Thus, the gap between two marginal values of wealth can serve as a index for risk sharing imperfection (i.e.

the inverse of risk sharing condition). I define

$$\Theta_t \equiv \log(\zeta_t^{\mathrm{e}}) - \log(\zeta_t^{\mathrm{h}}). \tag{32}$$

The quantity is called marginal value gap. It is obvious that $\Theta_t$ in always nonnegative in the equilibrium. This is because experts have the access to investing in assets in place and growth options in the spot capital market, whereas households are excluded from such investment opportunities. As a result, experts always get more utility per unit of net worth than households. Thus, in equilibrium, it holds that $\Theta_t \equiv \log(\zeta_t^{\mathrm{e}}) - \log(\zeta_t^{\mathrm{h}}) \geq 0$.

Figure 5 illustrates the consumption dispersion and marginal value gap under the calibration summarized in Table 1. The uncertainty shocks deteriorate the risk sharing condition by increasing the consumption dispersion and the marginal value gap. The effect of uncertainty shocks on the consumption dispersion increases nonlinearly as the risk sharing becomes more limited (i.e., $x_t$ decreases). The reason is that the expert's net worth has larger exposure to the idiosyncratic shocks when $x_t$ is lower. This is particularly true for growth uncertainty shocks' impact on consumption dispersions.

**Imperfect risk sharing on uncertainty shocks: from an optimal portfolio perspective.** When uncertainty rises, the idiosyncratic risk premia go up. However, experts are the only ones who can take advantage of the higher idiosyncratic risk premia by investing more in real assets. As a result, experts' investment environment deteriorates relative to households. Therefore, the risk sharing between experts and households is endogenously imperfect due to the incomplete market faced by experts. The imperfect risk sharing on uncertainty shocks can be seen from the optimal portfolio holdings of households, which deviate from the market portfolio by significant hedging components. More precisely, each household's portfolio holding can be characterized by $\left(\varphi\sigma, \varsigma_{n,t}^{\mathrm{h},(\nu_{g,t},\nu_g)}, \varsigma_{n,t}^{\mathrm{h},(\nu_{c,t},\nu_c)}\right)$. The risk sharing on $Z_t$ is perfect and thus the optimal holding is the market portfolio or the myopic component. However, the optimal exposure to uncertainty shocks features significant hedging components with the following analytical expressions:

$$\varsigma_{n,t}^{\mathrm{h},(\nu_t,\nu)} = \underbrace{\varsigma_t^{Q,(\nu_t,\nu)}}_{\text{market portfolio}} + \underbrace{x_t \frac{\gamma - 1}{\gamma} \varsigma_t^{\Theta,(\nu_t,\nu)}}_{\text{hedging for relative environment}},$$

where $\varsigma_t^{\Theta,(\nu_t,\nu)}$ is the effect of uncertainty shocks on the marginal value gap $\Theta_t$. Under the benchmark calibration, it holds that $\varsigma_t^{\Theta,(\nu_g^L,\nu_g^H)} > 0$ and $\varsigma_t^{\Theta,(\nu_c^L,\nu_c^H)} > 0$. It means that higher growth uncertainty or cash-flow uncertainty compromises the relative investment environment of households. If $\gamma > 1$ (intra-temporal wealth effect dominates), households have hedging motives to rising uncertainty; that is, $x_t \frac{\gamma - 1}{\gamma} \varsigma_t^{\Theta,(\nu_g^L,\nu_g^H)} > 0$ and $x_t \frac{\gamma - 1}{\gamma} \varsigma_t^{\Theta,(\nu_c^L,\nu_c^H)} > 0$.

There are three points that are worth mentioning. First, when $x_t$ is large and thus the risk sharing is high, the hedging components for the relative investment environment are almost gone because $\varsigma_t^{\Theta,(\nu_t,\nu)} \approx 0$ for all $\nu_t, \nu$. Thus, the optimal holdings go back to the market portfolio $\varsigma_t^{Q,(\nu_t,\nu)}$. In the extreme where the risk sharing is perfect, households only hold the market portfolio. Second, hedging motives mirrored in the framework of inter-temporal capital asset pricing (ICAPM) models (e.g. Merton, 1973*a*; Campbell, 1993), the equity risk premium and the value premium are theoretically and quantitatively accounted by the covariance between stock returns and relative investment environment of experts $\Theta_t$ which is negatively driven by uncertainty shocks. Third, the market portfolio $\varsigma_t^{Q,(\nu_t,\nu)}$ does not only contain the myopic component but also hedging component, since uncertainty shocks affect the overall investment environment.

**Displacement risks: why growth uncertainty can be so fearful?** Compared to cash-flow uncertainty shocks, higher growth uncertainty causes an additional risk to experts, the risk of increasing inequality in the distribution of innovation benefits from growth options. The skewness in the distribution of innovation benefits matters when the risk sharing on idiosyncratic investment shocks is limited. Thus, this risk becomes particularly devastating when risk sharing condition is poor. The intuition can be seen clearly from the impulse-response analysis illustrated in Figure 6. Panel A shows the growth uncertainty shock that hits the economy. It is a temporal shock with half life 1.6 years. Panels B, C, and D show the responses of experts' aggregate consumption, experts' consumption dispersion, and the median of the cross-section of experts' consumption shares, respectively. The blue solid curve is the response when the risk sharing condition is good, while the red dashed curve is the response when the risk sharing condition is poor. In Panel A, it is clear that the growth uncertainty shock works as an investment shock which transfer current consumption to future with a larger amount when the risk sharing is not limited; the growth uncertainty shock destroys consumption and it takes a long time to recover when the risk sharing is seriously limited. In Panel C, the variance of the cross-section of experts' consumption shares increases dramatically with the growth uncertainty shock when risk sharing is limited; they are almost not affected when risk sharing is not limited. Panel D shows that the cross-sectional distribution of experts becomes permanently more skewed, though the conditional cross-sectional variance of consumption share growth comes back to the original level quickly. The distribution is extremely skewed when the risk sharing is limited. However, the distribution does not matter for the equilibrium. It is a manifestation of the skewed wealth transfer among experts. The reason for the skewed wealth transfer is that when risk sharing is limited, experts cannot efficiently insure the idiosyncratic risks in investment opportunities. Thus, most of the benefits from innovation accrue to a small fraction of experts, while the majority of experts bear the cost of creative destruction since they need to pay for the new assets in place. Essentially, the wealth is reallocated from those who do not invest to those who receive high-quality investment opportunities. This reallocation becomes more skewed when growth uncertainty is high, because of asymmetric benefits of growth options. In other words, each expert faces a more skewed idiosyncratic investment risk. Because experts are risk-averse, the higher skewness of idiosyncratic risk decreases the expert's certainty-equivalent wealth. The displacement risk interacting with financial constraints is amplified.

## 5 Empirical Evidence

In this section, I analyze the model's key predictions on the asset pricing implications of uncertainty shocks in the data, using empirical measures of growth uncertainty shocks and cash-flow uncertainty shocks. In Section 5.1, I empirically construct both growth uncertainty and cash-flow uncertainty based on idiosyncratic stock return volatilities in a panel of U.S. public firms. In Section 5.2, I provide an alternative measurement of growth uncertainty and cash-flow uncertainty based on the time-varying cross-sectional dispersion of fundamental cash flows and investments, respectively. I show that the two alternative approaches produce coherent measurements of uncertainty, as predicted by the model. In Section 5.3, I examine the role of risk sharing condition for determining the securities' exposures to two kinds of uncertainty shocks. I explore whether time-varying cross-sectional heterogeneous risk exposures to growth uncertainty shocks and cash-flow uncertainty shocks can rationale the observed differences in average returns between value and growth firms.

### 5.1 Measuring Uncertainty Based on Idiosyncratic Stock Returns

**The idiosyncratic return volatility on assets in place.** I denote $\mathrm{d}\tilde{R}^{\mathbf{k}}_{f,t}$ as the instantaneous return on the equity of assets in place. The return is exposed to all three aggregate shocks in the economy with the risk loadings determined in equilibrium. Its conditional expected return $\mathbb{E}_t\left[\mathrm{d}\tilde{R}^{\mathbf{k}}_{f,t}\right]$ is determined by these risk loadings and market price of risk for the aggregate shocks according to the beta pricing rule in Equations (30) and (31). The equity return exposes to the idiosyncratic cash flow shock $\mathrm{d}W_{f,t}$. In the model, the idiosyncratic equity return is captured by the term $\nu_{c,t}\mathrm{d}W_{f,t}$. The instantaneous equity return on assets in place is characterized as follows:

$$\mathrm{d}\tilde{R}^{\mathbf{k}}_{f,t} = \underbrace{\mathbb{E}_t\left[\mathrm{d}\tilde{R}^{\mathbf{k}}_{f,t}\right]}_{\text{expected return}} + \underbrace{\varphi\sigma\mathrm{d}Z_t + \sum\nolimits_{\nu\neq\nu_t}\varsigma^{q,(\nu_t,\nu)}\left(\mathrm{d}N_t^{(\nu_t,\nu)} - \lambda^{(\nu_t,\nu)}\mathrm{d}t\right)}_{\text{aggregate return risk}} + \underbrace{\nu_{c,t}\mathrm{d}W_{f,t}.}_{\text{idiosyncratic return risk}}$$

It is obvious that the idiosyncratic volatility of $\mathrm{d}\tilde{R}^{\mathbf{k}}_{f,t}$ is simply $\nu_{c,t}$. That is, if I denote, by $\sigma_{\mathbf{k},t}$, the idiosyncratic volatility of equity return on assets in place, the following relationship holds

$$\sigma_{\mathbf{k},t} \equiv \mathrm{ivol}_t\left(\mathrm{d}\tilde{R}^{\mathbf{k}}_{f,t}\right) = \nu_{c,t}. \tag{33}$$

The relation in (33) shows that the cash-flow uncertainty $\nu_{c,t}$ can be identified and measured by the idiosyncratic volatility of equity return on assets in place $\sigma_{\mathbf{k},t}$.

**The idiosyncratic return volatility on growth options.** I denote $\mathrm{d}\tilde{R}^{\mathbf{s}}_{f,t}$ as the instantaneous return on the equity of growth options. The return is exposed to all three aggregate shocks in the economy with the risk loadings determined in equilibrium. Its conditional expected return $\mathbb{E}_t\left[\mathrm{d}\tilde{R}^{\mathbf{s}}_{f,t}\right]$ is determined by these risk loadings and market price of risk for the aggregate shocks according to the beta pricing rule in Equations (30) and (31). The equity return exposes to the idiosyncratic investment shock $\pi_{f,t}\mathrm{d}N_t - \mathbb{E}^{\varepsilon}(\pi_{f,t})\lambda\mathrm{d}t$ which compounds the idiosyncratic investment opportunity shock $\mathrm{d}N_{f,t}$ with the idiosyncratic IST shock $\varepsilon_{f,t}$. In the model, the instantaneous equity return on growth options is

$$\mathrm{d}\tilde{R}^{\mathbf{s}}_{f,t} = \underbrace{\mathbb{E}_t\left[\mathrm{d}\tilde{R}^{\mathbf{s}}_{f,t}\right]}_{\text{expected return}} + \underbrace{\varphi\sigma\mathrm{d}Z_t + \sum_{\nu\neq\nu_t}\varsigma^{p,(\nu_t,\nu')}\left(\mathrm{d}N_t^{(\nu_t,\nu)} - \lambda^{(\nu,\nu)}\mathrm{d}t\right)}_{\text{aggregate return risk}} + \underbrace{\left(\pi_{f,t}\mathrm{d}N_{f,t} - \mathbb{E}^{\varepsilon}(\pi_{f,t})\lambda\mathrm{d}t\right)}_{\text{idiosyncratic return risk}}.$$

Denote, by $\sigma_{\mathbf{s},t}$, the idiosyncratic equity return volatility of growth options. It is defined as follows:

$$\sigma^2_{\mathbf{s},t} \equiv \mathrm{ivol}_t\left(\mathrm{d}\tilde{R}^{\mathbf{s}}_{f,t}\right)^2 = \mathrm{var}_t\left[\pi_{f,t}\mathrm{d}N_{f,t} - \mathbb{E}^{\varepsilon}(\pi_{f,t})\lambda\mathrm{d}t\right]/\mathrm{d}t, \tag{34}$$

where $\mathrm{var}_t\left[\pi_{f,t}\mathrm{d}N_t - \mathbb{E}^{\varepsilon}(\pi_{f,t})\lambda\mathrm{d}t\right]$ is the variance of $\pi_{f,t}\mathrm{d}N_t - \mathbb{E}^{\varepsilon}(\pi_{f,t})\lambda\mathrm{d}t$. In fact, under the model's assumptions, the idiosyncratic volatility can be expressed in terms of economically meaningful variables. I summarize this result in Proposition 7.

**Proposition 7** (Identification of Growth Uncertainty)**.** *Suppose $\alpha \in (0,1)$. Then, the idiosyncratic volatility of equity returns on growth options can be expressed in the following form:*

$$\sigma_{\mathbf{s},t} = \mathcal{I}_{\alpha}(\nu_{g,t}/\xi_t), \tag{35}$$

*where $\mathcal{I}_{\alpha}(\cdot)$ is a deterministic function which is strictly increasing.*

The shock in the growth uncertainty $\nu_{g,t}$ can be captured by

$$\Delta\log(\nu_{g,t}) = \Delta\log\left[\mathcal{I}_{\alpha}^{-1}(\sigma_{\mathbf{s},t})\right] + \Delta\log(\xi_t). \tag{36}$$

According to (6), the second term on the right hand of (36) can be expressed as follows

$$\Delta\log(\xi_t) = (1-\alpha)\Delta\log(\hat{q}_t/\hat{p}_t) + \alpha\Delta\log(\hat{\tau}_t/\hat{q}_t). \tag{37}$$

The variables in (36) and (37) can all be approximated empirically. The volatility of $\Delta\log(\xi_t)$ is smaller than that of $\Delta\log\left[\mathcal{I}_{\alpha}^{-1}(\sigma_{\mathbf{s},t})\right]$ by an order of magnitude in the data.[35] Thus, approximately, the growth uncertainty shocks can be constructed based only on the idiosyncratic equity return volatility $\sigma_{\mathbf{s},t}$:

$$\Delta\log(\nu_{g,t}) \approx \Delta\log\left[\mathcal{I}_{\alpha}^{-1}(\sigma_{\mathbf{s},t})\right]. \tag{38}$$

**Methodology.** Guided by the implications of the model above, I construct two sources of uncertainty shocks using the cross section of asset returns. The model suggests that the cash-flow uncertainty and the growth uncertainty are identified by the idiosyncratic volatilities of equity returns on assets in place

[35] I approximate $\sigma_{\mathbf{s},t}$ by using the lowest 10% U.S. public firms according to their book-to-market ratios. Their annual returns are available from Ken French's web site and the annual idiosyncratic return volatility can be constructed as in (39). The valuation ratio between assets in place and growth options $\hat{q}_t/\hat{p}_t$ can be approximated by using annual Compustat data. More precisely, I follow Fazzari, Hubbard and Petersen (1988) and Kaplan and Zingales (1997). The relative price of investment goods is measured by the relative price of new equipments as in Greenwood, Hercowitz and Krusell (1997, 2000), Cummins and Violante (2002), and Papanikolaou (2011). Specifically, I follow the method adopted by Israelsen (2010) to extend the quality-adjusted relative price of investment goods proposed by Gordon (1990) and Cummins and Violante (2002) to 2014.

(Equation (33)) and on growth options (Equation (38)), respectively. However, there is no equivalent to an equity on pure growth options or an equity on pure assets in place in the data. So, I appeal to stock returns of firms with low and high book-to-market ratios to approximate the equity returns on growth options and assets in place, respectively. An advantage of these measures is that they are available at high frequencies since they are based on financial data.

To be more precise, I sort firms into 10 portfolios on the basis of book-to-market ratios.[36] The basic idea is to use a firm's book-to-market ratio as an inverse measure of growth opportunities held by the firm. This idea follows the conventional wisdom that the market value of growth options is accounted in the market value of the firm, but not in the book value of assets. As a result, a firm's book-to-market ratio should be negatively associated with the firm's value of growth options relative to its total value. In a recent paper by Kogan and Papanikolaou (2014), the authors empirically validate the book-to-market ratio by comparing it with an alternative empirical measure of growth opportunities based on return sensitivity to investment-specific technology shocks. The break points for sorting firms are based on New York Stock Exchange deciles of book-to-market ratios, provided on Ken French's web site.

I first extract the idiosyncratic component of log returns. For each firm, the idiosyncratic component is constructed for every month. More precisely, to obtain the idiosyncratic component of firm $f$ within the month $t_m$,[37] I appeal to the following factor model

$$r_{f,t_d} - r_{t_d} = a_{f,t_m} + \beta_{f,t_m}^T \mathbf{F}_{t_d} + \epsilon_{f,t_d}, \tag{39}$$

where $\mathbf{F}_{t_d}$ denotes the vector of factors and $t_d$ denotes a daily observation in the past $L_m$ months indexed by $t_m, t_m - 1, \cdots, t_m - L_m + 1$. The idiosyncratic component of the log return is estimated by the regression residual $\epsilon_{f,t_d}$ in (39). In the benchmark case, I choose $L_m = 3$.[38] Also, I use Fama and French (1993) three factors for the factor structure, following the standard in literature. Robustness check shows that the main results are not altered if using market returns, four factors in Carhart (1997), or principal components similar to Herskovic et al. (2014).

The idiosyncratic volatility of firm $f$'s stock return in month $t_m$ is then calculated as the standard deviation of residuals $\epsilon_{f,t_d}$ within that month, not including residuals in months $t_m - L_m + 1, \cdots, t_m - 1$. However, the standard deviation of $\epsilon_{f,t_d}$ is the idiosyncratic volatility of leveraged stock returns instead of all-equity returns as in the model; in other words, it is an amplified volatility by firm's financial leverage. More precisely, the underlying idiosyncratic shock to the value of firm's assets is amplified by a factor of the leverage ratio and pass through to the idiosyncratic equity returns. So the idiosyncratic volatility of stock returns is amplified by a factor of the leverage ratio. The leverage ratio is constructed by the sum of the

[36] The book-to-market ratio is computed using book equity and market capitalization constructed from Compustat items. I strictly following Fama and French's methodology.

[37] A firm-month return observation is included if (i) the stock has CRSP share code 10 or 11, and (ii) the firm has at least 17 return observations within the month, and (iii) the firm has no missing returns for the past 36 months.

[38] The empirical results are robust to alternative choices $L_m = 1, 2, 4, 6$. The way I constructed idiosyncratic returns are similar to the approach used in Herskovic et al. (2014) except several divergences. First, we have different frequencies. I focus on monthly idiosyncratic volatilities, whereas they construct annually idiosyncratic volatilities. Second, my factor regressions have overlapping rolling windows, while theirs rolling windows for factor regressions do not overlap.

book value of debt and the market value of equity divided by the market value of equity, similar to Welch (2004).[39] To construct the idiosyncratic volatility of the all-equity firm's returns, I need to adjust $\text{std}(\epsilon_f)$ by the leverage ratio. As a result, a panel of firm-month idiosyncratic volatility estimates for all-equity firms are obtained. Building on this firm-month panel, I construct two monthly time series, denoted by $\sigma_{\mathbf{k},t_m}$ and $\sigma_{\mathbf{s},t_m}$.

For constructing the series $\sigma_{\mathbf{k},t_m}$, I use the equal-weighted average of the idiosyncratic volatilities of those firms whose book-to-market ratios lie in the top 30% quantiles.[40] On the other hand, for the series $\sigma_{\mathbf{s},t_m}$, I use the idiosyncratic volatilities of those firms whose book-to-market ratios lie within the bottom three deciles. I first compute the average idiosyncratic volatilities within each category. I then regress each of the three series of logged average idiosyncratic volatilities onto the series $\log(\sigma_{\mathbf{k},t_m})$. The first principle component of the three residual series accounts for over 76% of their total variation. I use the first principle component to construct $\log(\sigma_{\mathbf{s},t_m})$.

Eventually, I construct an index tracking time-variation of the cash-flow uncertainty $\nu_{c,t_m}$ by the series $\log(\sigma_{\mathbf{k},t_m})$ with the constant level and linear trend taken out.[41] An index of tracking the growth uncertainty $\nu_{g,t_m}$ can be constructed by using $\log(\sigma_{\mathbf{s},t_m})$ with the constant level and linear trend taken out. It is quite intuitive why we need to take out the effect of cash-flow uncertainty from the average idiosyncratic volatility of low book-to-market firm stock returns. It is simply because their idiosyncratic volatilities are inevitably affected by cash-flow uncertainty shocks.

**Results.** The quarterly uncertainty indices, denoted by $\nu_{g,t_q}$ and $\nu_{c,t_q}$, and annually uncertainty indices, denoted by $\nu_{g,t_y}$ and $\nu_{c,t_y}$, are simply defined as the average of monthly uncertainty indices within each quarter and each year, respectively. Figure 7 illustrates the time variation of the cash-flow uncertainty annual index and the growth uncertainty annual index. The segments represent estimated high/medium/low regimes of uncertainties. The levels of the segments in the plots are the estimated average levels of uncertainty within each regime. The regimes and levels are estimated using regime-switching models (e.g., Hamilton, 1989, 1994; Timmermann, 2000).[42] Here, I employ the simplest regime-switching model specification for the cash-flow uncertainty:

$$\nu_{c,t_y} = a(\omega_{c,t_y}) + \epsilon_{c,t_y} \tag{40}$$

where $\omega_{c,t_y}$ is a latent state variable that follows a Markov chain jumping over time and the constant $a(\omega_{c,t_y})$ characterizes the average uncertainty level within each regime. The residual $\epsilon_{c,t_y}$ is assumed to be

[39] If the leverage ratio is missing from Compustat, I use the overall average leverage ratio in the same category as an approximation.

[40] I also use the first principle component of the three average idiosyncratic volatilities for the firms with the highest 10%, with the second highest 10%, and with the third highest 10% book-to-market ratios. The first principle component accounts for over 73% of the total variation. It leads to almost the same result for the approximation of $\nu_{c,t}$.

[41] In order to focus on business-cycle behaviors, it is important to get rid of the long-run increasing trend in firm-level idiosyncratic volatilities of stock returns. There are indeed significant long-run increasing trends in both $\sigma_{g,t_m}$ and $\sigma_{c,t_m}$, consistent with empirical results in Campbell et al. (2001).

[42] The regime-switching model has not only proved its success in macroeconomics, but also been widely adopted in asset pricing and financial portfolio research (e.g., Ang and Bekaert, 2002; Dai, Singleton and Yang, 2007).

uncorrelated with the latent state variable $\omega_{c,t_y}$. This simplest regime-switching model basically provides a time-series clustering analysis, in which a particular year is classified as a high/medium/low regime when its likelihood of being in that regime is larger than 50%. This simple clustering analysis helps us with a better understanding of the uncertainty dynamics. The estimated regimes and their transition probabilities are useful in calibrating of the model's transition matrices of the Markov chains of uncertainty levels. The regime-switching model is estimated using the EM algorithm which maximizes the marginal likelihood of observable variables. For the growth uncertainty, its regime shifts are estimated similarly as in the specification (40). The point estimation of Markov transition probabilities are summarized in Table 6.

It is observed from Table 6 that the high growth uncertainty regime is more persistent compared with the high regime of cash-flow uncertainty. The conditional probability of staying in high state is 81.9% for the growth uncertainty and is 67.5% for the cash-flow uncertainty. Also, in the long run, the growth uncertainty stays in the high state much more often than the cash-flow uncertainty (47.6% versus 24.9%). The growth uncertainty on average lasts longer in the high state, because it is usually associated with political unstable periods, technological revolutions, and energy supply shifts; the resolution of the uncertainty about those events typically takes a long period.[43] Table 6 provides targets for the calibrations of $\mathcal{Q}_c$ and $\mathcal{Q}_g$ in Table 1.[44]

## 5.2 Over-identifications of Uncertainty: Idiosyncratic Dispersions

Because the idiosyncratic volatilities of value firm returns and growth firm returns proposed above are new measures for the cash-flow uncertainty and the growth uncertainty, it is important to first establish the validity of these measures. Specifically, by the definitions of two kinds of uncertainties, I derive more direct measures for the two kinds of uncertainties based on the cross-sectional distribution of idiosyncratic shocks in firm-level sales and capital expenditures. Particularly, the idiosyncratic dispersion in log sales growth rates should provide an ideal measure for the cash-flow uncertainty, while the idiosyncratic dispersion in log investment rates should provide an ideal measure for the growth uncertainty. The two measures based on the idiosyncratic dispersions strictly follow the formal definitions of the two sources of uncertainties; they are also consistent with the model's implications. However, the idiosyncratic-dispersion-based measures

[43]The episodes of high growth uncertainty in the 1950s are mainly due to the fact that the 1950s was the first decade of post-war era and the starting decade of the Cold War. The international and domestic political uncertainty stayed very high for U.S. over the period. The episode of high growth uncertainty around 1970 is due to a major technological revolution in history (e.g., Perez, 2002). As the time approached the end of the 1960s, the old industries of oil, automobiles and mass production became matured, and new industries of information technology and telecommunications began to take the place from 1971. The episode of high growth uncertainty starting from the end of 1970s and lasting until the mid-1980s is mainly due to the long-lasting high oil price volatility (e.g., Peter Ferderer, 1996; Jo, 2012). The high oil price volatility was triggered by Iranian revolution from late 1978 to 1979. The Iranian Revolution, which began in late 1978, resulted in a drop of 3.9 million barrels per day of crude oil production from Iran and a large drop of oil supply from OPEC from 1978 to 1981. In early 1981, the U.S. Government responded to the oil crisis by removing price and allocation controls on the oil industry, which made oil prices more volatile. The episode of high growth uncertainty in the late 1990s is the result of the internet revolution. In the mid-1990s, the civilian Internet was transformed from a military safety net. At that time, the enormous potential of the internet to change all other industries and businesses aroused great growth uncertainty.

[44]The details for the calibration of $\mathcal{Q}_c$ and $\mathcal{Q}_g$ based on the estimated transition probabilities of uncertainty states can be found in the online appendix.

are only available at low frequencies (annual or quarterly) for a period of fifty years.[45] Now, I statistically verify whether the shocks constructed using the idiosyncratic volatility of stock returns validly serve as proxies for the uncertainty shocks from two different origins.

**Idiosyncratic dispersions of sales growth rates.** The sales intensity of firm $f$ over $[t, t + \mathrm{d}t)$ is $y_{f,t}$. According to Proposition 1, the sales is linear in firm's assets in place: $y_{f,t} = y(w_t)k_{f,t}$. Because the equilibrium wage $w_t$ only depends on the aggregate state variables, it readily leads to the dynamics of log sales growth rates:

$$\mathrm{d}\log(y_{f,t}) = \underbrace{\mathrm{d}\log\left(y(w_t)\right) - \delta \mathrm{d}t + \sigma \mathrm{d}Z_t}_{\text{only depending on aggregate shocks}} + \underbrace{\nu_{c,t}\mathrm{d}W_{f,t}}_{\text{idiosyncratic shocks}}. \tag{41}$$

The only source of the cross-sectional heterogeneity comes from the idiosyncratic shocks $\nu_{c,t}\mathrm{d}W_{f,t}$. Thus, the interdecile range (IDR) in the cross section of log sales growth rates implied by the model is

$$\mathrm{IDR}\left[\mathrm{d}\log(y_{f,t})\right] = \aleph \nu_{c,t}, \tag{42}$$

where $\aleph$ is a universal constant that is approximately $\aleph \approx 2.5633$. Therefore, the cross-sectional dispersion in sales growth rates naturally identifies the cash-flow uncertainty, which basically justifies the name of such kind of uncertainty.

**Idiosyncratic dispersions of investment rates.** The firm-level investment rate, normalized by the aggregate investment rate, has the following expression in the model:

$$\frac{\tau_t g_{f,t}}{q_t k_{f,t}} / \frac{\tau_t g_t}{q_t k_t} = \lambda^{-1} \overline{\Gamma}_\alpha(\xi_t/\nu_{g,t})^{-1} \left(\frac{\varepsilon_{f,t}}{\nu_{g,t}}\right)^{\frac{1}{1-\alpha}} \mathbf{1}_{(\varepsilon_{f,t} \geq \xi_t)},$$

where $\frac{\tau_t g_{f,t}}{q_t k_{f,t}}$ is the firm-level capital expenditure normalized by tangible capital stock (the firm-level investment rate) and $\frac{\tau_t g_t}{q_t k_t}$ is the aggregate investment rate. The source of the cross-sectional heterogeneity comes from the idiosyncratic IST shock $\varepsilon_{f,t}$. The cross-sectional standard deviation (CSD) of idiosyncratic shocks in investment rates is characterized by a strictly increasing function of $\nu_{g,t}/\xi_t$. This is formally summarized in the following proposition with proofs given in the online appendix.

**Proposition 8** (Growth Uncertainty versus Dispersions of Investment Rates)**.** *In equilibrium, the dispersion of idiosyncratic shocks in investment rates depends on $\nu_{g,t}$ positively. That is,*

$$CSD\left[\frac{\tau_t g_{f,t}}{q_t k_{f,t}} / \frac{\tau_t g_t}{q_t k_t}\right] = \lambda^{-1} \mathcal{J}_\alpha(\nu_{g,t}/\xi_t),$$

*where $\mathcal{J}_\alpha(\cdot)$ is a deterministic strictly increasing function.*

[45] I construct them from Compustat datasets. For quarterly frequency idiosyncratic dispersions, the time series are only available as early as 1984 which gives about 30 year data.

Similarly, the exercising boundary $\xi_t$ is relatively much stable compared to $\nu_{g,t}$. As a result, the cross-sectional standard deviation CSD $\left[\frac{\tau_t g_{f,t}}{q_t k_{f,t}} / \frac{\tau_t g_t}{q_t k_t}\right]$ also provides an (approximate) identification for the growth uncertainty $\nu_{g,t}$.

**Methodology.** Now, I extract idiosyncratic shocks in sales growth rates and in investment rates. Once that is done, I compute their dispersions in the cross section of firms. In this empirical exercise of extracting the idiosyncratic unexpected component, I adopt the method similar to Purnanandam and Rajan (2014) in which the predictable component, the aggregate unexpected component, and the idiosyncratic unexpected component are statistically separated and estimated by using dynamic panel regression models (e.g., Holtz-Eakin, Newey and Rosen, 1988; Arellano and Bond, 1991).

I first measure the idiosyncratic unexpected component of firm's investment rates. For each firm $f$, the investment rate of year $t_y$, denoted by $IoK_{f,t_y}$, is computed as the capital expenditure $CapEx_{f,t_y}$ deflated by capital stock of tangible assets $K_{f,t_y-1}$ in the previous year, and then normalized by the aggregate investment-to-capital ratio $IoK_{t_y-1}$.[46] That is,

$$IoK_{f,t_y} \equiv \left(CapEx_{f,t_y}/K_{f,t_y-1}\right)/IoK_{t_y-1}.$$

Here, the capital expenditure of firm $f$ within year $t_y$ is measured by the Compustat item capx, and the capital stock of tangible assets is measured by the Compustat item ppent. In this empirical exercise, I use the following regression model to extract the idiosyncratic shock in $IoK_{f,t_y}$:

$$IoK_{f,t_y} = \underbrace{\lambda_{capx,t_y}}_{\text{agg. component}} + \underbrace{a_{capx,f} + \beta_{capx,1} IoK_{f,t_y-1} + \beta_{capx,2} CoK_{f,t_y-1} + \beta_{capx,3} MoB_{f,t_y-1}}_{\text{firm-level expected component}} + \underbrace{\epsilon_{capx,f,t_y}}_{\text{idio. shock}},$$

where $a_{capx,f}$ is the fixed effect capturing the firm-level predictability, $\lambda_{capx,t_y}$ is the year effect capturing aggregate time-varying effect (can be caused by some latent aggregate factors) and captures the aggregate shock, $CoK_{f,t_y}$ is the cash flow deflated by capital stock of tangible assets in the previous year, and $MoB_{f,t_y}$ is the market-to-book ratio of assets capturing the investment opportunity of firm $f$ in year $t_y$. The variables $CoK_{f,t_y}$ and $MoB_{f,t_y}$ are needed, particularly because the literature of the cash-flow-sensitivity of investment argues that cash flows can have impact on investment decisions. Though there are different ways to measure $CoK_{f,t_y}$ and $MoB_{f,t_y}$ in the data, my measures follow the cash-flow-sensitivity of investment literature (e.g., Fazzari, Hubbard and Petersen, 1988; Kaplan and Zingales, 1997).[47]

[46] In order to extract the idiosyncratic volatility of investment rates that only caused by the growth uncertainty $\nu_{g,t}$ (i.e. the idiosyncratic volatility), I need to remove the scaling effect time-varying volatility of aggregate investment rates. More precisely, the regression needs to make sure that the heteroskedasticity in the aggregate volatility of investment rate shocks does not alter the idiosyncratic shock $\epsilon_{capx,f,t_y}$ specified in the econometric model. Bachmann, Caballero and Engel (2006) show that the volatility in the aggregate investment rate ($IoK$) is high when the past aggregate investment rate is high. So, I normalize the firm-level investment rate by the aggregate one; it serves as the simplest way to guarantee the idiosyncratic shock $\epsilon_{capx,f,t_y}$ not to be affected by the past aggregate investment rates through current aggregate volatility. This normalization is also consistent with the implications of the model in Proposition 8. I follow Bachmann, Caballero and Engel (2006) and Favilukis and Lin (2013) to construct the aggregate investment rate using nominal annual private fixed nonresidential investment and the annual private nonresidential capital stock at year-end prices from the Bureau of Economic Analysis (BEA).

[47] More precisely, to construct $CoK_{f,t_y}$, the cash flow is measured by the sum of income before extraordinary items

The residuals $\epsilon_{capx,f,t_y}$ in the regression model above capture the idiosyncratic shock in investment rates. The result of this procedure is a firm-year panel of idiosyncratic shocks $\epsilon_{capx,f,t_y}$. I estimate the dynamic panel regression model using the GMM estimator proposed by Holtz-Eakin, Newey and Rosen (1988) and Arellano and Bond (1991), with the first lagged value of capital expenditure rate as a GMM instrument variable.

Second, I measure the idiosyncratic unexpected component in sales growth rates. Following the literature (e.g., Bloom, 2009; Bachmann and Bayer, 2014; Herskovic et al., 2014), the sales growth rate is measured as follows:

$$GoS_{f,t_y} \equiv \log\left(Sales_{f,t_y}/Sales_{f,t_y-1}\right).$$

Here, the variable $Sales_{f,t_y}$ is the sales of firm $f$ within year $t_y$ and I use the Compustat item sale for its values. I focus on the following regression model to extract the unexpected idiosyncratic component of $GoS_{f,t_y}$:

$$GoS_{f,t_y} = \underbrace{\lambda_{sale,t_y}}_{\text{agg. component}} + \underbrace{a_{sale,f} + \beta_{sale,1} GoS_{f,t_y-1}}_{\text{firm-level expected component}} + \underbrace{\epsilon_{sale,f,t_y}}_{\text{idio. shock}} \tag{43}$$

where $a_{sale,f}$ is the fixed effect capturing the firm-level predictability, and $\lambda_{sale,t_y}$ is the year effect capturing the aggregate component (even there are latent factors) which includes the aggregate shock. The residuals $\epsilon_{sale,f,t_y}$ captures the idiosyncratic shocks in sales growth rates. The result of this procedure is a firm-year panel of idiosyncratic shocks. Similarly, I estimate the dynamic panel regression model using the GMM estimator with the first lagged value of cash flow rate as a GMM instrument variable.

Now, after obtaining these two firm-year panels of idiosyncratic shocks, I construct two annual time series of cross-sectional dispersions, denoted by $\sigma_{capx,t_y}$ and $\sigma_{sale,t_y}$. The series $\sigma_{capx,t_y}$ are the cross-sectional standard deviations (CSD) of idiosyncratic shocks in investment rates across all firms within year $t_y$ following Bachmann and Bayer (2014), while the series $\sigma_{sale,t_y}$ are the interdecile ranges (IDR) of idiosyncratic shocks in sales growth rates across all firms within year $t_y$. Like the indices based on idiosyncratic stock volatilities, I focus on linearly-detrended series. In particular, I denote $\nu_{capx,t_y}$ and $\nu_{sale,t_y}$ the annual time series $\log\left(\sigma_{capx,t_y}\right)$ and $\log\left(\sigma_{sale,t_y}\right)$ with linear trends removed, respectively.

**Results.** Consistent with the predictions of my model (Equations (33) and (38)), the underlying shocks that drive the idiosyncratic sales dispersions ($\Delta\nu_{sale,t_y} \equiv \nu_{sale,t_y} - \nu_{sale,t_y-1}$) are particularly associated with the cash-flow uncertainty shocks ($\Delta\nu_{c,t_y} \equiv \nu_{c,t_y} - \nu_{c,t_y-1}$), but not with the growth uncertainty shocks ($\Delta\nu_{g,t_y} \equiv \nu_{g,t_y} - \nu_{g,t_y-1}$). On the contrary, as predicted by the model (Propositions 7, and 8), the shocks that drive the idiosyncratic investment dispersions ($\Delta\nu_{capx,t_y} \equiv \nu_{capx,t_y} - \nu_{capx,t_y-1}$) are particularly associated with the growth uncertainty shocks $\Delta\nu_{g,t_y}$, but not the cash-flow uncertainty shocks $\Delta\nu_{c,t_y}$. More precisely, Panel A of Figure 8 shows that the shocks of the idiosyncratic investments dispersions

(Compustat item ib) and depreciation (Compustat item dp), and the capital stock of tangible assets is measured by net property, plant and equipment (Compustat item ppent). To construct $MoB_{f,t_y}$, the market value of assets is measured by the book value of assets (Compustat item at) plus the market value of common stock (Compustat item prccf × Compustat item csho) less the sum of book value of common stock (Compustat item ceq) and balance sheet deferred taxes (Compustat item txdb).

$\Delta\nu_{capx,t_y}$ can be statistically explained by the growth uncertainty shocks $\Delta\nu_{g,t_y}$ with the estimated slope 1.84 and the t-statistic 2.93. Panel B of Figure 8 shows that the shocks of idiosyncratic sales dispersions $\Delta\nu_{sale,t_y}$ cannot be statistically explained by the growth uncertainty shocks $\Delta\nu_{g,t_y}$, because the estimated slope $-0.016$ is not statistically different from zero. Its t-statistic is $-0.028$. Panel C of Figure 8 shows that the shocks of idiosyncratic investment dispersions $\Delta\nu_{capx,t_y}$ cannot be statistically explained by the cash-flow uncertainty shocks $\Delta\nu_{c,t_y}$. The slope is estimated to be $-0.31$ with t-statistic $-0.62$. Panel D of Figure 8 shows that the shocks of idiosyncratic sales dispersions $\Delta\nu_{sale,t_y}$ can be statistically explained by the cash-flow uncertainty shocks $\Delta\nu_{c,t_y}$ with the slope estimated to be 1.11 and its t-statistic to be 2.75.

Therefore, empirical evidence supports using the idiosyncratic volatilities of equity returns on assets in place as a measure of the cash-flow uncertainty and on growth options as a measure of the growth uncertainty. In other words, the uncertainty shocks of different origins, as fundamental macroeconomic shocks, can be identified and measured using panels of asset returns. Importantly, the asset pricing data allows for high frequency proxies for these underlying macroeconomic shocks. From the asset pricing perspective, the results show that the macroeconomic uncertainty shocks can have direct and significant impacts on the cross-sectional behavior of asset returns.

**Discussion: cyclicality of cross-sectional dispersions.** The cyclicality of the idiosyncratic investment dispersion $\nu_{capx,t_y}$ and the idiosyncratic sales dispersion $\nu_{sales,t_y}$, as well as the growth uncertainty $\nu_{g,t_y}$ and the cash-flow uncertainty $\nu_{c,t_y}$, are reported in Table 7. There, the cyclical component of real GDP per capita is estimated by using the one-sided HP filter. There are three points which worth mentioning about the statistics in Table 7. First, consistent with the main findings of Bachmann and Bayer (2014) as reproduced in Table 4, the cross-sectional dispersion of investment rates is statistically significantly pro-cyclical. In fact, the results reported here (the Pearson and Kendall correlations are 0.31 and 0.20, respectively) reinforce theirs. This is because the firm-specific predictable component, the aggregate predictable component, and the potential scaling effect have been all removed when the idiosyncratic sale dispersion $\nu_{sale,t_y}$ and the idiosyncratic investment dispersion $\nu_{capx,t_y}$ are constructed. Second, the dispersion of idiosyncratic shocks in sales growth rates $\nu_{sale,t_y}$ is countercyclical (the Pearson and Kendall correlations are $-0.16$ and $-0.10$, respectively), though annual estimated correlations are not significant (the p-values for Pearson and Kendall correlations are 0.26 and 0.32, respectively).[48] Third, the growth uncertainty $\nu_{g,t}$ is pro-cyclical (the Pearson and Kendall correlations are 0.13 and 0.05, respectively), while the cash-flow uncertainty $\nu_{c,t}$ is strongly countercyclical (the Pearson and Kendall correlations are $-0.31$ and $-0.21$, respectively).

Importantly, my theoretical and empirical results provide a natural and robust reconciliation for the so-called investment dispersion puzzle in the macroeconomics literature (e.g., Bachmann and Bayer, 2014). Basically, I show what has been missing in the macroeconomic models with heterogeneous firms is the growth uncertainty shocks. It's been a substantial literature documenting that the cross-sectional dispersion of micro-level fundamentals vary dramatically over time. In particular, it's been a consensus that the underlying shocks driving dispersions of sales have strongly adverse macroeconomic effects (e.g., Bloom,

[48] The insignificance results can be a result of short sample length and high persistency in the annual level time series.

2009; Bloom et al., 2013; Herskovic et al., 2014). However, in a recent work by Bachmann and Bayer (2014), the authors find pro-cyclical dispersion of firm-level investment in Germany, the United States, and the United Kingdoms. Quantitatively, they examine whether shocks in the dispersion of sales growth rates can generate pro-cyclical investment dispersions; they build their quantitative exercise upon the framework of Khan and Thomas (2008), Bloom (2009) and Bachmann, Caballero and Engel (2013). They show that only very small shocks to sales growth dispersion can generate pro-cyclical investment dispersion, and shocks with such small scales fail to generate observed business cycles. These empirical patterns impose additional cross-equation restrictions on the properties of uncertainty shocks used in macroeconomic models; in particular, they pose quantitative challenges to the uncertainty-driven business cycle models, such as Bloom et al. (2013).

In my model with growth uncertainty shocks, as suggested theoretically and verified empirically, the dispersion of investment rates is mainly driven by the growth uncertainty, but not by the cash-flow uncertainty; on the contrary, the dispersion of sales growth rates is driven by the cash-flow uncertainty, but not by the growth uncertainty. Therefore, my model naturally reproduces the empirical patterns for the dynamics of sales dispersions and investment dispersions.

### 5.3 Inspecting the Mechanism: the Role of Risk Sharing Condition

In this section, I explore the empirical tests of the basic mechanism. The basic mechanism of the model is that the effects of growth uncertainty shocks on asset prices in the cross section are determined by the risk sharing condition in the economy. In particular, I focus on testing two most direct implications of the basic mechanism, which are summarized as follows. The first direct implication (Basic Implication I) is that in response to a positive growth uncertainty shock, the value of growth options increases relative to assets in place when the risk sharing condition is good, but decreases otherwise; the cash-flow uncertainty always tends to suppress the value of assets in place relative to the value of growth options, regardless of the risk sharing condition. The second direct implication (Basic Implication II) is that the growth uncertainty shock carries a positive market price of risk when risk sharing condition is good, but carries a negative one otherwise; the cash-flow uncertainty shock always carries a negative market price.

**Testing basic implication I.** I use the value spread, high minus low book-to-market portfolio returns, to approximate the relative value change of assets in place to growth options in the data. To add robustness of the testing results, I set up three tests using different econometric tools, and I also use three different measures of risk sharing conditions in the economy.

*Regime-switching models.* My model implies that the betas of value spreads with respect to growth uncertainty shocks are informative about the underlying state of risk sharing conditions. More precisely, when the beta of value spreads to growth uncertainty shocks is negative, the underlying risk sharing condition is likely to be good; alternatively, the underlying risk sharing condition is likely to be poor. In order to provide a direct test on this implication, I appeal to the regime-switching econometric model studied by Hamilton (1989, 1994) and Timmermann (2000). In my monthly regime-switching econometric specification, the underlying risk sharing condition is the latent state variable, denoted by $\omega_{x,t_m}$. The latent state variable $\omega_{x,t_m}$ is to be uncovered from the data. It is assumed that $\omega_{x,t_m}$ follows a two-state Markov chain process; it's transition probabilities are to be estimated using the observables in the model. The observables include monthly market excess returns $r_{M,t_m} - r_{f,t_m}$, uncertainty shocks $\Delta\nu_{g,t_m}$ and $\Delta\nu_{c,t_m}$, and monthly value spreads $r_{H,t_m} - r_{L,t_m}$ with $r_{H,t_m}$ and $r_{L,t_m}$ to be returns of high and low book-to-market portfolio, respectively. More precisely, the econometric model is specified as follows:

$$r_{H,t_m} - r_{L,t_m} = a_{v,t_m} + \beta_{v,z,t_m}\left(r_{M,t_m} - r_{f,t_m}\right) + \beta_{v,g,t_m}\Delta\nu_{g,t_m} + \beta_{v,c,t_m}\Delta\nu_{c,t_m} + \epsilon_{v,t_m} \tag{44}$$

where the coefficients are time-varying and depend on the latent state $a_{v,t_m} \equiv a_v\left(\omega_{x,t_m}\right)$ and $\beta_{v,\iota,t_m} \equiv \beta_{v,\iota}\left(\omega_{x,t_m}\right)$ for $\iota \in \{z, g, c\}$. The latent state variable $\omega_{x,t_m}$ takes values in $\{\mathsf{Good}, \mathsf{Bad}\}$. Here, Good (Bad) stands for the state in which the risk sharing condition is good (bad). The state of risk sharing condition is unobservable in the econometric model and the identification implied by the theory is that

$$\beta_{v,g}\left(\mathsf{Good}\right) < \beta_{v,g}\left(\mathsf{Bad}\right). \tag{45}$$

Moreover, statistically, it is assumed that the residual term $\epsilon_{v,t_m}$ is not only uncorrelated with the input variables but also independent of the latent state variable $\omega_{x,t_m}$.

I estimate the regime-switching model (44) using the EM algorithm that maximizes the marginal likelihood function of observables. The estimation results of the regime-switching model consist of two parts: one is the statistical inference about the coefficients which are summarized in Table 8; the other is the estimated likelihood of the risk sharing condition being Bad for every month. The estimated likelihoods are displayed in Figure 9.

In Table 8, Column (3) shows that the loadings of value spreads on growth uncertainty shocks change from negative ($\beta_{v,g}$ is estimated to be $-1.76$) to positive ($\beta_{v,g}$ is estimated to be 6.03) as the underlying state moves from Good to Bad. The signs are statistically significant at 75% confidence level. In the econometric analysis, the only restriction used for identifying the Good state is the inequality (45). There is no restriction imposed on the sign of growth uncertainty beta $\beta_{v,g}$ in the estimation. As a result, the sign switching itself empirically supports prediction of the theoretical model. It should be noted that the significance level of the coefficients tend to be understated compared to the econometric model in which the risk sharing condition is assumed to be known. This is because a large amount of randomness about the latent states have to be taken into account when drawing statistical inferences about the regression coefficients in the regime-switching model. Moreover, Column (4) of Table 8 verifies another prediction of the theory: the growth options always offer a hedge against the cash-flow uncertainty shock. More precisely, the coefficient $\beta_{v,c}$ is estimated to be negative in both states ($-13.08$ in Bad versus $-1.00$ in Good). In particular, the sign is significant at 95% confidence level in Bad and 75% confidence level in Good. However, it is still unclear whether the state Bad in the model truly corresponds to the state of poor risk sharing in the data. Thus, I need to compare the estimated Bad state with the measures of risk sharing conditions in the data.

In fact, the regime-switching econometric model does not offer an exact answer to the question which state the economy is in. Instead, it allows one to estimate the likelihood of the economy being in certain state.[49] In Figure 9, the estimated likelihood of being in Bad state is plotted in Panel D and is compared with three measures of risk sharing conditions in the data. The first empirical measure (in Panel A) is the Reinhart-Rogoff financial crisis index.[50] The second empirical measure (in Panel B) is the financial condition index based on broker-dealer leverages. The third empirical measure (in Panel C) is the credit spread index. Gilchrist and Zakrajsek (2012), Krishnamurthy and Muir (2015), and Ivashina and Scharfstein (2010) show that credit spreads can serve as a crucial gauge of the degree of strains in the financial system. The basic idea is that fluctuations in credit spreads reflect shifts in the effective supply of funds offered by financial intermediaries. They found that an adverse shock to the equity valuations of the highly-leveraged financial intermediaries, relative to the market return, leads to an immediate and persistent increase in credit spreads.

My estimated likelihood of being in Bad state is plotted in Panel D. To interpret the levels of the time series in Figure 9, I set zero as the benchmark state in which the risk sharing condition is at its medium level. According to their definitions, positive index values indicate worse financial conditions than the medium state; negative index values indicate better financial conditions than the medium state. The three indices in Panels A–C capture the periods of stressed financial sector. Figure 9 shows that the Bad state is actually associated with poor financial conditions. Comparing the estimated financial condition (in Panel D) with the Reinhart-Rogoff financial condition index (in Panel A), the broker-dealer leverage index (in Panel B), and the credit spread index (in Panel C), the estimation results (in Panel D) are clearly consistent with the observations in the data (in Panels A, B, and C). More precisely, the four time series capture the major periods of financial stress in the history of the United States; at the same time, they also agree with each other upon the major periods of excellent financial conditions for U.S. economy.[51]

Most importantly, the estimation of financial condition (in Panel D) only depends on stock returns and the model's prediction about the cross-sectional impacts of growth uncertainty shocks. In other words, the estimation has almost zero prior information about financial conditions, which reinforces the power of the empirical result as a support for my theory. Now, I formally quantify the statistical association between the empirical measures of risk sharing conditions and the estimated likelihood of Bad state, which are reported in Table 9. I use both the Pearson correlation and the Kendall rank correlation to quantify the associations. As reported in Columns (1) and (2) of Table 9, the credit spread index is used as the benchmark, and it is significantly correlated with both the Reinhart-Rogoff financial condition index, the

[49] Of course, the state of the economy can be estimated based on the estimated likelihood. In practice, the economy is labeled by a particular state when the estimated likelihood of being that state is higher than a predetermined threshold. For example, 50% is used as the threshold, like in Figure 7.

[50] It is constructed based on U.S. banking/currency crisis, U.S. stock market crashes, U.K. banking/currency crisis, German banking/currency crisis, and France banking/currency crisis. I use a simplest nonlinear filter to form U.S. investors' expectation about financial sector conditions. If there are two or more crisis, investors have a bad outlook for financial conditions; if there is zero crisis, investors form a promising outlook for financial conditions; otherwise, they form a medium outlook.

[51] My estimation, together with the three empirical measures, capture the financial crises around 1976, around 1990, around 2003, and around 2008; they also capture the periods of excellent financial conditions including the late 1990s, the periods around 2005, and the periods after 2014. In the online appendix, I also compare my estimation with other empirical measures of financial conditions including the financial condition index proposed by Brave and Butters (2011).

broker-dealer leverage index, and the estimated likelihood of Bad state. In Columns (3) and (4) of Table 9, I also report the corresponding statistical associations of the financial condition indices in my model based on simulated samples. Because there is no corporate bond in my model, I use the equity premium as the proxy for credit spread. The risk sharing condition in the simulated data is measured by using $\Theta_t$ in (32).[52] The likelihood of Bad is estimated for the simulated data in the same way as for the real data. The associations between the simulated indices are comparable to those in the real data.

*Uncertainty betas of book-to-market sorted portfolios.* I also verify the theoretical implication by looking into the loadings of book-to-market sorted portfolios on uncertainty shocks $\Delta\nu_{c,t_m}$ and $\Delta\nu_{g,t_m}$ in subsamples corresponding to the periods of good or bad financial conditions. I first use the Reinhart-Rogoff index as the measure of risk sharing condition in the economy to construct subsamples. The betas of the book-to-market portfolios with respect to the market excess return $r_{M,t_m} - r_{t_m}$, the growth uncertainty shock $\Delta\nu_{g,t_m}$, and the cash-flow uncertainty shock $\Delta\nu_{c,t_m}$ are estimated within each of the two subsamples: one subsample includes the periods of good financial conditions; the other subsample includes periods of financial stress. The estimated betas are reported in Table 10. Panel A reports the beta estimates when the risk sharing condition is poor, while Panel B reports the beta estimates when the risk sharing condition is good. Comparing Columns (2) and (3) with Columns (5) and (6) in Table 10, the empirical results are almost perfectly in line with the theoretical prediction about the loadings on two sources of uncertainty shocks. More precisely, the beta on $\Delta\nu_{g,t}$ increases from $-3.11$ to $0.21$ for the stock returns of the firms with the lowest 10% book-to-market ratios (growth firms) versus those with highest 10% book-to-market ratios (value firms) when risk sharing condition is bad, as shown in Panel A with the sorting scheme #1; however, the growth uncertainty beta decreases from $2.40$ to $-12.66$ for growth firms versus value firms when risk sharing condition is good, as shown in Panel B with the sorting scheme #1. Moreover, according to the sorting scheme #1, the beta on $\Delta\nu_{c,t_m}$, for the stock returns of growth firms versus value firms, decreases from $2.56$ to $-17.85$ when the risk sharing condition is bad and decreases from $14.10$ to $-8.36$ when the risk sharing condition is good. Importantly, as shown in Figure 10, the empirical findings are robust to various sorted book-to-market portfolios (e.g. the sorting schemes #2 and #3).

I then use the financial condition index based on broker-dealer leverages as the measure of risk sharing condition in the economy. The estimated betas are reported in Table 11. According to the sorting scheme #1, the beta on $\Delta\nu_{g,t_m}$ for growth firms versus value firms increases from $-2.81$ to $1.10$ when the risk sharing condition is bad (in Panel A), while it decreases from $-1.61$ to $-3.74$ when the risk sharing condition is good (in Panel B). Moreover, under the sorting scheme #1, the beta on $\Delta\nu_{c,t_m}$ for growth firms versus value firms decreases from $6.80$ ($11.27$) to $-21.37$ ($-1.34$) when the risk sharing condition is bad (good). The empirical results are robust across various sorting schemes (#2 and #3). Therefore, the results in Table 11 show that the empirical findings in Table 10 are quite robust against other measures of risk sharing conditions.

At last, I use the credit spread index as the measure of risk sharing condition in the economy. I fit the

[52] I can also use the endogenous state variable $x_t$ or the consumption share dispersion $\Theta_t^e$ quantify risk sharing condition in the simulated data, because they are equally valid as the measure of risk sharing condition in my model.

credit spread into a simplest three-state regime-switching model like in (40). The estimation results using 50% to be the threshold is to cluster each quarter into three categories: high/median/low credit spread levels.[53] The estimated betas are reported in Table 12. According to the sorting scheme #1, the beta on $\Delta\nu_{g,t_m}$ for growth firms versus value firms increases from $-2.81$ to 8.08 when the risk sharing condition is bad (in Panel A), while it decreases from 2.37 to $-1.34$ when the risk sharing condition is good (in Panel B). Moreover, under the sorting scheme #1, the beta on $\Delta\nu_{c,t_m}$ for growth firms versus value firms decreases from 0.78 (1.61) to $-10.13$ ($-9.45$) when the risk sharing condition is bad (good). The empirical results are robust across various sorting schemes (#2 and #3). Therefore, the results in Table 12 reinforce that the empirical findings in Table 10 are quite robust against other measures of risk sharing conditions.

However, statistically, it is still unclear how significantly the role of risk sharing conditions in altering the impact of growth uncertainty on the value of growth options relative to assets in place. To investigate the statistical significance, I compute the t-statistics for the estimated betas of extreme book-to-market-sorted portfolios. In Table 13, it shows that the sign changes (Column (1) versus Column (3)) are significant based on one-sample statistical tests; the statistical result is particularly strong when using the credit spread index as the measure for risk sharing conditions (in Panel C).

*Linear models with interaction terms.* Now, I set up a linear regression model in which the dependent variable is the value spread and the independent variables include the interaction terms between the uncertainty shocks and the risk sharing condition. The risk sharing condition is measured by the Reinhart-Rogoff financial condition index (reported in Columns (5) and (6)) or by the financial condition index based on broker-dealer leverages (reported in Columns (7) and (8)) or by the credit spread index (reported in Columns (3) and (4)). The regression model with interaction terms is specified as follows:

$$\begin{aligned} r_{H,t_y} - r_{L,t_y} = a_{vi} &+ \beta_{vi,z}\left(r_{M,t_y} - r_{t_y}\right) + \beta_{vi,g}\Delta\nu_{g,t_y} + \beta_{vi,c}\Delta\nu_{c,t_y} + \beta_{vi,x}\textsf{regime-x}_{t_y} \\ &+ \gamma_{vi,g}\left[\Delta\nu_{g,t_y} \times \textsf{regime-x}_{t_y}\right] + \gamma_{vi,c}\left[\Delta\nu_{c,t_y} \times \textsf{regime-x}_{t_y}\right] + \epsilon_{vi,t_y} \end{aligned} \tag{46}$$

where $vi$ in the subscript of coefficients means that they are coefficients for the value spread regression with interactions. Here, $r_{M,t_y} - r_{t_y}$ is the market excess return, $\Delta\nu_{g,t_y}$ and $\Delta\nu_{c,t_y}$ are uncertainty shocks, and $r_{H,t_y} - r_{L,t_y}$ is the value spread with $r_{H,t_y}$ and $r_{L,t_y}$ to be the returns of high and low book-to-market-portfolio returns, respectively. The variable $\textsf{regime-x}_{t_y}$ is an aggregate state variable characterizing the condition of risk sharing in the economy.

The focus of this test has two folds: one is to test whether the coefficient $\gamma_{vi,g}$ in (46) is significantly positive; the other is to test whether the coefficient $\beta_{vi,c}$ is significantly negative. The regression (46) provides a formal statistical framework for testing whether the switching signs between Column (2) and Column (5) in Tables 10, 11, and 12 are statistically significant. In computing the t-statistics of coefficients, I appeal to Newey and West (1987, 1994) for the robust covariance matrix estimation with one year lag.

[53]The periods of low risk sharing condition (i.e. high credit spread) include 1974Q4 – 1976Q3, 1980Q2 – 1983Q3, 1988Q4 – 1992Q2, 2002Q1 – 2003Q4, 2008Q1 – 2009Q4, 2010Q2 – 2010Q4, and 2011Q4 – 2012Q4. On the other hand, the periods of high risk sharing condition (i.e. low credit spread) include 1973Q1 – 1974Q3, 1977Q3 – 1979Q3, 1987Q3 – 1988Q3, 2000Q1 – 2000Q3, 2004Q4 – 2005Q2, 2004Q4 – 2005Q2, and 2013Q4 – 2014Q3.

In Table 14, Column (1) shows that the value premium exists; it is about 5.65% and statistically significant. More importantly, Column (1) shows that the market excess return fails to explain the value premium, because the intercept term is significantly nonzero and the $F-$statistic is insignificant. Column (2) shows that the uncertainty shocks have large explanatory power for value spreads, since the $F-$statistic for the regression (2) has significance less than 0.5%. It also shows that the impact of cash-flow uncertainty on value spreads is significantly negative, which is consistent with the theoretical prediction of my model. Regression (3) shows that the risk sharing condition helps explain the value spread when it interacts with growth uncertainty shocks. Perfectly in line with the prediction of the model, the coefficient of the interaction term $\Delta\nu_{g,t_y} \times \mathsf{regime\text{-}x}_{t_y}$ is significantly positive, with estimate 407.44 and t-statistic 2.61. Moreover, the adjusted $R^2$ increases from 19.13% to 22.75% from the regression (2), and the intercept term becomes insignificantly positive. In Column (4), I further add in the interaction term between the risk sharing condition and the cash-flow uncertainty shock. The regression results of Column (3) are almost unaffected. The coefficient of the extra interaction term $\Delta\nu_{c,t_y} \times \mathsf{regime\text{-}x}_{t_y}$ is insignificantly positive, with estimate 66.93 and t-statistic 0.72. The regression results in Columns (5)–(6) and the regression results in Columns (7)–(8) show the robustness of the regression results in Columns (3) and (4) when the measure of risk sharing conditions changes to the Reinhart-Rogoff financial condition index and the broker-dealer leverage index displayed in Figure 9, respectively. Furthermore, as shown in Table 14, the loadings of value spreads on cash flow shocks, denoted as $\beta_{vi,c}$ in (46), are significantly negative across all regressions and different measures for risk sharing conditions.[54]

**Testing basic implication II.** To verify the model predictions on stochastic discount factors, I explore the possibility of the cross section of stock returns. Because the prediction is specifically on the market price of risk for the growth uncertainty shock and the cash-flow uncertainty shock, I focus on portfolios of firms' stocks sorted based on the two uncertainty shocks, separately. As long as the loadings of firm stock returns on the uncertainty shocks are fairly persistent, the ex ante differential sensitivity to uncertainty shocks will lead to the ex post differential sensitivity. The differential average returns of sorted portfolios then are informative about these market price of risk associated with the uncertainty shocks.

In Table 15, the average returns for uncertainty-sorted portfolios are reported for the full sample (in Columns (5) and (6)) and two subsamples (in Columns (1) – (4)). One subsample corresponds to the periods of poor risk sharing conditions (reported in Columns (1) and (2)), while the other subsample corresponds to the periods of good risk sharing conditions (reported in Columns (3) and (4)). In Panel A, the risk sharing condition is measured by the Reinhart-Rogoff financial condition index; in Panel B, the risk sharing condition is gauged by the broker-dealer leverage index; in Panel C, the risk sharing condition is gauged by the credit spread index. Across all columns in Table 15, it shows that the firms with a higher exposure to the cash-flow uncertainty shock, on average, gain lower returns; it thus implies that cash-flow uncertainty shocks tend to carry a negative market price of risk, no matter whether the risk sharing condition is good or bad. In particular, over the whole sample, the valuation spread between

[54] They are all significant except regressions in Columns (7)–(8) in which the risk sharing condition is measured by the financial condition index based on broker-dealer leverages.

the firms with a high exposure to the cash-flow uncertainty shock versus those with a low exposure is statistically significantly negative; the spread is $-5.11\%$ with the t-statistic equal to $-3.02$.

However, the firms with a higher exposure to the growth uncertainty shock, on average, gain lower returns when risk sharing is limited (see Columns (1)); in contrast, they gain higher average returns otherwise (see Columns (3)). More precisely, if the Reinhart-Rogoff financial condition index is used as the measure for risk sharing conditions (in Panel A), the spread between high versus low $\nu_g$-sorted portfolios changes from $-2.30$ (with t-statistic $-1.24$) to 1.96 (with t-statistic 0.91) when the risk sharing condition improves. This empirical pattern is robust against different choices of measures of risk sharing conditions. Particularly, if the broker-dealer leverage index is used to construct the regimes of risk sharing conditions (in Panel B), the spread between high versus low $\nu_g$-sorted portfolios changes from $-3.00$ (with t-statistic $-2.444$) to 3.73 (with t-statistic 3.33) as the risk sharing condition improves; if the credit spread index is used to construct the regimes of risk sharing conditions (in Panel C), the spread between high versus low $\nu_g$-sorted portfolios changes from $-5.34$ (with t-statistic $-1.14$) to 4.36 (with t-statistic 0.96) as the risk sharing condition improves. This suggests that the growth uncertainty shock tends to carry a negative market price of risk when the risk sharing condition is bad and a positive market price of risk otherwise.

# 6 Conclusion

I have studied an investment-based general equilibrium model with two sources of uncertainty shocks and endogenous imperfect risk sharing. The model provides a fundamental mechanism which can help reconcile seemingly contradictory empirical findings in asset pricing and macroeconomics under a unified framework.

There are two main new insights provided by this paper. First, the source of uncertainty shocks matters, since they affect the economy through different asset classes. The characteristics of the assets determine the impact of uncertainty shocks from certain origin on asset prices and investment. In particular, the growth uncertainty shocks can increase asset prices and investment because of the option feature embedded in growth options. Second, the risk sharing condition plays a vital role in shaping the impact of uncertainty shocks. When risk sharing is severely limited, a rise in uncertainty distorts agents' real investment decisions and portfolio allocations in an inefficient manner. If agents' preference over smoothing consumption across time (governed by the elasticity of intertemporal substitution) is not very strong, even the growth uncertainty shock can suppress asset prices, decrease investment, deteriorate risk sharing conditions, and hence carry a negative market price of risk.

This paper, moreover, discovers the linkage between the cross section of asset returns and uncertainty shocks from different origins. Because different sources of uncertainty shocks do not affect firms symmetrically, then the cross section of asset returns can help identify the source of uncertainty shocks. Financial data with a larger cross section and a higher frequency can serve well for uncovering the uncertainty shocks used in macroeconomic models. Moreover, as shown theoretically and empirically, the cross sectional exposures of asset returns to growth uncertainty shocks are largely driven by the risk sharing condition; hence the time-varying cross-sectional exposures to the growth uncertainty shock are informative about the underlying economy state of risk sharing.

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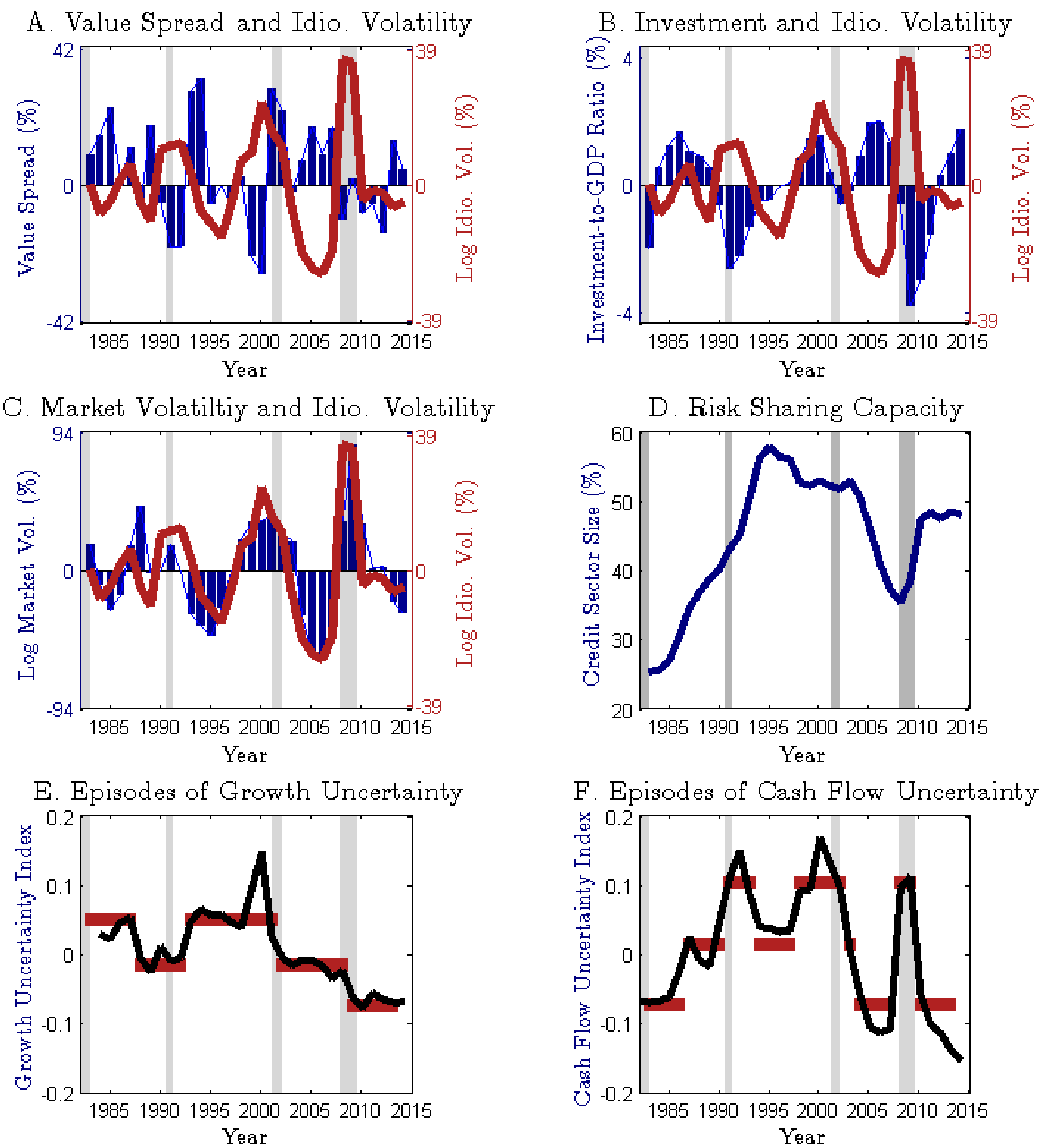
Figure 1: Impact of Idiosyncratic Volatility




Note: This figure illustrates the dynamics of idiosyncratic volatility of stock returns. It highlights the comovement pattern of the average idiosyncratic volatility with the cross-sectional spread between value and growth stock returns (i.e., value spreads) and the aggregate investment. In panel A, the red solid curve characterizes the cyclical dynamic of the average idiosyncratic volatility of U.S. stock returns. It is the cyclical component of the average idiosyncratic volatility extracted based on the HP filter. The average idiosyncratic volatility comoves, almost perfectly, with the common component in idiosyncratic volatilities (CIV) proposed by Herskovic et al. (2014). The bars characterize the spread between value and growth stock annual returns (value spreads) based on Fama-French 10 book-to-market sorted portfolios. More precisely, the value spread is the highest 10% minus the lowest 10%. In Panel B, the bars characterize the HP-filtered aggregate investment-to-output ratios constructed using BEA time series. In Panel C, the bars characterize the monthly market volatility computed based on daily returns on the CRSP value-weighted stock index using all firms listed on NYSE/AMEX/NASDAQ/ARCA. In Panel D, the curve characterizes the credit sector size (i.e., total corporate credit deflated by total corporate net worth) based on Flow of Funds data following Longstaff and Wang (2012). Panel E and Panel F display the growth uncertainty index and the cash-flow uncertainty index, respectively. Their construction are described in Section 5.1. The data sources and construction methods for Panels A - D are in the appendix.

Table 1: Baseline Parametrization

| Parameter | Symbol | Value |
|---|---|---|
| A. Preferences | | |
| Subjective discount rate | $\rho$ | 0.0111 |
| Relative risk aversion | $\gamma$ | 6 |
| EIS coefficient | $\psi$ | 2 |
| B. Assets in Place in Consumption Goods Sector | | |
| Capital share in production function | $\varphi$ | 0.3 |
| Assets in place depreciation rate | $\delta$ | 15% |
| Aggregate volatility | $\sigma$ | 10% |
| Cash-flow uncertainty | $\nu_c^L/\nu_c^H$ | 10%/50% |
| Transition of cash-flow uncertainty | $\lambda^{(\nu_c^L,\nu_c^H)}/\lambda^{(\nu_c^H,\nu_c^L)}$ | 0.111/0.39 |
| C. Growth Options in Consumption Goods Sector | | |
| Investment goods share in production function | $\alpha$ | 0.9 |
| Growth uncertainty | $\nu_g^L/\nu_g^H$ | 10%/49% |
| Investment opportunity arrival rate | $\lambda$ | 3.33 |
| Transition of growth uncertainty | $\lambda^{(\nu_g^L,\nu_g^H)}/\lambda^{(\nu_g^H,\nu_g^L)}$ | 0.1/0.44 |
| Fixed adjustment cost rate | $\varpi$ | 0.0083 |
| Aggregate growth options | $\overline{s}$ | 1 |
| D. Investment Goods Sector | | |
| Average productivity level | $z_\iota$ | 1.03 |
| E. Labor Market | | |
| Population share | $\varkappa$ | 2.04% |
| Average lifespan | $\mu$ | 1/40 |
| F. Financial Market | | |
| Severity of agency problem | $\phi$ | 0.4 |
| Pledgeability of human capital | $\varrho$ | 5% |

Note: This table reports the calibrated parameters of the model. The annualized values are used in the table for the dynamic parameters. When choosing the values of the parameters, both inside and outside-model data are employed.

Table 2: Model versus Data: Unconditional Moments of Macroeconomic Cycles

| | Mean | Stdev | AC(1) | Correlation | | |
|---|---|---|---|---|---|---|
| | | | | $\Delta$ log $(c_{t+1})$ | $\Delta$ log $(c_t)$ | $\Delta$ log $(y_t)$ |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| | | | A. Data | | | |
| $\Delta \log (c_t)$ | 1.46 [0.98, 1.93] | 3.77 [3.31, 4.17] | 0.36 [-0.13, 0.17] | | | |
| $\Delta \log (y_t)$ | **1.67** [0.65, 2.63] | **5.78** [3.43, 7.49] | 0.28 [0.00, 0.42] | 0.34 [-0.03, 0.50] | 0.92 [0.89, 0.94] | |
| $\Delta \log (i_t)$ | 1.35 [-2.58, 5.20] | **36.00** [10.92, 49.55] | 0.43 [0.29, 0.56] | 0.38 [0.09, 0.45] | 0.83 [0.78, 0.87] | 0.87 [0.85, 0.93] |
| | | | B. Model | | | |
| $\Delta \log (c_t)$ | 1.92 [0.74, 3.09] | 3.96 [2.96, 4.33] | 0.32 [0.11, 0.50] | | | |
| $\Delta \log (y_t)$ | **1.92** [0.73, 3.06] | **4.01** [3.35, 4.70] | 0.50 [0.34, 0.65] | 0.62 [0.49, 0.73] | 0.52 [0.33, 0.70] | |
| $\Delta \log (i_t)$ | 2.36 [-0.47, 5.73] | **55.38** [32.41, 77.63] | 0.30 [-0.00, 0.49] | 0.23 [0.07, 0.39] | -0.44 [-0.26, -0.61] | 0.30 [0.17, 0.43] |

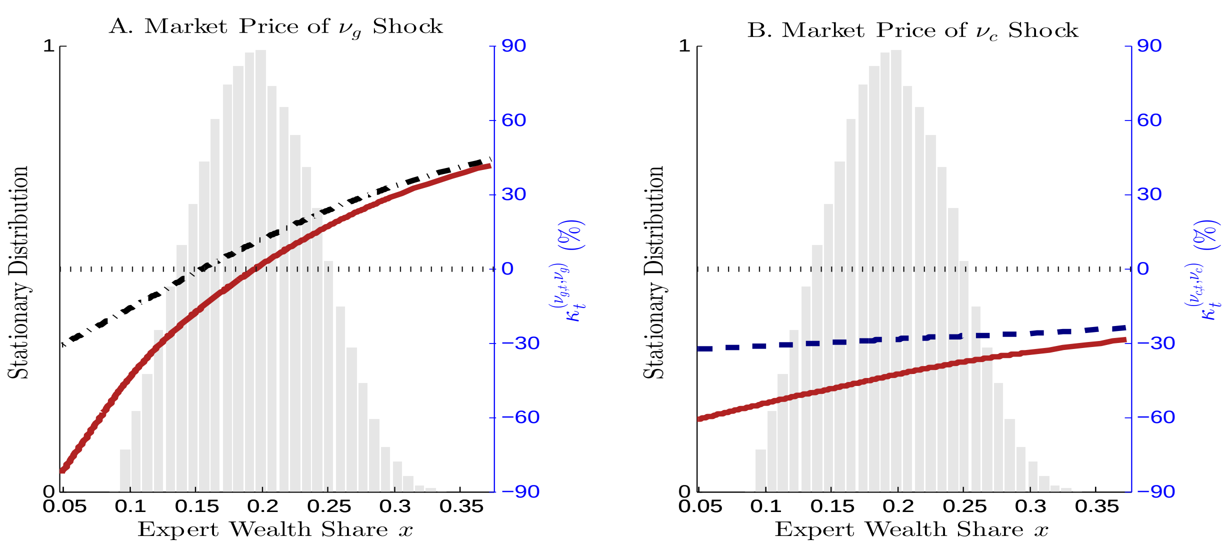
Notes: The table compares unconditional moments of the data to their simulated analogies in the model. Panel A reports the mean, standard deviation, and autocorrelation of U.S. output (y), consumption (c), and net investment (i) log growth rates, as well as their cross-correlation coefficients. All variables are real (adjusted by CPI) and scaled by U.S. population. The 95% confidence intervals are reported in brackets; they are obtained by applying stationary block bootstrap method in which the block size is random (see Politis and Romano, 1994*a*,*b*). The average block size is determined by the adaptive block length selection procedure of Politis and White (2004) and Patton, Politis and White (2009). Data are sampled at the annual frequency. Their sources and construction details are explained in the appendix. All variables are reported in percentage points, except for the autocorrelation and cross-correlation coefficients. The moments of the consumption growth and the output growth are from the extended long sample of Barro and Ursúa (2008) with sample period 1790 – 2014. The sample periods of net payout growth and investment are 1929 – 2014, and the labor supply growth is only available during 1948 – 2014. Panel B reports simulated moments in the model. I simulate the model at the weekly frequency and then time-aggregate the simulated data to construct annual observations. In brackets, they are the 5% and 95% quantiles across 1,000 independent simulations, each with a length of 80 years. The net investment is constructed using real private fixed investment plus real durable consumption minus real depreciation normalized by population.

Table 3: Model versus Data: Unconditional Moments of Macroeconomic Ratios

| Ratios (%) | A. Data | | | B. Model | | |
|---|---|---|---|---|---|---|
| | Mean (1) | Stdev (2) | AC(1) (3) | Mean (4) | Stdev (5) | AC(1) (6) |
| Investment to Output | **16.47** | 5.23 | 0.90 | **16.60** | 6.41 | 0.70 |
| | [13.85, 18.79] | [2.84, 6.23] | [0.60, 0.87] | [14.74, 18.28] | [4.83, 7.89] | [0.58, 0.83] |
| Net Payout to Consumption | **5.46** | 3.07 | 0.85 | **6.30** | 4.03 | 0.70 |
| | [3.93, 7.00] | [2.22, 3.42] | [0.59, 0.83] | [5.03, 7.49] | [2.42, 5.28] | [0.55, 0.84] |
| Wage Income to Output | **75.26** | 4.02 | 0.96 | **75.25** | 2.04 | 0.71 |
| | [72.94, 77.69] | [2.71, 4.48] | [0.74, 0.94] | [74.55, 75.89] | [1.55, 2.49] | [0.58, 0.83] |
| Capital to Output | **169.24** | 46.96 | 0.90 | **196.20** | 48.35 | 0.71 |
| | [144.40, 195.55] | [29.85, 53.13] | [0.66, 0.89] | [186.77, 204.51] | [33.96, 65.33] | [0.48, 0.82] |

Notes: The table compares unconditional moments of the data to their simulated analogies in the model. Panel A reports the mean, standard deviation, and autocorrelation of U.S. net investment/output ratio, net payout/consumption ratio, wage income/output ratio, and capital/output ratio. The 95% confidence intervals are reported in brackets; they are obtained by applying stationary block bootstrap method in which the block size is random (see Politis and Romano, 1994$a$,$b$). The average block size is determined by the adaptive block length selection procedure of Politis and White (2004) and Patton, Politis and White (2009). Data are sampled at the annual frequency. Their sources and construction details are explained in the appendix. All variables are reported in percentage points, except for the autocorrelation coefficients. The sample period is 1929 – 2014. Panel B reports simulated moments in the model. I simulate the model at the weekly frequency and then time-aggregate the simulated data to form annual observations. In brackets, they are the 5% and 95% quantiles across 1,000 independent simulations, each with a length of 80 years.

Figure 2: Policy Functions: market price of risk




Notes: This figure illustrates the market price of risk for two uncertainty shocks under the calibration summarized in Table 1. Panel A shows the market price of risk for growth uncertainty shocks; Panel B is about the market price of risk for cash-flow uncertainty shocks. The red solid curve corresponds to the normal state of the world where both uncertainties are at low levels; the blue dashed curve corresponds to state of high growth uncertainty; and, the black dashed-dotted curve corresponds to the state of high cash-flow uncertainty. The grey distribution in the background is the stationary distribution of the endogenous state variable $x_t$.

Table 4: Model versus Data: Fundamental Dispersions

| Dispersions (%) | Mean (1) | Stdev (2) | AC(1) (3) | Corr $\Delta \log(y_t)$ (4) |
|---|---|---|---|---|
| | A. Data | | | |
| IDR Sales Growth | **49.02** | **12.32** | 0.80 | -17.32 |
| | [42.20, 55.90] | [8.39, 13.71] | [0.64, 0.89] | [-36.85, 3.80] |
| CSD Investment Rate | **40.85** | **7.25** | 0.66 | 43.28 |
| | [37.03, 44.71] | [5.22, 7.93] | [0.48, 0.77] | [30.45, 59.99] |
| | B. Model | | | |
| IDR Sales Growth | **53.02** | **16.03** | 0.69 | -27.66 |
| | [42.20, 61.90] | [9.84, 23.19] | [0.57, 0.80] | [-45.51, -13.83] |
| CSD Investment Rate | **45.12** | **13.50** | 0.71 | 23.82 |
| | [39.13, 49.98] | [10.31, 16.37] | [0.43, 0.79] | [1.89, 40.35] |

Notes: The table compares unconditional moments of the data to their simulated analogies in the model. Panel A reports, in the data, the mean, standard deviation, autocorrelation, and cyclicality of Compustat sales dispersion measured by the cross-sectional interdecile range (IDR) and Compustat capital expenditures dispersion measured by the cross-sectional standard deviation (CSD). The sales are deflated by the sales in the previous year, and capital expenditure is deflated by capital stock in the previous year. Sales is constructed using item sales, capital expenditure is constructed using item capx, and capital stock is constructed using item ppent. The 95% confidence intervals are reported in brackets; they are obtained by applying stationary block bootstrap method in which the block size is random (see Politis and Romano, 1994*a*,*b*). The average block size is determined by the adaptive block length selection procedure of Politis and White (2004) and Patton, Politis and White (2009). Data are sampled at the annual frequency. Their sources and construction details are explained in the appendix. All variables are reported in percentage points, except for the autocorrelation coefficients. All variables have the sample period of 1966 – 2014. Panel B reports simulated moments in the model. I simulate the model at the weekly frequency and then time-aggregate the simulated data to form annual observations. In brackets, they are the 5% and 95% quantiles across 1,000 independent simulations, each with a length of 80 years.

Table 5: Model versus Data: Unconditional Asset Pricing Moments

| | A. Data | | | B. Model | | |
|---|---|---|---|---|---|---|
| Returns (%) | Mean (1) | Stdev (2) | AC(1) (3) | Mean (4) | Stdev (5) | AC(1) (6) |
| Equity Premium | 4.47 | 20.83 | 0.03 | 4.95 | 19.25 | 0.12 |
| | [0.85, 8.07] | [18.08, 22.99] | [-0.19, 0.16] | [3.17, 6.58] | [11.37, 27.91] | [-0.10, 0.33] |
| Interest Rate | **1.31** | 2.71 | 0.62 | **1.53** | 2.92 | 0.71 |
| | [0.63, 2.13] | [2.07, 3.19] | [0.39, 0.92] | [0.49, 2.66] | [2.41, 3.43] | [0.58, 0.83] |
| Net Payout Yield | 2.25 | 3.60 | 0.79 | 3.67 | 4.14 | 0.68 |
| | [1.78, 2.71] | [2.73, 3.91] | [0.56, 0.88] | [2.34, 4.71] | [3.31, 4.92] | [0.55, 0.81] |
| Value Spread | 5.05 | 25.21 | 0.11 | 7.57 | 16.84 | -0.02 |
| | [0.57, 9.57] | [21.14, 28.54] | [-0.16, 0.20] | [5.88, 9.57] | [12.39, 21.39] | [-0.29, 0.26] |

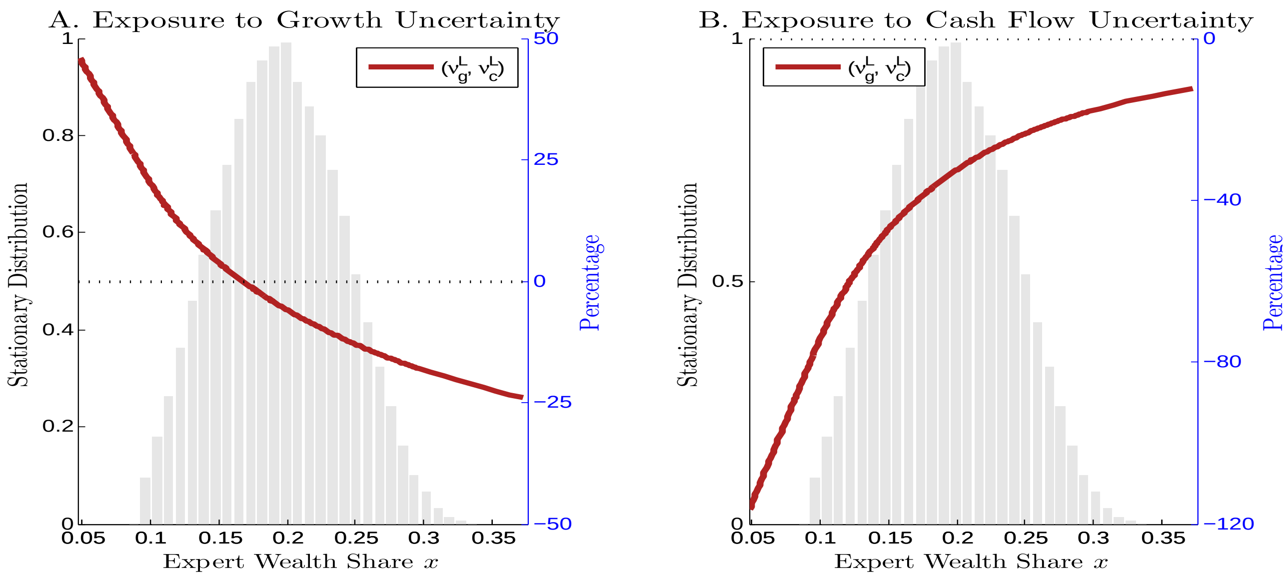
Notes: The table compares unconditional moments of the data to their simulated analogies in the model. Panel A reports the mean, standard deviation, and autocorrelation of U.S. equity premium, the real interest rate, the net payout yield, and the value spread which is the return spread between two portfolios of firms with the top and bottom decile of book-to-market ratios. The 95% confidence intervals are reported in brackets; they are obtained by applying stationary block bootstrap method in which the block size is random (see Politis and Romano, 1994*a*,*b*). The average block size is determined by the adaptive block length selection procedure of Politis and White (2004) and Patton, Politis and White (2009). Data are sampled at the annual frequency. Their sources and construction details are explained in the appendix. All variables are reported in percentage points, except for the autocorrelation coefficients. All variables have the sample period of 1929 – 2014. Panel B reports simulated moments in the model. I simulate the model at the weekly frequency and then time-aggregate the simulated data to form annual observations. In brackets, they are the 2.5% and 97.5% quantiles across 1,000 independent simulations, each with a length of 80 years.

Table 6: Estimated Transition: Uncertainty Regimes

| | Markov Transition Probabilities (%) | | | | | |
|---|---|---|---|---|---|---|
| | G-Uncert | | | C-Uncert | | |
| (states) | High | Medium | Low | High | Medium | Low |
| High | 81.9 | 16.3 | 1.8 | 67.5 | 20.1 | 12.4 |
| Medium | 21.6 | 70.5 | 7.9 | 13.9 | 80.5 | 5.6 |
| Low | 6.6 | 13.3 | 80.1 | 8.6 | 2.7 | 88.8 |
| Stationary Dist. | 47.6 | 34.4 | 18.0 | 24.9 | 31.7 | 43.4 |

Notes: This table reports point estimation of Markov transition probabilities of the latent states for two kinds of uncertainty, respectively. G-Uncert stands for growth uncertainty, and C-Uncert stands for cash-flow uncertainty. The numbers are estimates of the regime-switching model in (40) using the EM algorithm. The estimation is based on annual sample from 1953 to 2014.

Figure 3: Value Spread's Uncertainty Exposures



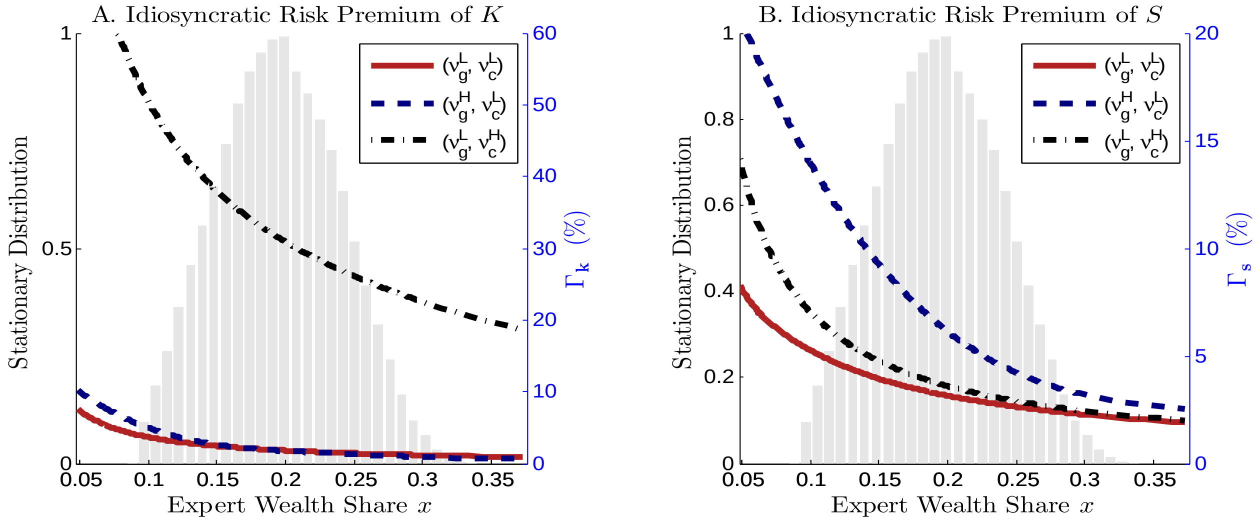

Notes: This figure illustrates the uncertainty exposure of value spreads under the calibration summarized in Table 1. Panel A shows the exposure of value spreads to growth uncertainty shocks; Panel B shows the exposure of value spreads to cash-flow uncertainty shocks. The red solid curve corresponds to the normal state of the world where both uncertainties are at low levels. The grey distribution in the background is the stationary distribution of the endogenous state variable $x_t$.

Table 7: Cyclicality: Uncertainty and Idiosyncratic Dispersion

| Correlations with The Cyclical Component of Output: | | | | |
|---|---|---|---|---|
| | A. Dispersion | | B. Uncertainty | |
| | Pearson | Kendall | Pearson | Kendall |
| | (1) | (2) | (3) | (4) |
| Investment | 0.31<br>(0.03) | 0.20<br>(0.04) | 0.13<br>(0.39) | 0.05<br>(0.62) |
| Cash Flow | −0.16<br>(0.26) | −0.10<br>(0.32) | −0.31<br>(0.03) | −0.21<br>(0.03) |

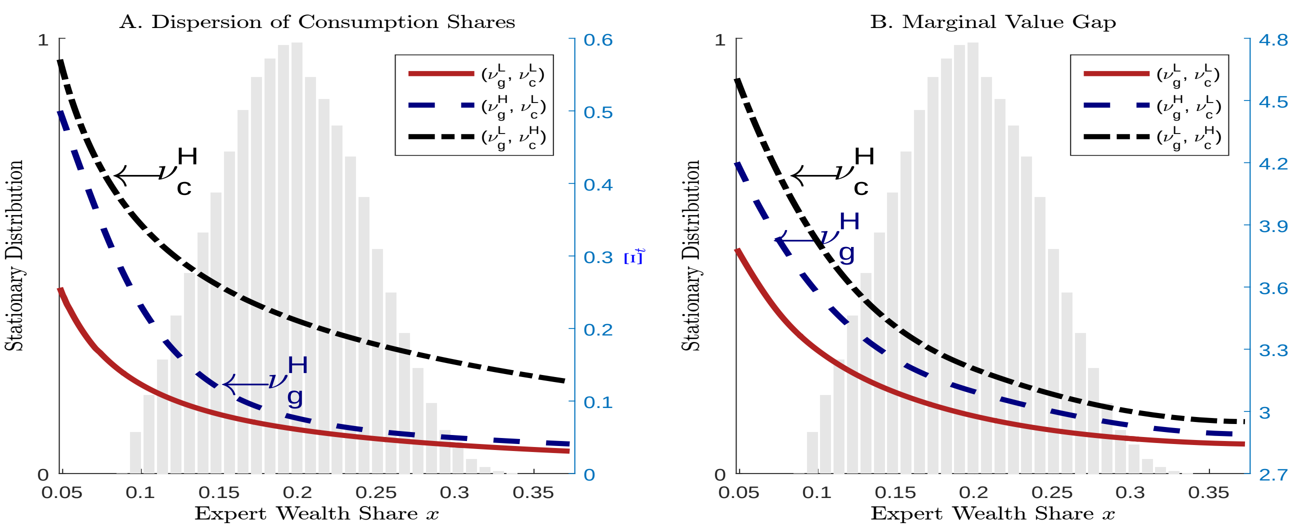
Notes: This table reports correlations of uncertainty indices and idiosyncratic dispersions with the cyclical component of U.S. real GDP per capita. The cyclical component is extracted from the log real GDP per capita by using the one-sided Hodrick-Prescott (HP) filter. Panel A reports the cyclicality of the dispersion of idiosyncratic shocks in sales growth rates $\nu_{sales,t_y}$ (Cash Flow row) and the idiosyncratic shocks in investment rates $\nu_{capx,t_y}$ (Investment row). Panel B reports the cyclicality of the cash-flow uncertainty $\nu_{c,t_y}$ (Cash Flow row) and the growth uncertainty $\nu_{g,t_y}$ (Investment row). Column (1) and Column (3) report the Pearson correlations, while Column (3) and Column (4) report the Kendall rank correlations. The Kendall rank correlation measures the similarity of the orderings of the data when ranked by each of the quantities. Thus, it provides a non-parametric measure of the association of two time series. The validity of the Pearson correlation is more dependent on the parametric gaussian assumption. The p values are reported inside the parentheses. The sample is annual from 1966 to 2014. The reliable dispersion estimates are only available after 1966 in annual Compustat fundamentals.

Figure 4: Idiosyncratic Risk Premia



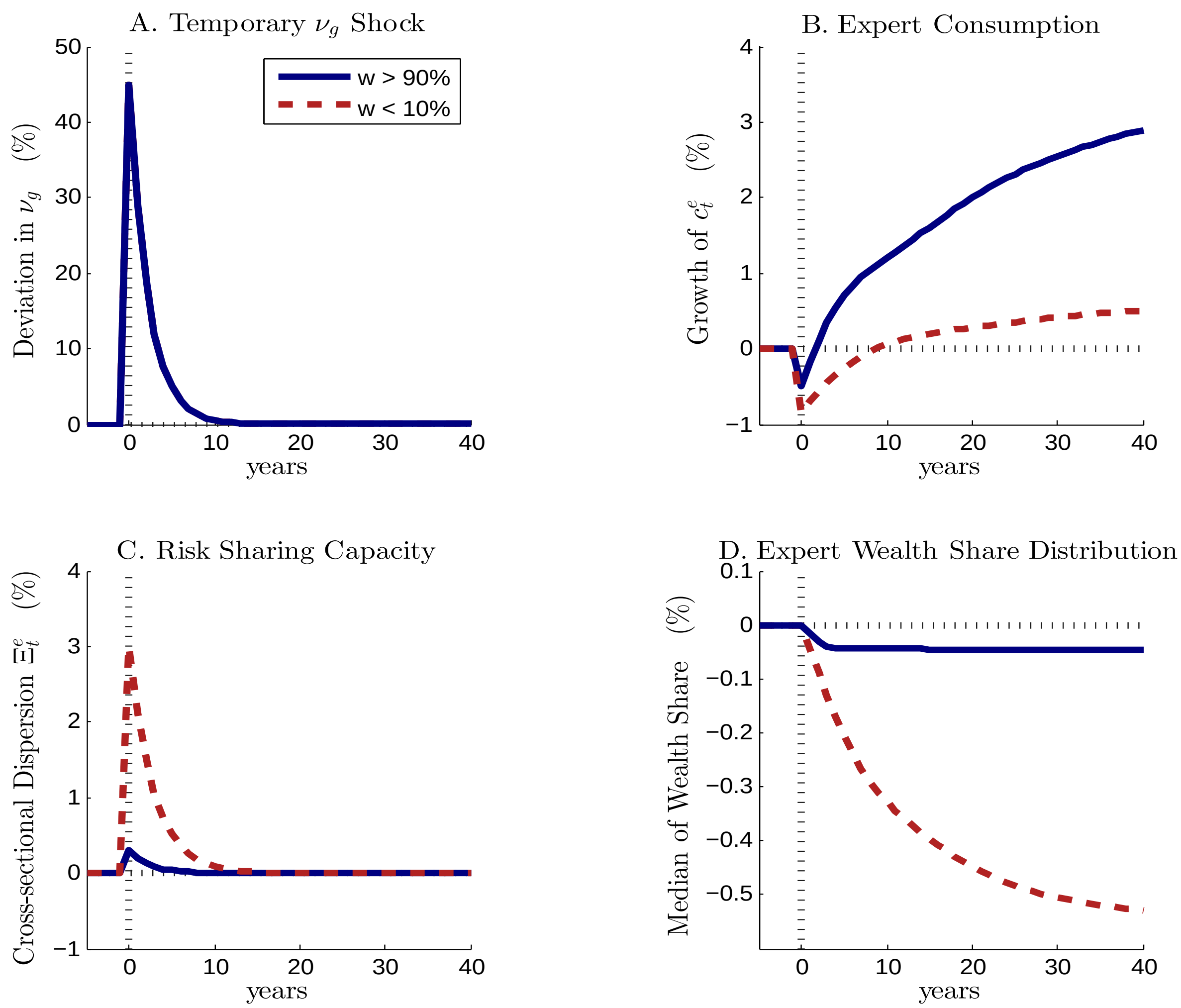

NOTES: This figure illustrates the idiosyncratic risk premia under the calibration summarized in Table 1. Panel A is about the premium on the idiosyncratic cash flow risk $dW_{f,t}$, and Panel B is about the premium on the idiosyncratic investment risk associated with $\varepsilon_{f,t}$ and $dN_{f,t}$. The red solid curve corresponds to the normal state of the world where both uncertainties are at low levels; the blue dashed curve corresponds to state of high growth uncertainty; and, the black dashed-dotted curve corresponds to the state of high cash-flow uncertainty. The grey distribution in the background is the stationary distribution of the endogenous state variable $x_t$.

Figure 5: Limited Risk Sharing



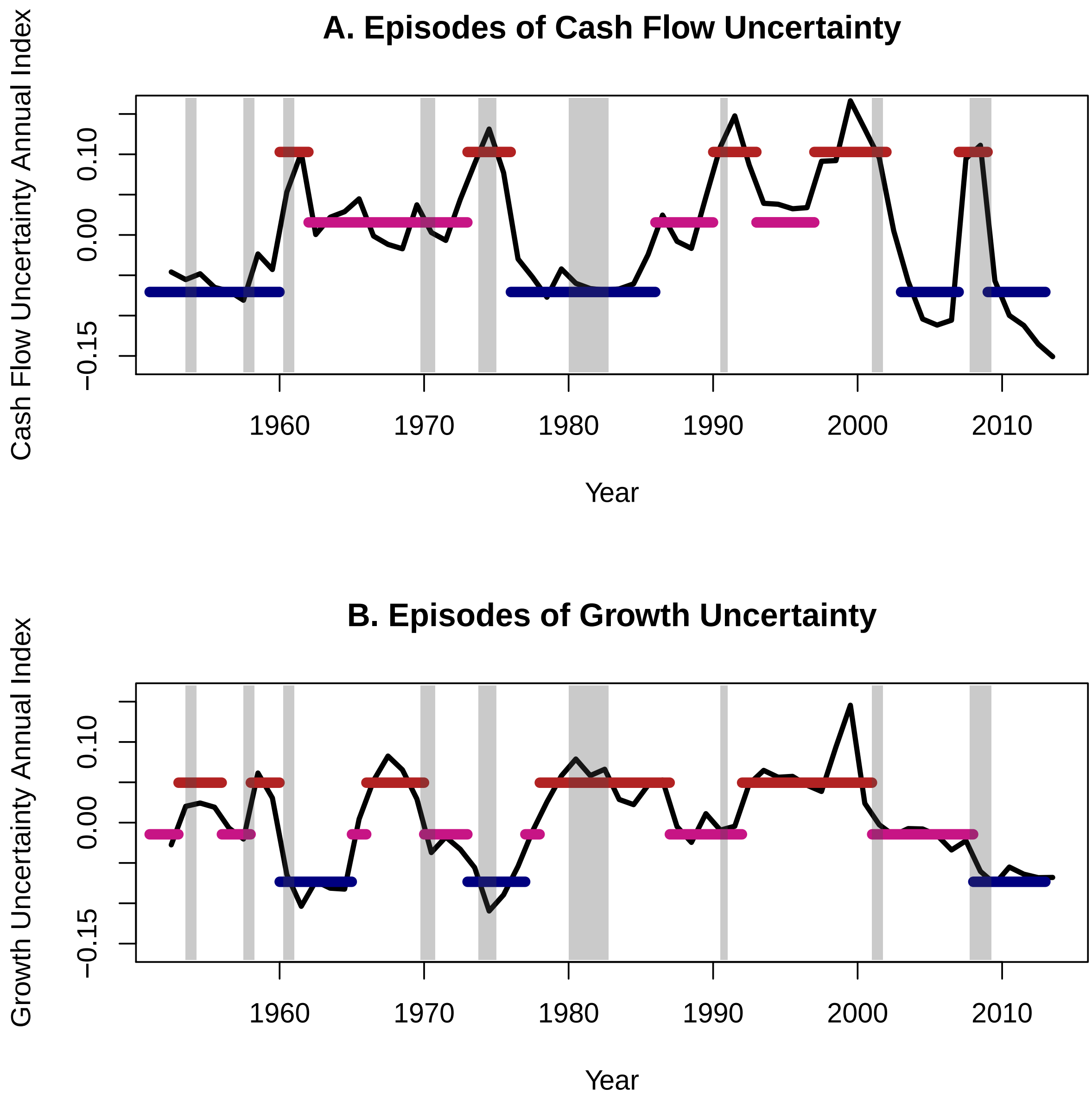

NOTES: This figure illustrates the consumption dispersion and marginal value gap under the calibration summarized in Table 1. Panel A is about the premium on the idiosyncratic cash flow risk $dW_{f,t}$, and Panel B is about the premium on the idiosyncratic investment risk associated with $\varepsilon_{f,t}$ and $dN_{f,t}$. The red solid curve corresponds to the normal state of the world where both uncertainties are at low levels; the blue dashed curve corresponds to state of high growth uncertainty; and, the black dashed-dotted curve corresponds to the state of high cash-flow uncertainty. The grey distribution in the background is the stationary distribution of the endogenous state variable $x_t$.

Figure 6: Impulse Responses to Growth Uncertainty Shocks



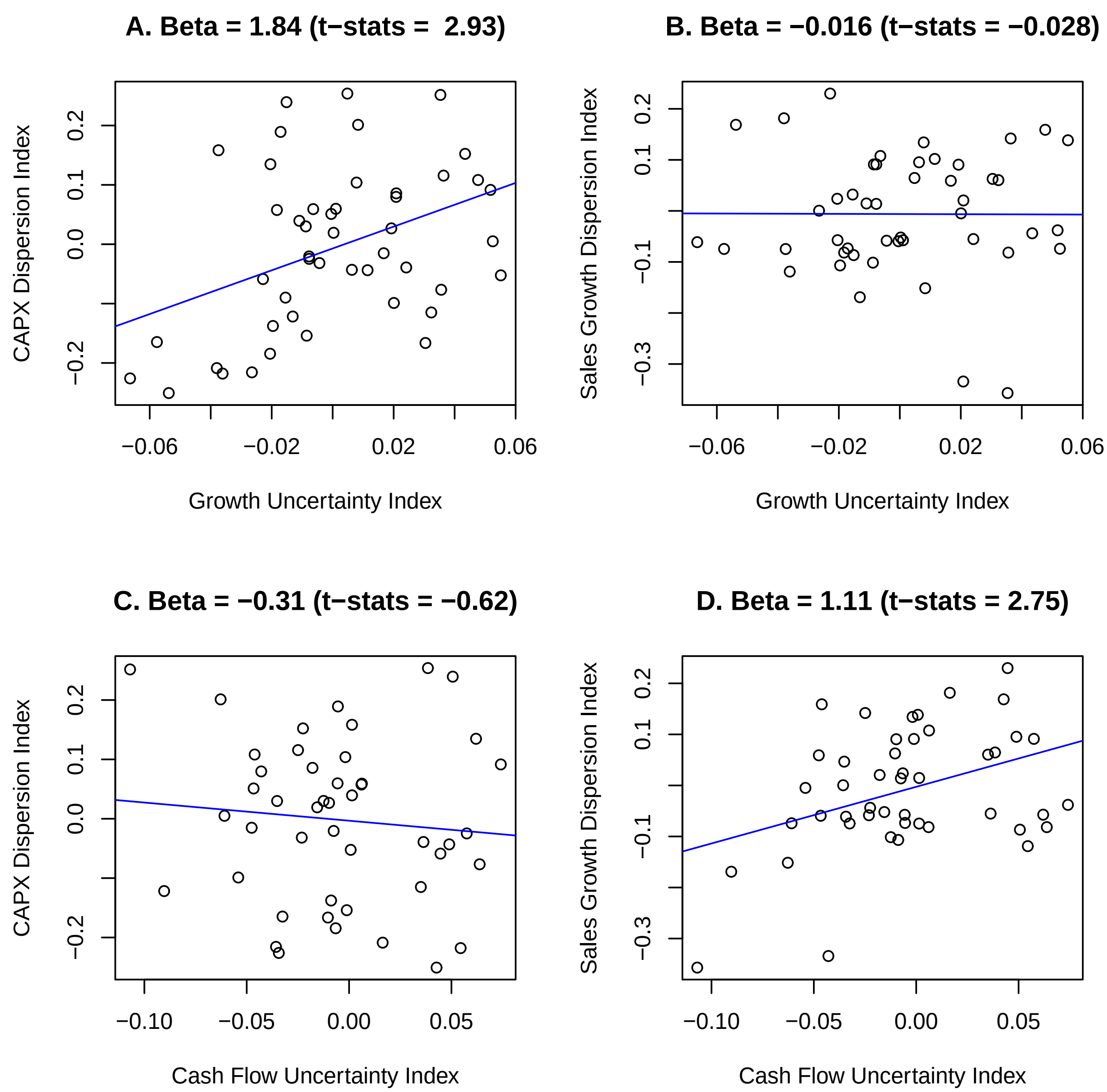

Notes: This figure illustrates the impulse-response to a temporary growth uncertainty shock under the calibration summarized in Table 1. Panel A shows the temporal shock as an impulse; Panel B is about the responses of experts' aggregate consumption; Panel C is about the responses of conditional cross-sectional variance of consumption share growth, and Panel D is about median of cross-section of experts' consumption shares. The blue solid curve corresponds to the states of good risk sharing conditions; the red dashed curve corresponds to states of high poor risk sharing conditions. The grey distribution in the background is the stationary distribution of the endogenous state variable $x_t$.

Figure 7: Annual Uncertainty indices, Estimated Uncertainty Regimes and U.S. Recessions




NOTES: This figure plots the annual indices of the cash-flow uncertainty and the growth uncertainty. The annual index is defined as the average of twelve monthly index values within each year. The monthly indices of uncertainty are constructed as described in Section 5.1. The horizontal segments represent the episodes of uncertainty conditions of U.S. economy. Their levels are the averages over time within each regime. The regimes are estimated under the framework of regime-switching models (e.g., Hamilton, 1989, 1994; Timmermann, 2000) by using the EM algorithm which maximizes the marginal likelihood of observable variables. Three regimes are assumed in the regime-switching model; the threshold for a regime to be taken in certain year is set at the likelihood of 50%. The estimation is based on annual sample from 1953 to 2014. The shaded areas represent the NBER-dated U.S. recessions. Panel A shows the cash-flow uncertainty annual index and its estimated high/medium/low episodes. Panel B shows the growth uncertainty annual index and its estimated high/medium/low episodes.

Figure 8: Statistical Associations: Uncertainty versus Cross-Section Dispersion




Notes: This figure plot the annual changes of idiosyncratic sales dispersions and idiosyncratic investment dispersions against annual changes of uncertainty indices $\Delta\nu_{c,t_y}$ and $\Delta\nu_{g,t_y}$. The annual index is defined as the average of twelve monthly index values within each year. The monthly indices of uncertainty are constructed as described in Section 5.1. The idiosyncratic sales dispersions $\nu_{sales,t_y}$ and the idiosyncratic investment dispersions $\nu_{capx,t_y}$ are calculated in Section 5.2. The sample is annual from 1979 to 2014. Although the Compustat annual fundamental panel starts in 1952, my sample for constructing cross-sectional dispersions starting from 1966. This is because the number of firms becomes large enough (over 1000 firms) to provide reliable cross-section distribution estimates only from 1979. The fitted lines are estimated using least-square regression with Grubbs (1950) robustness for outliers. Panel A shows the scatter plot of changes in idiosyncratic investment dispersions $\Delta\nu_{capx,t_y}$ against growth uncertainty index changes $\Delta\nu_{g,t_y}$. The slope is estimated to be 1.84 with t-statistic 2.93. Panel B shows the scatter plot of changes in idiosyncratic sales dispersions $\Delta\nu_{sales,t_y}$ against growth uncertainty index changes $\Delta\nu_{g,t_y}$. The slope is estimated to be $-0.016$ with t statistic $-0.028$. Panel C shows the scatter plot of changes in idiosyncratic investment dispersions $\Delta\nu_{capx,t_y}$ against cash-flow uncertainty index changes $\Delta\nu_{c,t_y}$. The estimated slope is $-0.31$ with t statistic $-0.62$. Panel D shows the scatter plot of changes in idiosyncratic sales dispersions $\Delta\nu_{sales,t_y}$ against cash-flow uncertainty index changes $\Delta\nu_{c,t_y}$. The estimated slope is 1.11 with t statistic 2.75.

Table 8: The Regime-Switching Model: Estimated Coefficients

| Estimated Coefficients in the Regime-Switching Model (44): | $a_v$ | $\beta_{v,z}$ | $\beta_{v,g}$ | $\beta_{v,c}$ |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| Bad state | 0.47 | 0.41 | 6.03 | −13.08 |
| 95% CI | [ 0.09, 1.07] | [−0.27, 0.55] | [ 0.29, 11.21] | [−19.92, −0.16] |
| 75% CI | [ 0.28, 0.67] | [ 0.27, 0.48] | [ 2.60, 6.84] | [−13.88, −5.47] |
| Good state | 0.46 | −0.41 | −1.76 | −1.00 |
| 95% CI | [ 0.11, 1.99] | [−0.57, 0.89] | [−10.42, 1.07] | [−7.44, 0.73] |
| 75% CI | [ 0.26, 0.59] | [−0.49, −0.26] | [−4.19, −0.99] | [−5.27, −0.80] |

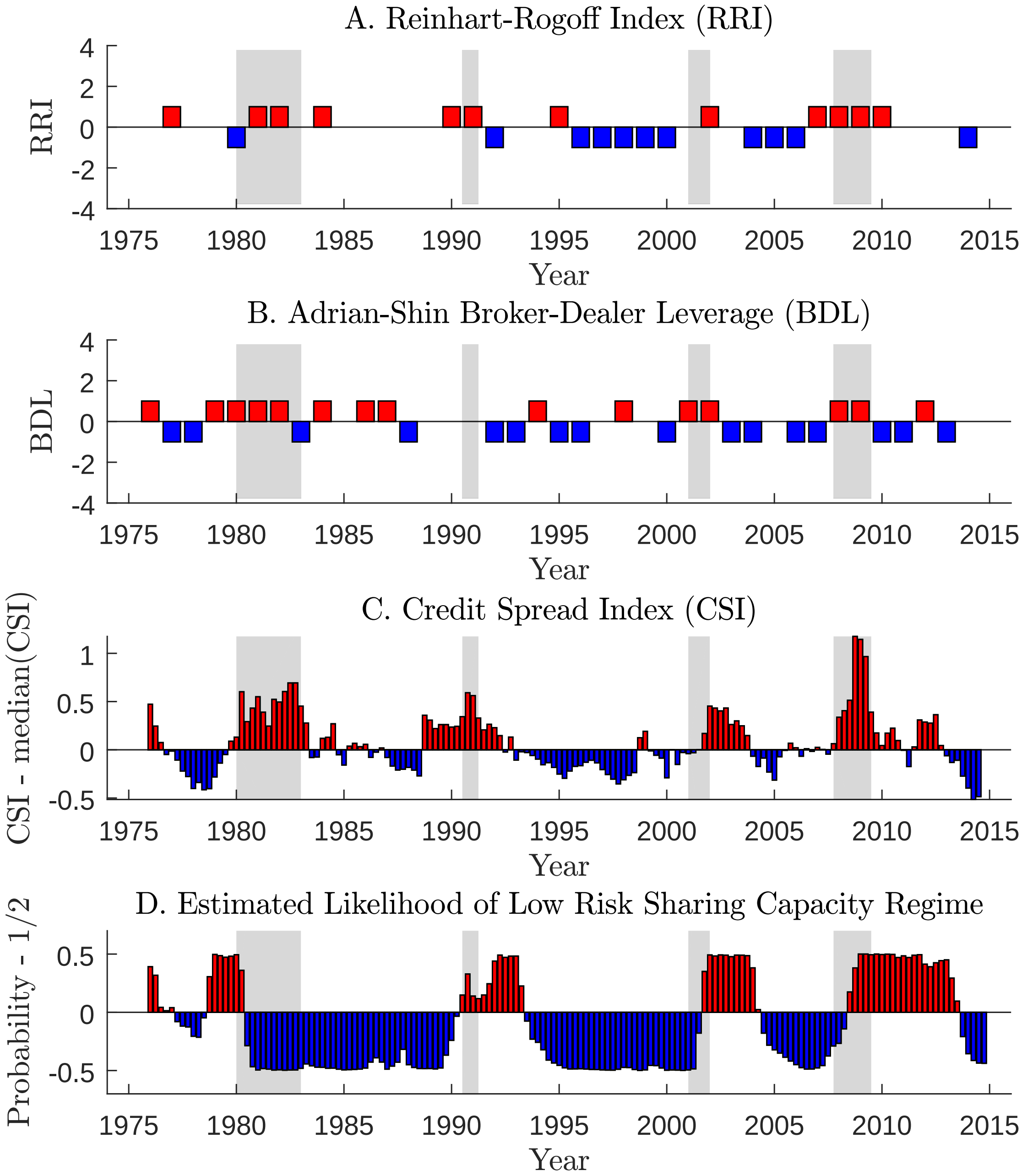
Notes: This table reports the estimation results of the regime-switching model (44). The model is estimated using the EM algorithm which maximizes the marginal likelihood of observables. The 95% and 75% confidence intervals are reported in brackets; they are obtained by applying stationary block bootstrap method in which the block size is random (see Politis and Romano, 1994*a*,*b*). The average block size is determined by the adaptive block length selection procedure of Politis and White (2004) and Patton, Politis and White (2009). Data are sampled at the monthly frequency from January 1953 to December 2014.

Table 9: Statistical Associations: Estimation and Measures of Risk Sharing Conditions

| Correlation of Credit Spread Index with: | A. Data | | B. Model | |
|---|---|---|---|---|
| | Pearson | Kendall | Pearson | Kendall |
| | (1) | (2) | (3) | (4) |
| Estimated Likelihood of Being in State Bad | 0.38 | 0.23 | 0.36 | 0.29 |
| | (0.00) | (0.00) | (0.00) | (0.00) |
| Reinhart-Rogoff Financial Index | 0.52 | 0.39 | 0.27 | 0.18 |
| | (0.00) | (0.01) | (0.00) | (0.00) |
| Broker-Dealer Leverage Index | 0.42 | 0.29 | −− | −− |
| | (0.00) | (0.07) | −− | −− |

Notes: This table reports the statistical association between the credit spread index, the Reinhart-Rogoff financial index, the Broker-Dealer leverage index, and the estimated likelihood of being in the Bad state plotted in Figure 9. Panel A shows the Pearson correlation (in Column (1)) and the Kendall rank correlation (in Column (2)) of the credit spread index with estimated likelihood of Bad state. At the same time, Panel B reports the corresponding statistical moments in the simulated data based on my model. Because there is no corporate bond in my model, I use the equity risk premium as the proxy for credit spread. The risk sharing condition in the simulated data is measured by using $\Theta_t$ in (32). The likelihood of Bad is estimated for the simulated data in the same way as for the real data. The Kendall rank correlation measures the similarity of the orderings of the data when ranked by each of the quantities. Thus, it provides a non-parametric measure of the association of two time series. The validity of the Pearson correlation is more dependent on the parametric gaussian assumption. The p values are reported inside the parentheses. The sample of indices are annual. They are time-aggregated from monthly or quarterly sample by averaging within each year. The simulated data are monthly and time-aggregated into quarterly frequency in the same way.

Figure 9: Measures and Estimation of Risk Sharing Condition of U.S. Economy




Notes: The figure presents three measures of risk sharing conditions in the data (in Panels A, B, and C) and the estimated likelihood of being in the Bad state (in Panel D). According to the theory, the periods of Bad states coincide with the periods of poor risk sharing conditions in the economy. Therefore, the empirical measures of risk sharing conditions should move together with the estimated likelihood of Bad state. Panel A shows the Reinhart-Rogoff financial crisis chronologies from Reinhart and Rogoff (2009). Panel B shows the chronologies of poor financial conditions based on the broker-dealer leverage studied by Adrian and Shin (2010) and Adrian, Etula and Muir (2014). The year is marked as poor financial conditions (+1) if there is a large quarterly drop or are at least three quarterly drops in broker-dealer leverage; it is marked as good financial conditions (−1) if there is no quarterly drop in broker-dealer leverage; it is marked as normal financial conditions (0) otherwise. Panel C plots the Baa-minus-Aaa corporate spread with linear trend removed allowing for a structural change following Andrews (2003) and Andrews and Ploberger (1994). The Baa-minus-Aaa corporate spread is the spread between yields on Baa- and Aaa-related long-term industrial corporate bonds. The credit spread index is one of the most widely used proxies for the financial condition in the literature (e.g., Gilchrist and Zakrajsek, 2012; Adrian, Etula and Muir, 2014). In Panel D, the estimated likelihood of Bad state is plotted. Sample in Panels C and D are quarterly and constructed from average of monthly sample for each quarter; sample in Panels A and B are annual. The sample is from the first quarter of 1976 to the fourth quarter of 2014.

Table 10: Uncertainty Betas: The Reinhart-Rogoff Financial Index

| | A. Bad Risk Sharing Condition (Reinhart-Rogoff Index is High) | | | B. Good Risk Sharing Condition (Reinhart-Rogoff Index is Low) | | |
|---|---|---|---|---|---|---|
| | Mkt ex-Ret | $\Delta\nu_g$ | $\Delta\nu_c$ | Mkt ex-Ret | $\Delta\nu_g$ | $\Delta\nu_c$ |
| Book-to-Market Sort | (1) | (2) | (3) | (4) | (5) | (6) |
| Sort #1: Six Portfolios | | | | | | |
| Lowest 10% (Growth) | 1.06 | −3.11 | 2.56 | 1.18 | 2.40 | 14.10 |
| 10% − 30% | 0.96 | −0.81 | 0.91 | 0.90 | −1.85 | −7.85 |
| 30% − 50% | 0.99 | 0.91 | 0.68 | 0.79 | −11.21 | −15.98 |
| 50% − 70% | 0.99 | 2.55 | −0.66 | 0.72 | −11.56 | −10.97 |
| 70% − 90% | 0.98 | 0.77 | −4.48 | 0.62 | −12.15 | −14.29 |
| Highest 10% (Value) | 1.22 | 0.21 | −17.85 | 0.64 | −12.66 | −8.36 |
| Sort #2: Five Portfolios | | | | | | |
| Lowest 20% (Growth) | 1.02 | −2.79 | 0.89 | 1.10 | 2.55 | 8.23 |
| 20% − 40% | 0.98 | 0.50 | 2.04 | 0.87 | −8.39 | −12.18 |
| 40% − 60% | 0.97 | 2.03 | −0.46 | 0.78 | −9.80 | −14.50 |
| 60% − 80% | 0.97 | 1.94 | −0.27 | 0.60 | −11.37 | −15.55 |
| Highest 20% (Value) | 1.05 | −0.98 | −9.69 | 0.63 | −11.40 | −12.74 |
| Sort #3: Three Portfolios | | | | | | |
| Lowest 30% (Growth) | 1.00 | −2.39 | 1.97 | 1.06 | 1.91 | 4.38 |
| 30% − 70% | 0.99 | 1.78 | 0.32 | 0.76 | −11.49 | −14.37 |
| Highest 30% (Value) | 1.97 | 0.32 | −4.98 | 0.62 | −12.59 | −13.49 |

Notes: The table reports estimated betas of book-to-market sorted portfolios with respect to the market excess return, the growth uncertainty shock, and the cash-flow uncertainty shock. In particular, it compares the estimation results of two subsamples. One subsample consists of the periods in which the risk sharing condition is good, while the other subsample consists of the periods in which the risk sharing condition is poor. The periods of good or poor risk sharing conditions are estimated using the Reinhart-Rogoff index shown in Panel A of Figure 9. The regression model for estimating the betas is $r_{BM,t_m} = a_{BM} + \beta_{BM,z}(r_{M,t} - r_{f,t}) + \beta_{BM,g}\Delta\nu_{g,t_m} + \beta_{BM,c}\Delta\nu_{c,t_m} + \epsilon_{BM,t_m}$, where $BM$ stands for a book-to-market portfolio and $r_{BM,t_m}$ is the return of the book-to-market portfolio labeled by $BM$. The reported estimates are obtained by using the ordinary-least-squares method. To account for the heteroskedasticity in stock returns, I also use the weighted-least-squares method with inverse market variance to be the weights. The estimation results are quite similar, because the regressions are totally separated for different subsamples. And, the heteroskedasticity does not show up dramatically and hence not bias the estimation within each subsample. The data are monthly from January of 1976 to December 2014.

Table 11: Uncertainty Betas: The Broker-Dealer Leverage Index

| | A. Bad Risk Sharing Condition (BD Leverage Index is High) | | | B. Good Risk Sharing Condition (BD Leverage Index is Low) | | |
|---|---|---|---|---|---|---|
| | Mkt ex-Ret | $\Delta\nu_g$ | $\Delta\nu_c$ | Mkt ex-Ret | $\Delta\nu_g$ | $\Delta\nu_c$ |
| Book-to-Market Sort | (1) | (2) | (3) | (4) | (5) | (6) |
| Sort #1: Six Portfolios | | | | | | |
| Lowest 10% (Growth) | 1.09 | −2.81 | 6.80 | 1.03 | −1.61 | 11.27 |
| 10% − 30% | 0.98 | 2.41 | −0.99 | 0.97 | −2.00 | 0.98 |
| 30% − 50% | 0.98 | 3.82 | −3.35 | 0.91 | −5.75 | −3.75 |
| 50% − 70% | 0.89 | 1.66 | −6.59 | 0.87 | −4.79 | −3.67 |
| 70% − 90% | 0.88 | 1.22 | −9.79 | 0.84 | −4.30 | −1.40 |
| Highest 10% (Value) | 1.03 | 1.10 | −21.37 | 1.06 | −3.74 | −1.34 |
| Sort #2: Five Portfolios | | | | | | |
| Lowest 20% (Growth) | 1.05 | −1.94 | 3.81 | 1.02 | −1.35 | 6.99 |
| 20% − 40% | 0.98 | 4.86 | −1.52 | 0.95 | −3.49 | −0.72 |
| 40% − 60% | 0.93 | 2.90 | −5.48 | 0.89 | −7.02 | −3.67 |
| 60% − 80% | 0.86 | 0.78 | −6.20 | 0.79 | −3.30 | −3.69 |
| Highest 20% (Value) | 0.92 | 0.65 | −14.44 | 0.95 | −3.82 | −2.83 |
| Sort #3: Three Portfolios | | | | | | |
| Lowest 30% (Growth) | 1.03 | −0.77 | 3.62 | 1.01 | −1.21 | 5.41 |
| 30% − 70% | 0.94 | 2.45 | −4.26 | 0.89 | −4.89 | −4.30 |
| Highest 30% (Value) | 0.91 | 1.71 | −10.30 | 0.88 | −3.86 | −1.95 |

Notes: The table reports estimated betas of book-to-market sorted portfolios with respect to the market excess return, the growth uncertainty shock, and the cash-flow uncertainty shock. In particular, it compares the estimation results of two subsamples. One subsample consists of the periods in which the risk sharing condition is good, while the other subsample consists of the periods in which the risk sharing condition is poor. The periods of good or poor risk sharing conditions are estimated using the Broker-Dealer Leverage Index shown in Panel B of Figure 9. The regression model for estimating the betas is $r_{BM,t_m} = a_{BM} + \beta_{BM,z}(r_{M,t} - r_{f,t}) + \beta_{BM,g}\Delta\nu_{g,t_m} + \beta_{BM,c}\Delta\nu_{c,t_m} + \epsilon_{BM,t_m}$, where $BM$ stands for a book-to-market portfolio and $r_{BM,t_m}$ is the return of the book-to-market portfolio labeled by $BM$. The reported estimates are obtained by using the ordinary-least-squares method. To account for the heteroskedasticity in stock returns, I also use the weighted-least-squares method with inverse market variance to be the weights. The estimation results are quite similar, because the regressions are totally separated for different subsamples. And, the heteroskedasticity does not show up dramatically and hence not bias the estimation within each subsample. The data are monthly from January of 1976 to December 2014.

Table 12: Uncertainty Betas: The Credit Spread Index

| | A. Bad Risk Sharing Condition (Credit Spread is High) | | | B. Good Risk Sharing Condition (Credit Spread is Low) | | |
|---|---|---|---|---|---|---|
| | Mkt ex-Ret | $\Delta\nu_g$ | $\Delta\nu_c$ | Mkt ex-Ret | $\Delta\nu_g$ | $\Delta\nu_c$ |
| Book-to-Market Sort | (1) | (2) | (3) | (4) | (5) | (6) |
| Sort #1: Six Portfolios | | | | | | |
| Lowest 10% (Growth) | 1.07 | −2.81 | 0.78 | 1.03 | 2.37 | 1.61 |
| 10% − 30% | 0.97 | 0.41 | 0.24 | 1.06 | 0.92 | −0.15 |
| 30% − 50% | 0.98 | 0.80 | 0.59 | 1.00 | −3.92 | 0.27 |
| 50% − 70% | 0.98 | 2.56 | −3.22 | 0.86 | −2.23 | −1.82 |
| 70% − 90% | 0.98 | 2.43 | 0.07 | 0.86 | −4.51 | −0.35 |
| Highest 10% (Value) | 1.24 | 8.08 | −10.13 | 0.98 | −1.34 | −9.45 |
| Sort #2: Five Portfolios | | | | | | |
| Lowest 20% (Growth) | 1.03 | −0.97 | 1.73 | 1.04 | 0.56 | 2.71 |
| 20% − 40% | 0.96 | 0.20 | −0.67 | 1.04 | 0.88 | −2.12 |
| 40% − 60% | 0.97 | 0.76 | −0.92 | 0.94 | −2.13 | −2.10 |
| 60% − 80% | 0.96 | 2.77 | −1.01 | 0.82 | −5.82 | −0.67 |
| Highest 20% (Value) | 1.06 | 4.69 | −3.81 | 0.90 | −2.00 | −0.45 |
| Sort #3: Three Portfolios | | | | | | |
| Lowest 30% (Growth) | 1.01 | −1.53 | 0.53 | 1.05 | 1.02 | 0.96 |
| 30% − 70% | 0.98 | 1.26 | −0.84 | 0.93 | −2.83 | −0.55 |
| Highest 30% (Value) | 1.03 | 3.08 | −0.77 | 0.88 | −3.83 | −2.96 |

Notes: The table reports estimated betas of book-to-market sorted portfolios with respect to the market excess return, the growth uncertainty shock, and the cash-flow uncertainty shock. In particular, it compares the estimation results of two subsamples. One subsample consists of the periods in which the risk sharing condition is good, while the other subsample consists of the periods in which the risk sharing condition is bad. The periods of good or bad risk sharing conditions are estimated by using the simplest three-state regime-switching model of the credit spread index. The periods of bad risk sharing conditions are those estimated to have high credit spread index level, while the periods of good risk sharing conditions are those estimated to have low credit spread index level. The regression model for estimating the betas is $r_{BM,t_m} = a_{BM} + \beta_{BM,z}(r_{M,t} - r_{f,t}) + \beta_{BM,g}\Delta\nu_{g,t_m} + \beta_{BM,c}\Delta\nu_{c,t_m} + \epsilon_{BM,t_m}$, where $BM$ stands for a book-to-market portfolio and $r_{BM,t_m}$ is the return of the book-to-market portfolio labeled by $BM$. The reported estimates are obtained by using the ordinary-least-squares method. To account for the heteroskedasticity in stock returns, I also use the weighted-least-squares method with inverse market variance to be the weights. The estimation results are quite similar, because the regressions are totally separated for different subsamples. The heteroskedasticity does not show up significantly and hence not bias the estimation within each subsample. The data are monthly from January of 1976 to December 2014.

Table 13: MODEL VERSUS DATA: UNCERTAINTY EXPOSURES

| | LOW RISK SHARING CONDITION | | HIGH RISK SHARING CONDITION | | ALL | |
|---|---|---|---|---|---|---|
| BOOK-TO-MARKET SORT | $\Delta\nu_g$ (1) | $\Delta\nu_c$ (2) | $\Delta\nu_g$ (3) | $\Delta\nu_c$ (4) | $\Delta\nu_g$ (5) | $\Delta\nu_c$ (6) |
| | A. DATA: REINHART-ROGOFF INDEX | | | | | |
| LOW 10% | −3.11 | 2.56 | 2.40 | 14.10 | −0.47 | 3.84 |
| | (−0.75) | (0.85) | (0.47) | (3.51) | (-0.08) | (0.62) |
| HIGH 10% | 0.21 | −17.85 | −12.66 | −8.36 | 3.96 | −30.01 |
| | (0.04) | (−3.35) | (−2.42) | (−2.53) | (0.64) | (−3.11) |
| | B. DATA: BROKER-DEALER LEVERAGE INDEX | | | | | |
| LOW 10% | −2.81 | 6.80 | −1.61 | 11.27 | −0.47 | 3.84 |
| | (−0.75) | (2.21) | (−0.34) | (3.32) | (−0.08) | (0.62) |
| HIGH 10% | 1.10 | −21.37 | −3.74 | −1.34 | 3.96 | −30.01 |
| | (0.23) | (−3.73) | (−0.36) | (−0.24) | (0.64) | (−3.11) |
| | C. DATA: CREDIT SPREAD INDEX | | | | | |
| LOW 10% | −1.25 | 3.33 | 18.82 | 26.38 | −0.47 | 3.84 |
| | (−0.15) | (0.37) | (2.33) | (3.12) | (−0.08) | (0.62) |
| HIGH 10% | 25.19 | −41.17 | −7.10 | −32.40 | 3.96 | −30.01 |
| | (2.05) | (−6.78) | (−0.90) | (−3.36) | (0.64) | (−3.11) |
| | D. MODEL | | | | | |
| GROWTH | −0.88 | 1.14 | 0.31 | 0.11 | −0.09 | 0.43 |
| | (−3.66) | (5.42) | (1.99) | (2.10) | (−0.16) | (3.03) |
| VALUE | −0.31 | −1.06 | −0.37 | −0.13 | −0.34 | −0.47 |
| | (−1.91) | (−5.13) | (−2.34) | (−2.22) | (−2.01) | (−3.49) |

NOTES: The table compares unconditional moments of the data to their simulated analogies in the model. It reports boot-to-market sorted portfolios' uncertainty betas for the whole sample and two subsamples. The t-statistics are reported in the parentheses. In computing the t-statistics, the standard errors are estimated using Newey and West (1987, 1994) method with one lag. Data are sampled at the monthly frequency. Their sources and construction details are explained in the appendix. The sample period is 1976 – 2014. The risk sharing regimes are measured by using the Reinhart-Rogoff Index (the Broker-Dealer Leverage Index) in the Panel A (Panel B), while the risk sharing regimes are measured by using the credit spread index in Panel C. Panel D reports the simulated results based on the model. I simulate at the weekly frequency and then time-aggregate the simulated data to form monthly observations. In parentheses, they are t-statistics computed using 1,000 independent simulations, each with a length of 400 years.

Table 14: Interactions: Risk Sharing Conditions and Uncertainty Shocks

| Return Spreads of Highest 10% and Lowest 10% Book-to-Market Portfolio (Value Spread) Are Regressed on | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| Risks (Input Variables) | | | | | | | | |
| Intercept ($\alpha_{vi}$) | 5.65 | 5.82 | 6.03 | 5.72 | 6.56 | 6.53 | 5.54 | 5.52 |
| | (2.45) | (2.12) | (1.39) | (1.27) | (2.37) | (2.59) | (3.95) | (6.39) |
| Mkt ex-Ret ($\beta_{vi,z}$) | 0.21 | 0.22 | 0.18 | 0.18 | 0.21 | 0.22 | 0.03 | 0.04 |
| | (1.91) | (1.60) | (1.54) | (1.46) | (3.06) | (2.55) | (0.22) | (0.89) |
| $\Delta\nu_g$ ($\beta_{vi,g}$) | | 59.45 | −93.55 | −85.98 | 8.39 | 4.98 | −28.12 | −37.90 |
| | | (1.26) | (−1.29) | (−1.14) | (0.29) | (0.20) | (−0.74) | (−1.61) |
| $\Delta\nu_c$ ($\beta_{vi,c}$) | | −183.33 | −181.78 | −151.40 | −212.63 | −213.63 | −55.35 | −19.12 |
| | | (−2.31) | (−3.29) | (−3.39) | (−4.63) | (−3.01) | (−1.27) | (−0.73) |
| regime-x ($\beta_{vi,x}$) | | | −3.59 | −3.15 | −2.35 | −2.64 | −11.40 | −11.22 |
| | | | (−0.53) | (−0.45) | (−0.74) | (−1.01) | (−3.86) | (−7.57) |
| $\Delta\nu_g$ × regime-x ($\gamma_{vi,g}$) | | | 407.44 | 390.06 | 90.89 | 100.56 | 80.48 | 88.64 |
| | | | (2.61) | (2.38) | (2.27) | (3.24) | (1.82) | (6.25) |
| $\Delta\nu_c$ × regime-x ($\gamma_{vi,c}$) | | | | 66.39 | | 37.45 | | 63.17 |
| | | | | (0.72) | | (0.67) | | (2.67) |
| Adj-$R^2$ (%) | 1.50 | 19.13 | 22.75 | 21.66 | 18.42 | 17.32 | 16.63 | 14.27 |
| F-statistic | 1.92 | 5.73 | 4.53 | 3.76 | 3.66 | 3.06 | 3.29 | 2.75 |
| | (0.17) | (0.00) | (0.00) | (0.00) | (0.01) | (0.01) | (0.01) | (0.04) |

Notes: The table reports the results of regressions for value spreads. Column (1) reports the results of regressing value spreads on the constant term and the excess market return. Column (2) the uncertainty shocks into the regression. In Columns (3) and (4), the risk sharing condition (regime-$x_{t_y}$) is measured by the credit spread index; in Columns (5) and (6), the risk sharing condition (regime-$x_{t_y}$) is measured by the Reinhart-Rogoff Index; and, in Columns (7) and (8), the risk sharing condition (regime-$x_{t_y}$) is measured based on the Broker-Dealer Leverage Index. In Columns (3), (5), and (7), an extra independent variable regime-$x_{t_y}$ and its interaction term with the growth uncertainty shock $\Delta\nu_{g,t_y}$ × regime-$x_{t_y}$. In Columns (4), (6), and (8), the interaction terms between the state of risk sharing condition and the cash-flow uncertainty shock are added on the top of the regression (3), (5) and (7), respectively. The regressions are annual, because the state variable regime-$x_{t_y}$ is quite slow moving and monthly returns cause too much unnecessary noise for the inference about the slow moving state variable. The annual indices are constructed by averaging monthly or quarterly indices within each year. The coefficients are estimated based on weighted-least-square estimation where weights are inverse market return variance. The weighted-least-squares method is necessary, since heteroskedasticity shows up largely in this unified regression and it is correlated with the explanatory state variable regime-$x_{t_y}$. The data are from 1953 to 2014 for regressions in (1), (2), (5), and (6); due to restrictions of availability, the data are from 1976 to 2014 for regressions in (3), (4), (6), and (8).

Table 15: Model versus Data: Uncertainty-Beta Sorted Portfolios

| | Bad Risk Sharing Condition | | Good Risk Sharing Condition | | All | |
|---|---|---|---|---|---|---|
| Uncertainty-Beta Sort | $\Delta\nu_g$ (1) | $\Delta\nu_c$ (2) | $\Delta\nu_g$ (3) | $\Delta\nu_c$ (4) | $\Delta\nu_g$ (5) | $\Delta\nu_c$ (6) |
| | A. Data: Reinhart-Rogoff Index | | | | | |
| Low 20% | 3.77 | 6.89 | 13.79 | 16.79 | 12.38 | 17.45 |
| | (0.87) | (1.11) | (3.23) | (2.86) | (6.49) | (8.18) |
| High 20% | 1.46 | 0.21 | 15.75 | 14.11 | 12.34 | 12.34 |
| | (0.37) | (0.03) | (3.98) | (2.26) | (7.98) | (6.62) |
| High − Low | −2.30 | −6.68 | 1.96 | −2.27 | −0.04 | −5.11 |
| | (−1.24) | (−1.69) | (0.91) | (−0.42) | (−0.02) | (−3.02) |
| | B. Data: Broker-Dealer Leverage Index | | | | | |
| Low 20% | 7.99 | 10.19 | 12.76 | 14.91 | 12.38 | 17.45 |
| | (1.31) | (1.69) | (4.13) | (3.60) | (6.49) | (8.18) |
| High 20% | 4.98 | 5.21 | 16.49 | 14.12 | 12.34 | 12.34 |
| | (1.21) | (0.81) | (5.92) | (3.33) | (7.98) | (6.62) |
| High − Low | −3.00 | −4.99 | 3.73 | −0.80 | −0.04 | −5.11 |
| | (−2.44) | (−1.19) | (3.33) | (−0.27) | (−0.02) | (−3.02) |
| | C. Data: Credit Spread Index | | | | | |
| Low 20% | 14.32 | 13.04 | 13.41 | 19.09 | 12.38 | 17.45 |
| | (2.31) | (1.64) | (2.07) | (2.59) | (6.49) | (8.18) |
| High 20% | 8.98 | 9.45 | 17.77 | 15.37 | 12.34 | 12.34 |
| | (1.31) | (1.16) | (2.43) | (3.10) | (7.98) | (6.62) |
| High − Low | −5.34 | −3.59 | 4.36 | −3.72 | −0.04 | −5.11 |
| | (−1.13) | (−0.60) | (0.96) | (−1.23) | (−0.02) | (−3.02) |
| | D. Model | | | | | |
| Low | 16.97 | 18.33 | 11.22 | 16.03 | 13.73 | 16.33 |
| | (6.66) | (6.84) | (4.49) | (6.42) | (8.11) | (9.65) |
| High | 7.79 | 7.34 | 15.79 | 11.39 | 13.34 | 11.76 |
| | (2.60) | (2.76) | (6.28) | (4.56) | (7.88) | (6.95) |
| High − Low | −9.18 | −10.99 | 4.57 | −4.64 | −0.39 | −4.57 |
| | (−3.06) | (−3.98) | (1.79) | (−1.86) | (−0.23) | (−2.70) |

Notes: The table compares unconditional moments of the data to their simulated correspondences in the model. Within each Panel, it reports the average returns of uncertainty-beta sorted portfolios for the whole sample and two subsamples. The difference is that Panel A (Panel B) uses the Reinhart-Rogoff Index (Broker-Dealer Leverage Index) to measure risk sharing conditions, while Panel C uses the credit spread index to measure risk sharing conditions. The t-statistics are reported in the parentheses. Data are sampled at the monthly frequency. Their sources and construction details are explained in the online appendix. The sample period is 1976 – 2014. Panel D reports simulated average returns based on uncertainty-beta sorted portfolios in the model. I simulate the model at the weekly frequency and then time-aggregate the simulated data to form monthly observations. In parentheses, the numbers are t-statistics computed using 1,000 independent simulations, each with a length of 400 years.