Comparing Solution Methodologies for Macro-Finance Models with Nonlinear Dynamics*

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We provide a global solution method for a class of macroeconomic models featuring nonlinear dynamics and compare the global method with first order, second order, and occasionally binding local perturbation methods. This class of models includes macroeconomic models for unconventional monetary policies with financial intermediaries. Within this framework, we show that the solving the model globally is essential for policy function and impulse response analysis when the intermediary's financial constraint only binds occasionally, and when risk plays an important role in the economy. We also present an economy in which local methods fail when there does not exist a steady state with binding constraint.

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1 Introduction

The recent financial crisis and the Great Recession of 2007–2009 revealed serious gaps in commonly used approaches to define, measure, and manage financial sector activities that pose risks to the macroeconomy as a whole.

One emerging narrative is that macroeconomic models commonly employed at policy institutions for evaluating monetary policy lack the analytical specificity to account for important financial sector influences on the aggregate economy. A new generation of enhanced models and advanced empirical and quantitative methodologies are needed by policymakers and need to be provided by researchers to better study the impact of shocks that are initially large or build up endogenously over time.

This paper provides nonlinear global solution methods that are necessary, if one wishes to guarantee that key nonlinear dynamics in the financial market and the macroeconomy are eventually captured in quantitative analysis.

To illustrate the general solution method and algorithm, we present a fully specified canonical example with financial sectors in Section 2 that readers can work with immediately.The model in Section 2 is solved globally. We hope the contribution of our code and global solution method to this review may be of general interest to a broader group of researchers in the macrofinancial and monetary economics community. We calibrate this model using historical data in Section 3.1, and explore the empirical implications of our canonical model.

Our model is a variation of the standard Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) model. We solve the model with different solution methods, including first-order pertubation, second-order pertubation, occasionally binding pertubation by Guerrieri and Iacoviello (2015) (in short, OccBin), and global method. Under each method, we plot the policy functions and impulse responses of important economics variables, including consumption, investment, risk premium, price of capital, and credit policy. Compared to the more challenging global solution method, the simple and widely used first order and second order pertubation methods are limited when the economy features occasionally binding constraint, which is regarded to be a realistic feature of the functioning of the financial sector. The OccBin method is able to feature occasionally binding constraint, but the currently readily available algorithm and computation package (on Matteo Iacoviello's webpage) is only applicable to first order approximation. Extention to higher order is not a trivial task. Therefore, the method is limited not to consider the role of risk in driving macroeconomic dynamics and asset prices in the economy.

We evaluate the approximation errors to the Euler equations, market clearing conditions, and intratemporal pricing relations in the economy. First order (including OccBin) has relatively large Euler errors as the higher order terms are neglected. Second order approximation does well when the constraint binds, but has larger errors when then constraint is slack. Moreover, all the pertubation based methods heavily rely on the existence of a deterministic steady state in which the constraint always binds. We show that in an economy where the equilibrium investment rate is low, such deterministic steady state does not exist and the constraint binds only occasionally. This case is relevant given the large literature on financial crisis and "sudden stop". In this case, solving the model globally is the only choice.

Literature Review. From methodological perspective, there are mainly two classes of macro finance models. The first class incorporates financial frictions into a macroeconomic model and studies the economy with the constraint being (nearly) always binding. They mainly focus on the role of financial sector in amplifying business cycle fluctuations. This class includes Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Bernanke et al. (1999), Gertler and Karadi (2011), Jermann and Quadrini (2012), etc. In the second class of models, the economy behaves similarly to a frictionless one in normal times, but experiences a sharp decline (sudden stop) in the crisis. They focus on the strong nonlinear dynamics during the crisis, and the precautionary economic behaviors during normal times. Examples include Mendoza (2010), Bianchi (2011), Bianchi and Mendoza (2015), He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014), etc.

In terms of solution technique, the first class of models usually assume the constraint always binds and are solved with perturbation method (Bocola (2016) is an exception, which solves the model globally to emphasize the "risk channel"). The second class of models must be solved globally, which is more challenging.

There is a vast literature on numerically solving macroeconomic models both locally and globally, including pertubation, projection, and value function iteration methods. Fernández-Villaverde et al. (2016) provides the most recent comprehensive survey on this topic. Textbook treatment of numerical methods include Judd (1998), Miranda and Fackler (2004), Heer and Maussner (2009), Novales et al. (2008).

Perturbation is the most widely used and convenient method of solving dynamic models. It makes a Taylor expansion of policy functions around some deterministic steady state and transforms the nonlinear optimality conditions into (log-)linear systems. Judd and Guu (1993) shows how to apply perturbation methods in economic problems. Jin and Judd (2002) introduces higher-order perturbation, and Den Haan and De Wind (2012) improves the stability of high-order approximation. Judd (2002) compares two alternative ways of perturbation: using Taylor series in variable x and in variable $\log x$, and Fernández-Villaverde and Rubio-Ramírez (2006) explores the optimal change of variables to reduce approximation error. Andreasen et al. (2013) improves the stability of economic system with higher-order approximations with a pruned state space system. This strand of literature explores how to compute the solutions to economic problems more accurately with higher-order and better local approximation techniques. Schmitt-Grohé and Uribe (2004) provides a general introduction of using perturbation method.

The perturbation method is advantageous in its flexibility. It can handle an economic system with a large number of state variables with very nice global accuracy, as shown by Aruoba et al. (2006). The results are easy to interpret and computation is simple, especially with the usage of softwares such as dynare and dynare++. However, there are limitations to perturbation methods. First of all, it only provides accurate solutions around the deterministic steady state, around which we expand the policy function, so that it works worse if there exists a highly nonlinear relationship between economic variables. An example is the net foreign asset dynamics in open economy models, shown by Mendoza et al. (2016). Second, it is hard to handle problems in which policy function has kinks. These problems are very common in macroeconomic applications, including occasionally binding borrowing constraint, zero lower bound, corporate and sovereign default. Third, it heavily relies on the existence of a deterministic steady state.

Projection is an alternative way of solving macroeconomic models. We project policy function of the model onto some basis functions. The basis can be chosen globally (being nonzero and smooth for most of the domain) or locally (being zero for most of the domain). The former is also called spectral method, while the latter is called finite element method. Judd (1992) shows how to apply spetral method in solving macroeconomic models. One commonly used basis functions is Chebyshev polynomial basis. Finite element method chooses local basis functions and can well capture the local behavior of economic variables to high accuracy. However, the projection methods suffer from curse of dimensionality. Krueger and Kubler (2004) and Malin et al. (2011) apply Smolyak collocation method to simplify computation in multidimensional cases.

The value function iteration method (or the time-iteration method) is another widely used global method for solving macroeconomic problems that can be formulated into a contraction mapping. Conditions for contraction mapping can be found in Lucas and Stokey (1989). We can deal with kinks in policy function, heterogenous agents, and nonexistence of deterministic steady state with value function iteration. However, the method is also subject to the curse of dimensionality. It is crucial to have a relatively small number of endogenous state variables, and computing time increases exponentially if more state variables are introduced. The choice of grids crucially determines the computational complexity. There are several methods of choosing grid points to simplify computation: quadrature method by Tauchen and Hussey (1991), randomized grid method by Rust (1997), endogenous grid by Carroll (2006) and Barillas and Fernández-Villaverde (2007). Value function iteration is especially useful in solving models with heterogenous agents, such as Aiyagari (1994) and Krusell and Smith (1998). Reiter (2010) and Den Haan and Rendahl (2010) provide alternative algorithms. Den Haan (2010) makes comparisons of these algorithms. Brunnermeier and Sannikov (2016) proposed a value function iteration method to solve the continuous-time model in Brunnermeier and Sannikov (2014). When solving equilibria for economies with incomplete markets and nonlinear dynamics, incorporating the idea of projection method in each step of iterations can boost the value function iteration method.

2 A Canonical Macro-Finance Model

The purpose of this section is to provide a benchmark model for unconventional monetary policy analysis. This model will incorporate two defining features: the nonlinear dynamics of risk premia and the endogenous financial risks originating from imperfect intermediation. The financial crisis of 2008 and the accompanying Great Recession have highlighted the need for such models. Monetary authorities have become particularly aware of nonlinear risk premia and the real investment dynamics caused by dysfunctional financial intermediaries. As a result, unconventional monetary policies have been brought into the limelight by the monetary authorities following the financial crisis, and their role has fast become a focus of academic research.

Our model is a simplified version of the New Keynesian DSGE model proposed by Gertler and Kiyotaki (2010), yet extending it with regard to asset pricing dynamics. We solve the model globally based on time iteration projection procedure, as well as using local pertubation methods that are much easier to execute. The simple model enables us to compare solutions by different methods, and show when the local pertubation methods are limited. We believe the global method developed here is of general interest as well as an useful asset for the macro finance and monetary economics communities. In fact, we show that the global solutions are necessary if we are to have a model with occasionally binding financial constraints as well as important role of risk. Particularly, standard first and second order pertubation assumes the constraint always binds and get solutions deviating from the global ones when the constraint is slack. The OccBin method fails to capture the role of risk in the economy.

2.1 Households

We begin with a description of households in the model, and then turn to firms and intermediaries. There is a continuum of households of unit mass. The members of each household are either workers or bankers. Although there are two types of household members, and certain portfolio constraints among them, we assume the representative household framework following Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) by assuming the household members to be part of a large family, sharing everything or, equivalently, assuming that the full set of Arrow-Debreu securities are available to the members within each household (but not across households), so that the idiosyncratic consumption risks can be fully insured, and all household members have identical preferences. A fraction ϖ of the members of the household are bankers. At any time, a fraction $1-\theta$ of randomly selected existing bankers exit and become workers, and return their net worth to their households. At the same time, an equal number of workers become bankers within each household, so the proportion of workers and bankers remains fixed. The new bankers receive some start-up funds from their household, which we describe below. The "perpetual youth" assumption in our model is purely technical, with the purpose of guaranteeing the survivorship of workers and preventing the economy from evolving into a degenerate situation.[1] It can be seen analytically from the condition (40).

Each banker within a household manages a financial intermediary. Workers deposit funds into these financial intermediaries. Household members do not hold capital directly by themselves. Instead, these financial intermediaries hold equity claims on a firm's capital; their funding, in turn, comes partly from the deposits put down by household members. At the same time, all household members provide labor to the firm for production. The firm and intermediaries will be described in details in Sections 2.2 and 2.4, respectively.

Since all household members have identical preferences, there are no incentives for bankers to pay dividends from their financial intermediaries. Rather, bankers would choose to accumulate the net worth of the financial intermediary up to a critical level from which the financial intermediation will be out of the credit constraints and stay there forever. If the critical value is the total value of all assets, the workers will be eliminated from the economy in the long run. As mentioned previously, to avoid

[1] Another way to prevent the over-accumulation of intermediary net worth is to assume efficiency losses, as in Bolton et al. (2011).

this outcome, we assume that bankers and workers switch roles with probabilities $1-\theta$ and $(1-\theta)\frac{\varpi}{1-\varpi}$, respectively. When a banker switches roles, she pays all the accumulated net worth to her household. On the other hand, when a worker becomes a banker, she needs funds to operate. To be precise, the start-up fund is transferred from the household, and it is equal to a fraction $\frac{\aleph}{(1-\theta)\varpi}$ of the aggregate asset value for each new banker. The parameter $\aleph > 0$ characterizes the intensity of funding transfers from workers to bankers. We denote Π_t the net transfer from bankers to workers, which will be defined later.

There are two points worth mentioning. First, the members of each household in this economy are divided into bankers and workers, both of whom supply labor, but only bankers own capital. In this way, bankers decide how much capital to accumulate given the capital adjustment costs. The heterogeneity of agents, however, mainly serves as an interpretational device.[2] Second, a potentially important deficiency is that the role of bankers is hardwired into the model: there is no other way to provide capital finance in financial markets. This sidesteps potentially important opportunities for flexibility in funding sources, an issue discussed more substantially in de Fiore and Uhlig (2011) and de Fiore and Uhlig (2015), for example.

The preferences of the household are given by

$$\mathbb{E}_t\left[\sum_{\tau=0}^{\infty}\beta^{\tau}\frac{C_{t+\tau}^{1-\gamma}}{1-\gamma}\right], \tag{1}$$

where C_t is the consumption and L_t is the labor supply at time t.

We denote by $R_{f,t}$ the real interest rate. Let B_t denote the quantity of the risk-free bank debt held by the household at the end of period t, and $B_{g,t}$ denote the quantity of the risk-free government debt held by households at the end of period t. Bank debt and government are both risk free, so they are perfect substitutes to the

[2] For example, the equilibrium and its implications should not be affected if all households are homogeneous. Each household manages an intermediary. Households can invest in a firm's equity only through their intermediaries. Each household randomly terminates its intermediary and transfers its net worth to all households. Afterwards, they immediately start new intermediaries with funds collected from the households.

households, but bank debt enables banks to lever up and invest in capital privately, while government debt finances government purchase of capital. The household then faces a state-by-state aggregate budget constraint

$$C_t = W_t L_t + \Pi_t - T_t + (1 + R_{f,t-1})(B_{t-1} + B_{g,t-1}) - (B_t + B_{g,t}), \tag{2}$$

where W_t is the real wage, Π_t is the net profit from exiting intermediaries. T_t is the real lump sum taxes, and $R_{f,t-1}$ is the net real risk-free rate from the end of period $t-1$ to the end of period t. We assume each household provides one unit of labor inelastically, and thus the total labor supply remains $L_t \equiv 1$. Implicitly, we assume that households (workers and bankers) can trade a full set of Arrow-Debreu securities so that consumption risk is perfectly insured. The total payoffs of Arrow-Debreu securities are zero in aggregate. The intertemporal Euler equation for risk-free bond holding is

$$1 = \mathbb{E}_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} (1 + R_{f,t}) \right], \quad \text{where } \Lambda_t \equiv \beta^t (C_t)^{-\gamma}. \tag{3}$$

Here, Λ_t is the marginal utility of consumption C_t at date t.

2.2 Consumption Goods Sector

There is a continuum of firms of mass unity in the consumption goods sector. Each firm produces its output using an identical Cobb-Douglas production function with capital and labor as its input. The labor market is perfectly competitive, and labor is perfectly mobile across firms. As a result, there exists a representative firm with the same Cobb-Douglas production function:

$$Y_t = A_{c,t} K_t^{\alpha} L_{c,t}^{1-\alpha}, \quad 0 < \alpha < 1, \tag{4}$$

where $A_{c,t}$ is an exogenous and stochastic total factor productivity (TFP) parameter for consumption goods production, K_t is aggregate capital installed at the end of period $t-1$, and $L_{c,t}$ is aggregate labor demand in the consumption goods sector. Denote $a_t \equiv \ln A_{c,t}$. The TFP and the capital quality evolve as

$$a_t = a_{t-1} + \sigma_a \epsilon_{a,t}, \tag{5}$$

where $\epsilon_{a,t}$ are i.i.d. standard normal variables.

Firms are owned by intermediaries. There is no friction between firms and intermediaries. A firm's investment is still subject to constraints, however, since intermediaries can be financially constrained. Firms always choose to pay their earnings to their intermediaries since the marginal value of cash for intermediaries is never less than that for firms. In addition, the capital structure of firms is irrelevant, since the firm's leverage may always be neutralized by the intermediary leverage, and it is the total leverage that is eventually subject to the credit constraint. Therefore, similar to most macroeconomic and asset pricing models, it is assumed that firms are all-equity firms, and pay out all their earnings. Such a firm has no wealth of its own, i.e. retained earnings. In period t, it issues new equity to intermediaries and uses the proceeds to purchase capital I_t, to be used for production in the next period[3] $t+1$. The number of shares issued by the firm is normalized to one, although equity issuances occur over time.

Let us describe more details about the firm's production, hiring, and investment decisions along the timeline. Shocks are realized at the very beginning of each period. Observing the shocks, the firm hires labor at a perfectly competitive wage W_t and uses the capital K_t chosen at the end of period $t-1$ to produce the consumption goods using the production function specified in (4).

After production takes place, the firm makes its investment by converting investment goods into new capital, and trading with other firms in a capital spot market. Together with the newly created capital, the depreciated old capital is traded freely

[3] Note that we adopt the more conventional timing assumption, and index capital with the date when it is used in production, not with the date of the decision. As is well known, one needs to be careful when implementing this in solution software such as Dynare or Uhlig's Toolkit. See Uhlig (1999).

in the spot market, and the amount of capital stock is optimally chosen for the next period. We denote the aggregate investment as I_t and the depreciation rate as δ. The law of motion for aggregate capital stock is given by

$$K_{t+1} = I_t + (1-\delta)K_t. \tag{6}$$

There are convex adjustment costs for the rate of investment, I_t/K_t. We assume that the cost for creating I_t units of new capital in terms of investment goods is

$$\Upsilon_t = \Upsilon(I_t; K_t) \equiv I_t + g(I_t, K_t), \quad \text{where } g(I_t, K_t) \equiv \frac{\vartheta}{2}\left(\frac{I_t}{K_t}\right)^2 K_t, \tag{7}$$

where $\vartheta > 0$ is a constant. The investment goods are produced by investment good firms. In the simpler cases adopted by macroeconomic asset pricing models (e.g. Gomes et al., 2003; Uhlig, 2007; Guvenen, 2009), a firm's investment converts consumption goods directly into new capital. By introducing an investment goods sector which exogenously maintains a stable scale relative to the whole economy, we can show that there exists a competitive equilibrium fluctuating around the balanced growth path. Similar methods have been adopted by Kogan et al. (2015) and Dou (2016). Details about the investment goods sector are introduced in Section 2.3.

Let us introduce the payout and valuation of firms. Arriving in period t with capital stock K_t chosen in $t-1$, the firm will choose labor L_t and investment I_t optimally. The profit of each firm is

$$X_t = \underbrace{\left[A_{c,t}K_t^{\alpha}L_{c,t}^{1-\alpha} - W_t L_{c,t}\right]}_{\text{production profits}} + \underbrace{\left[Q_t I_t - P_t \Upsilon(I_t; K_t)\right]}_{\text{investment profits}}, \tag{8}$$

where Q_t and P_t are the equilibrium spot prices of capital and investment goods, respectively. The net capital gain at the end of period t is $Q_t(1-\delta)K_t = Q_t K_{t+1} - Q_t I_t$, since all firms trade in a perfectly competitive spot market of capital with equilibrium spot price Q_t. In effect, there is zero net trade among firms in equilibrium, since all firms are assumed to be homogeneous, even in the ex post situation.

The value of capital after depreciation, $Q_t(1-\delta)K_t$, can be viewed as the net capital gain of holding the "corporate sector". We do not explicitly model assets in place and growth options separately on firm balance sheets. More precisely, the capital in stock K_t implicitly contains both assets in place and growth options, and thus the value Q_t contains two components: the value of assets in place and the value of growth opportunities. Thus, the stock return can be represented as follows:

$$\begin{aligned} 1+R_{k,t+1} &\equiv \underbrace{\frac{Y_{t+1}-W_{t+1}L_{c,t+1}+Q_{t+1}(1-\delta)K_{t+1}}{Q_tK_{t+1}}}_{\text{return due to assets in place}} + \underbrace{\frac{Q_{t+1}I_{t+1}-P_{t+1}\Upsilon(I_{t+1},K_{t+1})}{Q_tK_{t+1}}}_{\text{return due to growth options}} \\ &= \frac{X_{t+1}}{Q_tK_{t+1}} + \underbrace{\frac{Q_{t+1}(1-\delta)K_{t+1}}{Q_tK_{t+1}}}_{\text{net capital gain return}}, \end{aligned} \tag{9}$$

where Q_tK_{t+1} is the value of assets at the end of period t. Capital gains from holding corporate shares are included in the profit X_t. To sort out the consumption component in the return, we introduce the "dividend" of firms:

$$D_t \equiv Y_t - W_tL_{c,t} - P_t\Upsilon(I_t,K_t). \tag{10}$$

The stock return can be rewritten in terms of dividends as

$$1+R_{k,t+1} = \underbrace{\frac{D_{t+1}}{Q_tK_{t+1}}}_{\text{total dividend return}} + \underbrace{\frac{Q_{t+1}K_{t+2}}{Q_tK_{t+1}}}_{\text{total capital gains return}}. \tag{11}$$

It is worth mentioning that our model follows Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), and includes a separate capital goods sector, a sector which produces capital and sells to consumption good firms. Basically, the model, rather than taking the macro view of stock returns, takes the portfolio view, as defined in

Larrain and Yogo (2008).[4] Here in this model, consumption good firms are assumed to be short-lived, and the value of their outstanding shares is assumed to be equal to the size of capital.

The adjustment cost function has no intertemporal feature in itself; as a result, the intertemporal and dynamic aspects of investment decisions are captured by the forward-looking capital price Q_t. The trading in the competitive spot market of capital breaks the direct link between the current investment I_t and the capital stock for next period's production K_{t+1} for each firm. Thus, the current decision I_t has no effect on the following decisions I_{t+1} through K_{t+1}. As a result, the investment decision is not dynamic, and the standard q theory of Hayashi (1982) holds. To see this more clearly, consider the firm's optimization problem at the end of period t:

$$
\begin{aligned}
&K_{t+1}Q_t \geq \max_{L_{c,t+1},I_{t+1}} \mathbb{E}_t \left\{ \mathcal{M}^{\mathcal{I}}_{t,t+1} \left[D_{t+1} + Q_{t+1}(1-\delta)K_{t+1} \right] \right\} \qquad (12)\\
&= \max_{L_{c,t+1},I_{t+1}} \mathbb{E}_t \left\{ \mathcal{M}^{\mathcal{I}}_{t,t+1} \left[Y_{t+1} - W_{t+1}L_{c,t+1} + Q_{t+1}\left(I_{t+1} + (1-\delta)K_{t+1}\right) - P_{t+1}\Upsilon(I_{t+1};K_{t+1}) \right] \right\}
\end{aligned}
$$

where $\mathcal{M}^{\mathcal{I}}_{t,t+1}$ is the effective intertemporal marginal rate of substitution (IMRS) of financial intermediaries. The equilibrium asset pricing condition depends on inequality (i.e. the supermartingale condition), instead of equality (i.e. the martingale condition), due to the credit constraints. Furthermore, due to the intermediary's credit constraint, the intermediary's IMRS can be different from the household's actual IMRS $\mathcal{M}_{t,t+1} \equiv \Lambda_{t+1}/\Lambda_t$. We denote $\Omega^{\mathcal{I}}_{t,t+1}$ as the wedge between the intermediary's effective IMRS ($\mathcal{M}^{\mathcal{I}}_{t,t+1}$) and the household's IMRS ($\mathcal{M}_{t,t+1}$); more precisely,

$$
\mathcal{M}^{\mathcal{I}}_{t,t+1} = \Omega^{\mathcal{I}}_{t,t+1}\mathcal{M}_{t,t+1}. \qquad (13)
$$

[4]The difference between two views as regards stock returns is the growth option component. To be more precise, the stock return under the macro view is $1 + R^{\mathrm{m}}_{k,t+1} \equiv \dfrac{Y_{t+1} - W_{t+1}L_{c,t+1} + Q_{t+1}(1-\delta)K_{t+1}}{Q_tK_{t+1}} + \dfrac{Q_{t+1}I_{t+1} - \Upsilon(I_{t+1},K_{t+1})}{Q_tK_{t+1}}$. In Gertler and Kiyotaki (2010) and Gertler and Karadi (2011) which takes the portfolio view, the stock return is $1 + R^{\mathrm{p}}_{k,t+1} \equiv \dfrac{Y_{t+1} - W_{t+1}L_{c,t+1} + Q_{t+1}(1-\delta)K_{t+1}}{Q_tK_{t+1}}$. Effectively, the gap between $R^{\mathrm{m}}_{k,t+1}$ and $R^{\mathrm{p}}_{k,t+1}$ is the net return due to growth options.

The wedge $\Omega^{\mathcal{I}}_{t,t+1}$ is derived and discussed in Section 2.4. Briefly, the wedge $\Omega^{\mathcal{I}}_{t,t+1}$ characterizes the economic tightness of the credit constraint. The wedge $\Omega^{\mathcal{I}}_{t,t+1}$ becomes larger when the credit constraint is binding more tightly for financial intermediaries. In addition, when the credit constraints are not binding for intermediaries, the equality holds in the asset pricing condition (12). In fact, the Hamilton-Jacob-Bellman (HJB) equation in (12) can be rewritten in terms of stock returns as

$$1 \geq \mathbb{E}_t \left[\mathcal{M}^{\mathcal{I}}_{t,t+1}(1 + R_{k,t+1}) \right]. \tag{14}$$

Finally, let us characterize the equilibrium relationships between optimal investment, hiring, and consumption. Given the capital price Q_{t+1} and the price of investment goods P_{t+1}, the problem of optimal investment for firms can be decomposed into a sequence of state-by-state static (intratemporal) optimization problems:

$$\max_{I_{t+1}} Q_{t+1} I_{t+1} - P_{t+1} \Upsilon(I_{t+1}; K_{t+1}). \tag{15}$$

The state-by-state first-order condition with respect to I_{t+1} gives

$$Q_{t+1}/P_{t+1} = 1 + \vartheta i_{t+1}, \quad \text{where} \;\; i_{t+1} \equiv \frac{I_{t+1}}{K_{t+1}}. \tag{16}$$

This is the standard q theory of investment developed by Hayashi (1982), in which the investment decision I_{t+1}/K_{t+1} is directly linked to the marginal q (marginal value) of the capital.

Similarly, the optimal labor demand can also be derived from the state-by-state (static) optimization problem:

$$\max_{L_{c,t+1}} A_{c,t+1} K^{\alpha}_{t+1} L^{1-\alpha}_{c,t+1} - W_{t+1} L_{c,t+1} \tag{17}$$

The first-order condition with respect to $L_{c,t+1}$ gives

$$L_{c,t+1} = \left[(1-\alpha)\frac{A_{c,t+1}}{W_{t+1}}\right]^{1/\alpha} K_{t+1}. \tag{18}$$

The consumption goods are non-durable, and thus the market clearing condition implies the characterization for aggregation consumption goods:

$$Y_t = D_t + W_t L_{c,t} + W_t L_{\iota,t}. \tag{19}$$

2.3 Investment Goods Sector

There is a continuum of investment good firms which produce investment goods using labor. These firms are identical, and they have the same production function:

$$\Upsilon_t = A_{\iota,t} L_{\iota,t} \tag{20}$$

where $A_{\iota,t}$ is the productivity of investment goods production, and $L_{\iota,t}$ is the labor demand in the investment goods sector. We assume that the scale of the investment goods section is co-integrated with the scale of the consumption goods sector. More precisely, we simply assume that $A_{\iota,t} = Z_{\iota,t} K_t$ where $Z_{\iota,t}$ follows a stationary stochastic process. For simplicity, the process $Z_{\iota,t}$ is assumed to be constant. It is worth mentioning that K_t is the total physical capital stock that cannot be internalized by single investment good firms, and thus it is the exogenous scale of the investment goods sector. This guarantees that the balanced growth path is $A_{c,t} K_t^{\alpha}$.

This assumption creates an externality for making investment. Installing more capital not only brings higher cash flow and larger growth option to the investment firm, but also enhances the productivity of the investment good production sector. Investment firms fail to internalize this extra benefit when making investment decisions. Therefore, investment is suboptimal even without the financial frictions. The externality works in the same way as Romer (1986).

Appendix A.3 provides detailed comparison between social planner's optimality conditions and optimality conditions in the decentralized equilibrium. The difference can be clearly seen from the first order condition (94) in Appendix A.3. The marginal value of capital to the social planner does not exclude the labor cost in producing investment goods. This part of labor cost is offset by the productivity enhancing effect of capital. All other optimality equations in the social planner's problem are exactly identical to those in the decentralized equilibrium.

This assumption is useful to ensure a balanced growth path $A_{c,t}K_t^{\alpha}$, while not keeping K_t as an endogenous state variable. In the numerical analysis of the model with financial frictions, the net worth share of intermediaries plays a key role as state variable. It greatly simplifies our numerical exercise by keeping endogenous state variable minimal, i.e., the net worth share of intermediaries being the single state variable.

On the other hand, this assumption is innocuous for the purpose of our paper. Our paper aims to present and compare numerical solutions to the general class of macro finance models. The parsimony our model displays is helpful in making it transparent and clear. By comparing solutions to our model in the main text with frictions to solutions to the benchmark model, we can easily identify the source of inefficiencies caused by financial frictions.

The market clearing condition for labor market requires

$$L_{c,t} + L_{\iota,t} = L_t \quad \text{for all } t. \tag{21}$$

All the investment good firms produce and sell investment goods competitively. As a result, they are zero-profit firms. We denote the competitive price of investment goods as P_t.

2.4 Financial Intermediaries

Financial intermediaries borrow funds from households at a risk-free rate, pool the funds with their own net worth and invest the sum in the equity of the representative consumption good firm. The balance sheet of intermediary j at the end of time t is given by

$$Q_t K_{t+1} S_{j,t} = N_{j,t} + B_{j,t}, \tag{22}$$

where Q_t is the price of the firm's equity, $S_{j,t}$ is the quantity of equity held by the intermediary, $N_{j,t}$ is the net worth, and $B_{j,t}$ is the deposits raised from households. The intermediary earns a return $R_{k,t+1}$ from the equity investment at time $t+1$, and must pay the interest, $R_{f,t}$, on the deposit. The net worth of the intermediary, therefore, evolves as

$$\begin{aligned} N_{j,t+1} &= (1 + R_{k,t+1}) Q_t K_{t+1} S_{j,t} - (1 + R_{f,t}) B_{j,t} \\ &= (R_{k,t+1} - R_{f,t}) Q_t K_{t+1} S_{j,t} + (1 + R_{f,t}) N_{j,t}. \end{aligned} \tag{23}$$

where $N_{j,t+1}$ is intermediary j's net worth at the end of period $t+1$.

The intermediaries face a constraint on raising deposits from households. They cannot raise deposits beyond a certain level, which is determined endogenously in the equilibrium. We shall describe this constraint in more detail below. Since bankers own the intermediaries, we use the bankers' IMRS, which coincides with the IMRS of the representative household, $\mathcal{M}_{t,t+1} \equiv \Lambda_{t+1}/\Lambda_t$, to determine the value of assets held by an intermediary according to the cash flows received by the bankers.

The following schematic (Figure 1) is the timing convention for financial intermediaries and firms, to help explain the ordering of these events within the model.

Since an intermediary exits exogenously in each period with probability $1-\theta$, the value of intermediary j's terminal wealth to its household is given by

$$V_{j,t} = \max_{\{S_{j,t+\tau}, B_{j,t+\tau}\}_{\tau \geq 1}} \mathbb{E}_t \left[\frac{\Lambda_{t+\tilde{\tau}_j}}{\Lambda_t} N_{j,t+\tilde{\tau}_j} \right], \tag{24}$$

where $\tilde{\tau}_j$ is the stochastic stopping time for the financial intermediary j to exit and

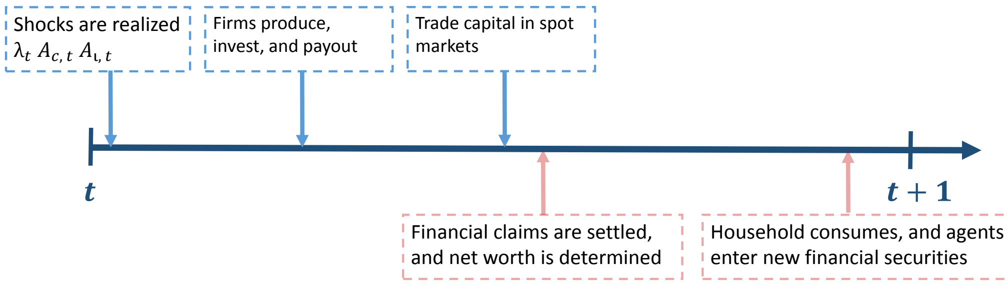


Figure 1: Timeline convention.

pay out the net worth $N_{\tilde{\tau}_j}$ to its banker. Thus, the value of the financial intermediary j can be expressed as weighted average of discounted possible "payouts" (net worth $N_{t+\tau}$):

$$\begin{aligned} V_{j,t} &= \max_{\{S_{j,t+\tau},B_{j,t+\tau}\}_{\tau\geq 1}} \sum_{\tau=1}^{+\infty} \mathbb{P}(\tilde{\tau}_j = \tau)\mathbb{E}_t\left[\frac{\Lambda_{t+\tau}}{\Lambda_t} N_{j,t+\tau}\right] \\ &= \max_{\{S_{j,t+\tau},B_{j,t+\tau}\}_{\tau\geq 0}} \sum_{\tau=1}^{+\infty} (1-\theta)\theta^{\tau-1}\mathbb{E}_t\left[\frac{\Lambda_{t+\tau}}{\Lambda_t} N_{j,t+\tau}\right] \\ &= \max_{S_{j,t+1},B_{j,t+1}} \mathbb{E}_t\left\{\frac{\Lambda_{t+1}}{\Lambda_t}\left[(1-\theta)N_{j,t+1} + \theta V_{j,t+1}\right]\right\}. \end{aligned} \tag{25}$$

The equation (25) is the HJB equation for the value of financial intermediary j.

In order to motivate the borrowing constraint faced by financial intermediaries, we introduce a simple moral hazard/costly enforcement problem following Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). We assume that the banker can choose to liquidate the financial intermediation and divert the fraction of available funds, λ_t, from the value of the financial intermediation.

The borrowing constraint is modeled as follows. At any time t, the banker of the intermediary can divert a fraction λ_t of the intermediary's assets for his own benefit, where λ_t is an exogenous parameter. The log of margin parameter $\ln\lambda_t$ follows a first-order Markov chain with long-term mean $\overline{\lambda}$, autocorrelation ρ_λ, and long-term variance $\overline{\sigma}_\lambda^2$. The quantity $1-\overline{\lambda}$ measures the steady-state pledgeability of the intermediary's asset. If the value of the intermediary falls below $\lambda_t Q_t S_{j,t}$, the banker will simply divert the assets and terminate the intermediary. In such a case, households will get a zero gross return from their deposits. In order for the households to have an incentive to deposit cash with the intermediary, the following condition must hold:

$$V_{j,t} \geq \lambda_t Q_t K_{t+1} S_{j,t}. \tag{26}$$

Because utility functions are homothetic, the optimal portfolios are linear in terms of the net worth. Thus, the value of financial intermediaries is also linear in the net worth and therefore can be characterized as follows:

$$V_{j,t} = \Omega_t N_{j,t} \tag{27}$$

where Ω_t is the marginal value of net worth for financial intermediaries. Since each financial intermediary is atomistic, it does not affect the equilibrium. Furthermore, the cross-sectional distribution of the net worths of the intermediaries does not affect the equilibrium either, due to the linearity of the optimal portfolio holdings of intermediaries guaranteed by the homothetic utilities. Instead, the total net worth share $n_t \equiv \frac{N_t}{Q_t K_{t+1}}$ is the only endogenous state variable needed for characterizing the equilibrium, where $N_t \equiv \int_j N_{j,t} \mathrm{d}j$ is the aggregate net worth in all intermediaries. Let $S_{p,t} \equiv \int_j S_{j,t} \mathrm{d}j$ be the aggregate outstanding shares of firms held by private financial intermediaries. The total supply of $S_{p,t}$ is determined by the credit policy of the government.

We conjecture that Ω_t only depends on the aggregate exogenous state λ_t and the aggregate endogenous state variable n_t. The aggregate net worth characterizes the average leverage of financial intermediaries, and thus the incentives for bankers to walk away from their financial intermediaries. As a consequence, it also determines intuitively the tightness of the credit constraint, and in turn, the expected returns to financial holdings. The multiplier Ω_t can be interpreted as the marginal value ("marginal q") of the net worth held by intermediaries.

Following (25) and (26), the portfolio problem of the financial intermediary with credit constraints can be written as,

$$\Lambda_t \Omega_t N_{j,t} = \max_{S_{j,t+1}, B_{j,t+1}} \mathbb{E}_t \left[\Lambda_{t+1} \left(1 - \theta + \theta \Omega_{t+1}\right) N_{j,t+1}\right] + \mu_{j,t} \Lambda_t \left(\Omega_t N_{j,t} - \lambda_t S_{j,t} Q_t K_{t+1}\right)$$

subject to

$$N_{j,t} = S_{j,t} Q_t K_{t+1} - B_{j,t}, \quad \text{and} \tag{28}$$

$$N_{j,t+1} = S_{j,t} Q_t K_{t+1}(1 + R_{k,t+1}) - B_{j,t}(1 + R_{f,t}) \tag{29}$$

and $\mu_{j,t} \geq 0$ and $\Omega_t N_{j,t} \geq \lambda_t S_{j,t} Q_t K_{t+1}$. Here, $\mu_{j,t}$ is the Lagrangian multiplier normalized by Λ_t; it is non-negative, and becomes positive if and only if the credit constraint becomes binding for intermediary j. This is the HJB equation which formulates the optimization problem of the intermediaries. Because the intermediaries are the only channel to hold risky assets of the economy, and they face a leverage constraint, the marginal value of cash of the intermediaries $\Lambda_t(1 - \theta + \theta\Omega_t)$ is always larger than the marginal value of cash outside the intermediaries Λ_t. As a result, the intermediaries are the natural borrowers in the economy. That is, $B_{j,t} \geq 0$ for each intermediary j. In aggregate, it holds that $0 < N_t \leq S_{p,t} Q_t K_{t+1}$ and thus $0 < n_t \leq 1$.

The first-order condition of substituting between $S_{j,t}$ and $B_{j,t}$ gives

$$0 \leq \mu_{j,t} \lambda_t \Omega_t^{-1} = \mathbb{E}_t \left[\mathcal{M}^{\mathcal{I}}_{t,t+1}(R_{k,t+1} - R_{f,t})\right], \tag{30}$$

where $\Omega^{\mathcal{I}}_{t,t+1} \equiv \dfrac{1 - \theta + \theta\Omega_{t+1}}{\Omega_t}$. The wedge $\Omega^{\mathcal{I}}_{t,t+1}$ is the core component of the so-called "intermediary asset pricing theory" and the effective IMRS of intermediaries is $\mathcal{M}^{\mathcal{I}}_{t,t+1} \equiv \mathcal{M}_{t,t+1}\Omega^{\mathcal{I}}_{t,t+1}$. The condition shows that $\mathbb{E}_t \left[\mathcal{M}^{\mathcal{I}}_{t,t+1}(R_{k,t+1} - R_{f,t})\right] > 0$ when the credit constraint is binding. This condition by itself appears to violate the supermartingale condition of self-financed cash flows. Thus, there appears to be a possible arbitrage opportunity by going long on equity and going short on risk-free bonds. The absence of arbitrage still holds since the intermediary cannot further increase its leverage when its credit constraint is binding.

Plugging (30) into the HJB equation for intermediaries leads to pricing rules for risk-free bonds and firm equity, respectively:

$$1 \geq 1 - \mu_{j,t} = \mathbb{E}_t\left[\mathcal{M}^{\mathcal{I}}_{t,t+1}(1 + R_{f,t})\right], \quad \text{and} \tag{31}$$

$$1 \geq 1 - \mu_{j,t}(1 - \lambda_t \Omega_t^{-1}) = \mathbb{E}_t\left[\mathcal{M}^{\mathcal{I}}_{t,t+1}(1 + R_{k,t+1})\right]. \tag{32}$$

The inequality in (32) holds because λ_t is between 0 and 1. The supermartingale conditions hold for equity returns and risk-free rates separately. From (31) and (32), we can see that the Lagrangian multipliers $\mu_{j,t}$ for intermediaries should be the same. Upon reflection, we turn our focus to a symmetric equilibrium. Denote $\mu_t \equiv \mu_{j,t}$ for all j. In the symmetric equilibrium, each intermediary chooses the shares of equity to hold proportionally to its net worth in the following sense:

$$S_{j,t} Q_t K_{t+1} = s_t N_{j,t}, \tag{33}$$

for all j and s_t only depending on aggregate state variables. Given equilibrium asset prices and returns, the optimal holding s_t can be characterized by only the market-clearing condition in the equity market. The supply to private financial intermediaries is $S_{p,t} = 1 - S_{g,t}$ and the market-clearing condition is

$$S_{p,t} Q_t K_{t+1} = s_t N_t. \tag{34}$$

Thus, the optimal holding increases in the total supply of equity ($S_{p,t}$), and decreases in the total net worth of intermediaries (x_t):

$$s_t = \frac{S_{p,t}}{n_t}. \tag{35}$$

2.5 Net Worth Evolution

After integrating the dynamic equations in (23) over all intermediaries and accounting for the net fund transfer, the aggregate net worth evolves as

$$N_{t+1} = \tilde{N}_{t+1} - \Pi_{t+1}$$

where:

$$\Pi_{t+1} = (1-\theta)\widetilde{N}_{t+1} - \aleph Q_{t+1} K_{t+2} \tag{36}$$

$$\widetilde{N}_{t+1} = (R_{k,t+1} - R_{f,t}) Q_t K_{t+1} S_{p,t} + (1 + R_{f,t}) N_t. \tag{37}$$

Here, N_t is the (end-of-period) aggregate net worth of of intermediaries in time t after intermediary payout. The quantity $\widetilde{N}_{t+1} = (R_{k,t+1} - R_{f,t})Q_t K_{t+1} S_{p,t} + (1 + R_{f,t})N_t$ is the aggregate net worth before intermediary payout, and a fraction $1 - \theta$ exits the market and pay out their net worth. In the meantime, $\aleph Q_{t+1} K_{t+2}$ of net startup fund are given to newly entered intermediaries in time $t + 1$.
Plug in the expression for payout, we have:

$$N_{t+1} = \theta[(R_{k,t+1} - R_{f,t}) Q_t K_{t+1} S_{p,t} + (1 + R_{f,t}) N_t] + \aleph Q_{t+1} K_{t+2} \tag{38}$$

Thus, the net worth share of intermediaries evolves as

$$n_{t+1} = \theta\left[(R_{k,t+1} - R_{f,t}) S_{p,t} + (1 + R_{f,t}) n_t\right] / G_{k,t+1} + \aleph \tag{39}$$

where $G_{k,t+1} \equiv \dfrac{Q_{t+1} K_{t+2}}{Q_t K_{t+1}}$ is the total capital gain of equity.

Let μ^* be the upper bound of the dividend-price ratio of stocks in the frictionless economy. Then the net worth share n_t is always less than 1 when

$$\mu^* < (1-\theta) - \aleph. \tag{40}$$

The specifications of μ^* can be found in Appendix A. In the case when (40) holds, there exists $\underline{n}_t \in (0, \lambda_t S_{p,t})$ characterizing the constraint-binding boundary such that

$$\Omega_t = \begin{cases} \dfrac{\lambda_t S_{p,t}}{n_t}, & \text{when} \quad n_t \in (0, \underline{n}_t]; \\ \Omega(n_t, \lambda_t) > \max\left\{\dfrac{\lambda_t S_{p,t}}{n_t}, 1\right\}, & \text{when} \quad n_t \in (\underline{n}_t, 1). \end{cases}$$

The net worth share n_t never reaches the limit 1 since there is no efficiency loss attached to the intermediary net worth. Solving the equilibrium is effectively solving for the functional form of $\Omega(n_t, \lambda_t)$.

2.6 Government Policies

The ultimate goal of our model is to analyze the effectiveness of unconventional monetary policies in fighting financial crises and their destructive impact on the macroeconomy as a whole.

According to Section 13.3 of the Federal Reserve Act, the Fed is allowed to take risky positions through making loans in the private sector (provided that they are not unduly so), under "unusual and exigent circumstances." This legislation basically makes the Fed the lender of last resort of the economy. Meanwhile, the Treasury, the Fed, the FDIC, and the bailout bills passed by Congress together took unconventional policy measures, including equity injection into the private sector, asset purchases from distressed banks, the lifting of caps on deposit insurance for certain bank accounts, and lending guarantees for certain types of bank loans. All these policies and interventions were structured to encourage firms to bring in private capital. For instance, it was intentionally designed that firms returning capital to the government by certain dates would get better terms for the government's stake. The central plank of all these unconventional measures was to attract private capital.

These different measures work together in practice, and were intentionally designed to complement each other. As a result, it is unrealistic to discuss them individually in a unified framework. Given their primary goal and common ideas, however, our model adopts a single abstract unconventional policy as a modeling device, yet one still relevant enough to serve an illustrative purpose. We assume the government is willing to buy the shares of the firm directly to facilitate lending. Such policies were studied by Gertler and Kiyotaki (2010), and Gertler and Karadi (2011). This captures the unconventional policy of purchasing risky, privately managed, non-government assets, implemented in the U.S., the U.K. and the eurozone in the wake of the financial crisis. The U.S. Federal Reserve's program of buying \$600 million of mortgage-backed securities in 2008-09 (QE-1) and the European Central Bank's Covered Bond Purchase Programs (CBPPs) for buying private sector debt are examples of such policies. Our intention of appealing to such a simple form of unconventional monetary policy (or a credit policy) is to develop a baseline model for analysis.

More precisely, in our model, the government buys a fraction $S_{g,t}$ of the total outstanding shares of firms (normalized to one), so that

$$Q_t K_{t+1} = S_{p,t} Q_t K_{t+1} + S_{g,t} Q_t K_{t+1}, \tag{41}$$

where $S_{p,t} \equiv \int S_{j,t} \mathrm{d}j$ is the total share of equity held privately, and the share of government-held equity is $S_{g,t} = 1 - S_{p,t}$. To conduct the credit policy, the government issues government debt to households that pay the risk-free rate $R_{f,t}$ and then lends the funds to firms or purchases the equity stakes of firms with returns $R_{k,t+1}$. The government credit has an efficiency cost of $\tau > 0$ units per unit of credit supplied. This deadweight loss may reflect the government's fundraising costs or its investment search costs.

We then introduce the key assumption which makes the government's balance sheet, and thus the credit policy, non-neutral as regards its macroeconomic implications. This is the only special feature of government intermediation in our model. A general discussion about the special characteristics that make a government's balance sheet relevant can be found in the companion review paper Dou et al. (2017). More precisely, government intermediation is not financially constrained in our model, unlike private financial intermediation. This can be justified by the assumption that the government always honors its debt, and thus incurs no agency problems between it and its household creditors.

We define the total leverage ratio $\phi_{c,t}$ as follows

$$Q_t K_{t+1} = \phi_{c,t} N_t. \tag{42}$$

The leverage ratio, $\phi_{c,t}$, is the leverage ratio for total intermediated funds, public as well as private, and has the following relation with the private leverage ratio, $\phi_t \equiv S_{p,t} Q_t K_{t+1} / N_t$, and the intensity of government credit intervention, $S_{g,t}$,

$$\phi_{c,t} = \frac{\phi_t}{1 - S_{g,t}}. \tag{43}$$

The government issues government bonds $B_{g,t}$ and collects tax T_t to fund the purchase of these shares (as well as other government spending G_t). The government thus earns $R_{k,t+1} - R_{f,t}$ per dollar purchuase every period.

We assume that at the onset of a crisis, which is defined loosely to mean a period when the log risk premium $\Xi_t \equiv \mathbb{E}_t \left[\ln\left(1 + R_{k,t+1}\right)\right] - \ln\left(1 + R_{f,t}\right)$ rises sharply and becomes much higher than the frictionless benchmark $\Xi^* \equiv \mathbb{E}_t \left[\ln\left(1 + R_k^*\right)\right] - \ln\left(1 + R_f^*\right)$, the government injects credit in response to movements in risk premia. Similar to a standard Lucas-tree economy, the log risk premium is

$$\Xi^* \approx \gamma \sigma_a^2 - \frac{1}{2}\sigma_a^2,$$

where $\frac{1}{2}\sigma_a^2$ is Jensen's term for the log return. The frictionless benchmark is described in Appendix A. We consider the credit policy that follows the rule for $S_{g,t} = 1 - S_{p,t}$:

$$S_{p,t} = \frac{1}{1 + \nu_g \times \left(\Xi_t - \Xi^*\right)}, \tag{44}$$

where the sensitivity parameter, ν_g, is positive. According to (44), the government expands credit as the risk premium gap increases. Our specification is a global version of the credit policy considered by Gertler and Kiyotaki (2010) and Gertler and Karadi

(2011). In the local-linear approximation when $\Xi_t - \Xi^*$ is small,

$$S_{g,t} = 1 - S_{p,t} \approx \nu_g \times (\Xi_t - \Xi^*) \,. \tag{45}$$

The rationale behind this policy specification is as follows. In the absence of financial friction that prevents the financial intermediaries from leveraging too much, the equilibrium outcome is efficient. The inefficiency arises due to the inability of households to buy the risky assets directly, and to the limit on the leverage of their financial managers. This inefficiency manifests itself in the form of large risk premia, since the financial intermediaries must be compensated adequately in the absence of high leverage. The government does not intervene when the risk premium is at its steady-state level, but it does intervene when the premium rises to increasingly inefficient levels beyond it.

We shall show that the global solution of this nonlinear system allows for a state-dependent sensitivity coefficient. For example, we can specify a policy rule as follows:

$$\nu_{g,t} = \nu_{g,0} + \nu_{g,1} \times \left(\frac{1}{n_t} - 1 \right), \tag{46}$$

with $\nu_{g,0} \geq 0$ and $\nu_{g,1} \geq 0$. The idea is that it should be better for the government to conduct more aggressive credit policy (i.e., the sensitivity ν_g is larger) when the financial system is already more fragile (i.e., n_t is smaller).

From (43), it is clear that when the private leverage ratio ϕ_t is kept fixed, the expanding credit policy $S_{g,t}$ increases the total leverage of intermediation, i.e., $\phi_{c,t}$ rises. This captures the idea that the government's balance sheet acts as an intermediary to channel household funds to the asset market when the financial intermediaries are constrained. The government's intermediation prevents asset prices from becoming overly distressed when this is caused by the inefficiency of financial intermediaries after a sequence of negative shocks.

2.7 Resource and Government Budget Constraints

The resource constraint for the final good in our model is given by

$$Y_t = C_t + G_t + \tau S_{g,t} Q_t K_{t+1}. \tag{47}$$

The government spends a fraction $\overline{g}$ of output Y_t in period t, where $\overline{g}$ is an exogenously specified constant. That is,

$$G_t = \overline{g} Y_t. \tag{48}$$

In addition to funding government expenditure, the government also needs to fund the central bank's purchase of shares by issuing purchasing bonds worth $B_{g,t} = S_{g,t} Q_t K_{t+1}$ and the efficiency loss by taxes. Its revenues include taxes, T_t, and the government's income from intermediation, $S_{g,t-1} Q_{t-1} K_t (R_{k,t} - R_{f,t-1})$. Thus, the government budget constraint is

$$G_t + (1 + \tau) S_{g,t} Q_t K_{t+1} = T_t + S_{g,t-1} Q_{t-1} K_t (R_{k,t} - R_{f,t-1}) + B_{g,t}. \tag{49}$$

Since the taxation T_t effectively takes up any slack that shows up on the government balance sheet, and given the existence of representative agents in the economy, the intertemporal budget constraint of the representative household and the intertemporal budget constraint of the government can be combined with taxes left out. Intuitively, then, by Walras' Law, both budget constraints are redundant in determining the equilibria. However, this is very different from saying that the size and composition of the government balance sheet are irrelevant for pinning down the equilibrium under efficient financial market conditions in the sense of Wallace (1981). This is simply because not all investors can purchase an arbitrary amount of the same assets at the same market prices as the government in this model. Put more precisely, unlike private financial intermediation, government intermediation is not constrained by the balance sheet.

3 Quantitative Analysis

3.1 Calibration

Our model can be used to understand the response of the economy to various shocks. We highlight the role of nonlinear dynamics of risk premia in determining the intensity of the credit policy. Furthermore, we show that the global solution is essential when the constraint is occasionally binding and risk plays an important role in the economy. Particularly, in these situations, the first and second order approximation will lead to inaccurate policy functions and severe approximation errors. On the other hand, despite being able to capture the occasionally binding feature, the OccBin method by Guerrieri and Iacoviello (2015) cannot be easily extended to higher order approximation. The role of risk will be ignored.

We use a calibrated version of the model, basing our parameter choices mainly on those in Gertler and Karadi (2011) and standard choices or estimates in the literature (e.g. Smets and Wouters, 2007). In our numerical analysis, the exogenous autoregressive processes are discretized into homogeneous Markov chains according to the procedure proposed by Rouwenhorst (1995). The calibrated parameters for our baseline analysis are summarized in Table 1.

3.2 Policy Function Analysis

We solve the model using four different solution methods: first-order perturbation method, second-order perturbation method with pruning [5], the OccBin method [6], and the global solution method based on time-iteration projection procedure. We group key variables of the model into four categories: real variables (investment i and consumption c), financial variables (risk premium Ξ and capital price q), variables associated with the constraint (value of net worth Ω and constraint multiplier μ), and credit policy S_g. In this economy, there are two state variables: constraint margin λ, and intermediary net worth share n.Figure 2 to Figure 5 display the policy

[5]It is important to use pruning to kill the explosive higher order term. Otherwise, the system will explode.

[6]Appendix D provides a detailed description of the OccBin method.

Table 1: Baseline Parameters (Monthly)

Parameter	**Symbol**	**Value**	**Source**
Household preference			
Discount rate	β	$0.99^{\frac{1}{12}}$	Standard
Relative risk aversion	γ	6	Standard
Total labor supply	$\bar{L}$	1	Standard
Financial intermediaries			
Steady-state fraction of divertible capital	$\bar{\lambda}$	0.381	GK (2011)
Proportional transfer to new bankers	$\aleph$	$0.002 \times \frac{1}{12}$	GK (2011)
Survival rate of bankers	θ	$1 - 0.24 \times \frac{1}{12}$	Non-degenerate condition
Consumption-good firms			
Effective capital share	α	0.33	Standard
Depreciation rate	δ	$0.06 \times \frac{1}{12}$	Standard
Adjustment cost coefficient	ϑ	20	Standard
Investment-good firms			
Efficiency of investment good production	Z_ι	$1 \times \frac{1}{12}$	Standard
Government policies			
Government expenditure ratio	$\overline{g}$	20%	Standard
Government efficiency loss	τ	$10\% \times \frac{1}{12}$	Calibration
Sensitivity coefficient	$\nu_{g,0}$	5	Calibration
Sensitivity coefficient	$\nu_{g,1}$	0	Calibration
Dynamic			
Volatility of TFP	σ_a	$0.03 \times \sqrt{\frac{1}{12}}$	Standard
Persistence of margin	ρ_λ	$0.6^{\frac{1}{12}}$	GK (2011)
Volatility of margin	σ_λ	$0.267 \times \sqrt{\frac{1-\rho_\lambda^2}{1-\rho_\lambda^{24}}}$	GK (2011)

NOTE: All parameters are standard except the credit-policy related parameters τ, $\nu_{g,0}$, and $\nu_{g,1}$. We pick $\nu_{g,0}$ and τ here to provide reasonable private holding shares of risky assets and average risk premia. The parameter $\nu_{g,1}$ is chosen to be zero in the baseline calibration, though we emphasize that the non-zero $\nu_{g,1}$ is important as part of the optimal policy given the nonlinear dynamic of risk premia and the economy. The closest comparison in the literature is Gertler and Karadi (2011); however, their model is solved using log-linearization techniques. The local approximation significantly understates the magnitude and the volatility of the conditional risk premium, even though the model is capable of generating both quantitatively. The implication of biased asset pricing makes quantitative discussion of unconventional monetary policies itself biased. For example, the suppressed risk premium and its nonlinear dynamics require extremely sensitive unconventional monetary policy in order to have a quantitatively significant stabilization effect on the aggregate quantities. In Gertler and Karadi (2011), the credit policy sensitivity parameter $\nu_{g,0}$ is chosen about 100 and the cost parameter of the intervention τ is chosen at an extremely small value 0.1%. Parameters on the intermediary side θ, ρ_λ, $\bar{\lambda}$, and $\bar{\sigma}_\lambda$ are calibrated monthly. An alternative specification of the efficiency of investment-good production we consider is $Z_\iota = 0.4 \times \frac{1}{12}$. It implies weaker investment opportunity in the economy compared to the baseline calibration.

functions for three possible states of the constraint margin: λ_L, λ_M, and λ_H. The discretized levels of λ are chosen at three different levels (λ_L, λ_M, and λ_H) according to discretization. Each row in the figures corresponds to λ_L, λ_M, and λ_H respectively. For example, the first row of Figure 2, consisting of Panel A and Panel B, display the policy functions of investment and consumption when the constraint margin λ is at a low level (i.e. the financial sector is relatively in a good shape). In Panel D of Figure 5, we show the histogram of the ergodic distribution of net worth share n. Appendix C describes in detail the procedure of obtaining policy functions with first and second order pertubation methods. Policy functions with OccBin method are plotted based on a simulation of 50,000 periods of the economy, with observations corresponding to λ values close to λ_L, λ_M, and λ_H. Therefore, the simulation based policy functions only covers the range of simulated net worth share.

Global Solutions and Economic Intuitions. First, let's focus on the policy functions delivered by the global solution method to understand how the economy works. The economy features occasionally-binding financial constraints. For any value of λ, when intermediary net worth share n is low, the intermediaries' financial constraints become binding, and n determines the risk-taking capacity of intermediaries; when n is high, the constraint is slack and the economy is close to a frictionless one. Therefore, when the intermediaries' net worth share is low, the marginal value of net worth is high, and the intermediaries require a higher compensation for taking risk, which suppresses the price of capital q. As a result, investment is reduced, risk premium Ξ rises, and the government conducts aggressive credit policy. The intermediaries earn an excess return in their assets over liabilities, so that the value of net worth Ω exceeds one. In terms of macroeconomic implications, this widely-adopted intermediary asset pricing model works purely through an investment-wedge channel. Like models based on the investment-wedge channel (e.g. investment-specific technological shocks and news shocks), consumption is negatively correlated with investment. This is because when investment is reduced, more labor is endogenously allocated to produce consumption goods.

As n increases, the constraint on intermediaries becomes looser, and price of capital

q and investment i rises, while risk premium Ξ drops. The credit policy S_g, value of net worth Ω, multiplier of the constraint μ, and consumption c all decrease with n. When n grows to be large enough, the intermediaries are not constrained any more and the economy converges to the frictionless case.

The values of constraint margin λ is also important in determining how tight the constraint is. The higher λ is, the intermediaries are required to have higher market value to hold the same portfolio. In other words, higher λ is associated with higher marginal value of net worth for the same level of n. As λ becomes higher, investment decreases and risk premium increases.

Policy Function Comparisons for Different Solution Methods. Now, we compare the policy functions delivered by four solution methods. When we use perturbation method (including the OccBin method) to solve the model, we need a deterministic steady state, around which we perturb the dynamic system. We choose the steady state where the constraint always binds. We emphasize that the deterministic steady state with binding financial constraint may not exist under the parametrization where the intermediaries are constrained infrequently in the equilibrium. Section 4.2 lays out such an economy, in which makes the local pertubation method fails.

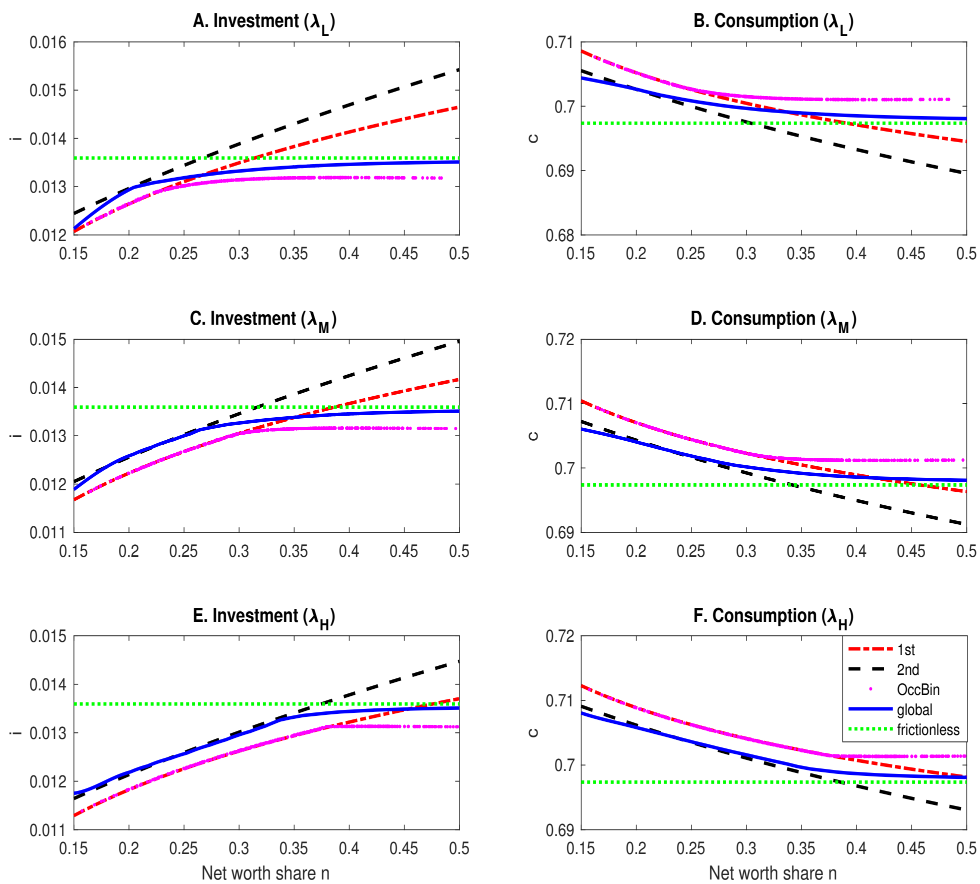
Figure 2 to Figure 5 plot the policy functions of the key variables obtained using different solution methods. Four observations stand out: (1) First and second order perturbation methods work well when n is small (For $\lambda = \lambda_M$, the constraint binds when n is smaller than 0.27). When n increases to get out of the constraint, the policy functions deviate from the global solution, because the constraint is forced to bind; (2) First and second order perturbation methods work better in the state of high λ than the state of low λ, since the region of binding constraint is wider. From the ergodic distribution of net worth share n in Figure 5, it centers around 0.25 and fluctuates between 0.15 and 0.4. The cutoff for the constraint to be slack when $\lambda = \lambda_H$ is close to $n = 0.4$, so that the constraint binds most of the time. However, when λ takes the value of λ_L, the cutoff is even smaller than 0.3, which means the constraint does not bind for about half of the time. This makes perturbation method work worse overall; (3) Using second order perturbation with pruning, we obtain policy functions that are closer to the ones delivered by the global solution than first order perturbation. The result is not surprising, as the first order approximation kills the covariance term in the Euler equation; (4) The OccBin method can capture the change of regime from binding constraint to slack constraint. When the constraint is binding, policy functions are identical to first order pertubation by construction, and the policy functions become flat when n grows larger and the constraint becomes slack. But the OccBin method is limited in first order approximation, thus not able to handle the potentially important role of risk in the economy.

3.3 Impulse Response Analysis

In this subsection, we turn to look at the impulse response functions of the key variables in the economy. Specifically, we examine the responses of various economic variables when λ has a one-time deviation from λ_M to λ_H as well as λ_L. The response to a positive shock and a negative shock of λ are symmetric if we use first order perturbation method, as the system is log-linearized into a VAR representation. Second order pertubation generates impulse responses that are almost the same as first order. The global method and OccBin method give us different results. The positive shock to λ tightens the constraint and price of capital will drop substantially. If λ is shocked negatively, the constraint turns slack. The response magnitude to a negative λ shock is smaller than responses to a positive λ shock. Comparing the two sets of impulse responses verifies this point.

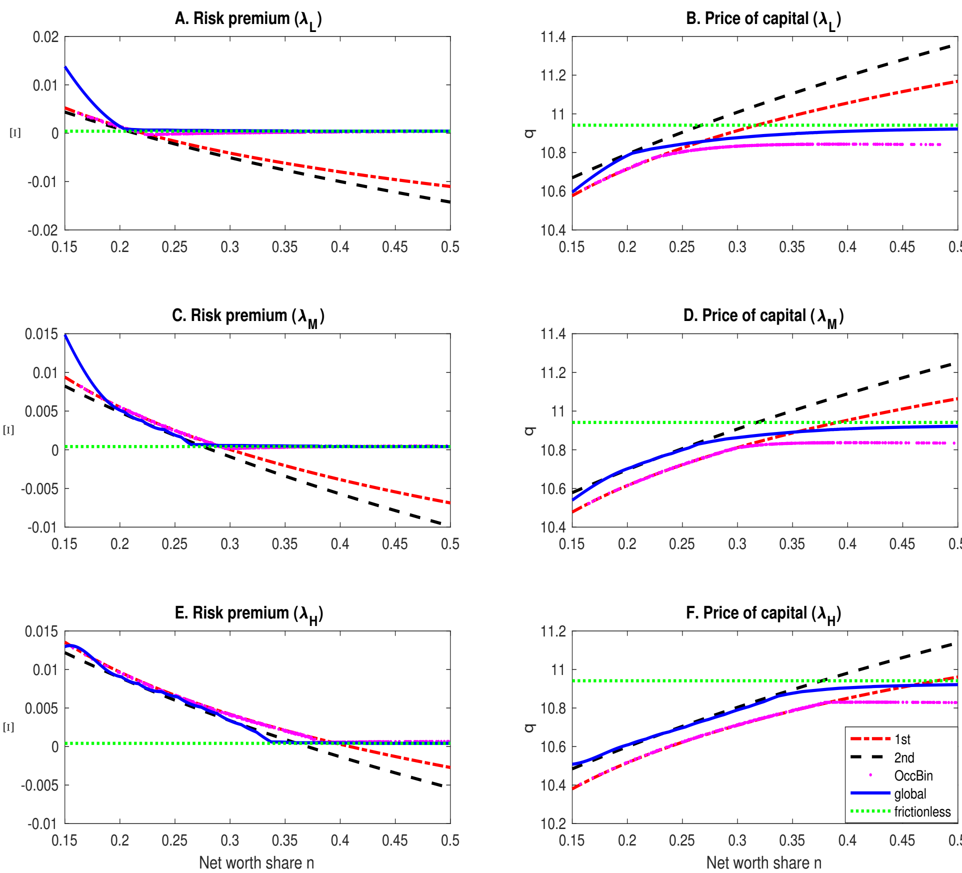
To compute the impulse response functions using global method, we first simulate N parallel economies ($N = 1000$) for T_1 periods ($T_1 = 200$) when fixing $\lambda = \lambda_M$. Then we let $\lambda = \lambda_H$ (or $\lambda = \lambda_L$) for all parallel economies at period $T_1 + 1$, and let the economy evolve afterwards for T_2 periods ($T_2 = 120$, 10 years) with λ drawn randomly from the discretized state space. We compute the mean of various variables after the shock from the N parallel economies. Figure 6 shows the impulse responses of variables to a positive shock to λ_H, and Figure 7 shows their impulse responses to a negative shock to λ_L.

Figure 2: Policy Functions of Real Variables



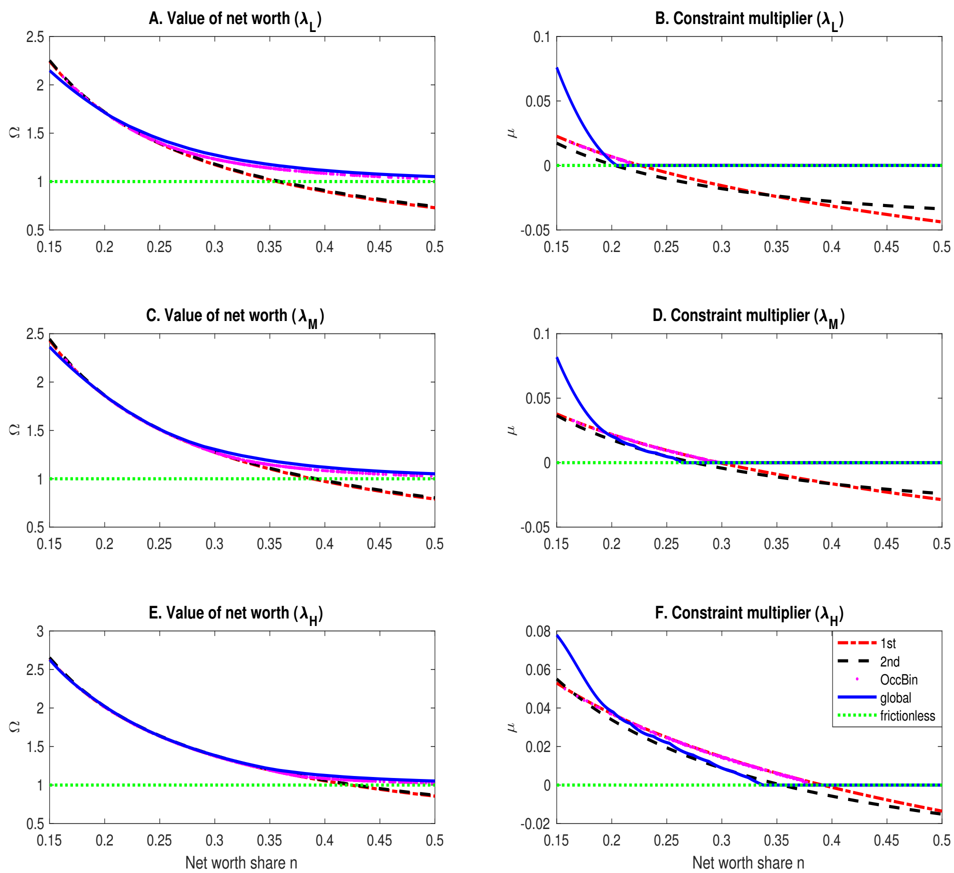
NOTE: This figure shows the policy function of real variables (investment i and consumption c) as a function of intermediary net worth share n for different states λ_L, λ_M, and λ_H. The dot-dashed red line shows the policy function obtained through first order perturbation method. The dashed black line displays the policy function obtained through second order perturbation method with pruning. The dotted pink line shows the policy function obtained through the OccBin method. The solid blue line plots the policy function obtained using global method. The green dots are values of corresponding variables in a frictionless economy.

Figure 3: Policy Functions of Financial Variables



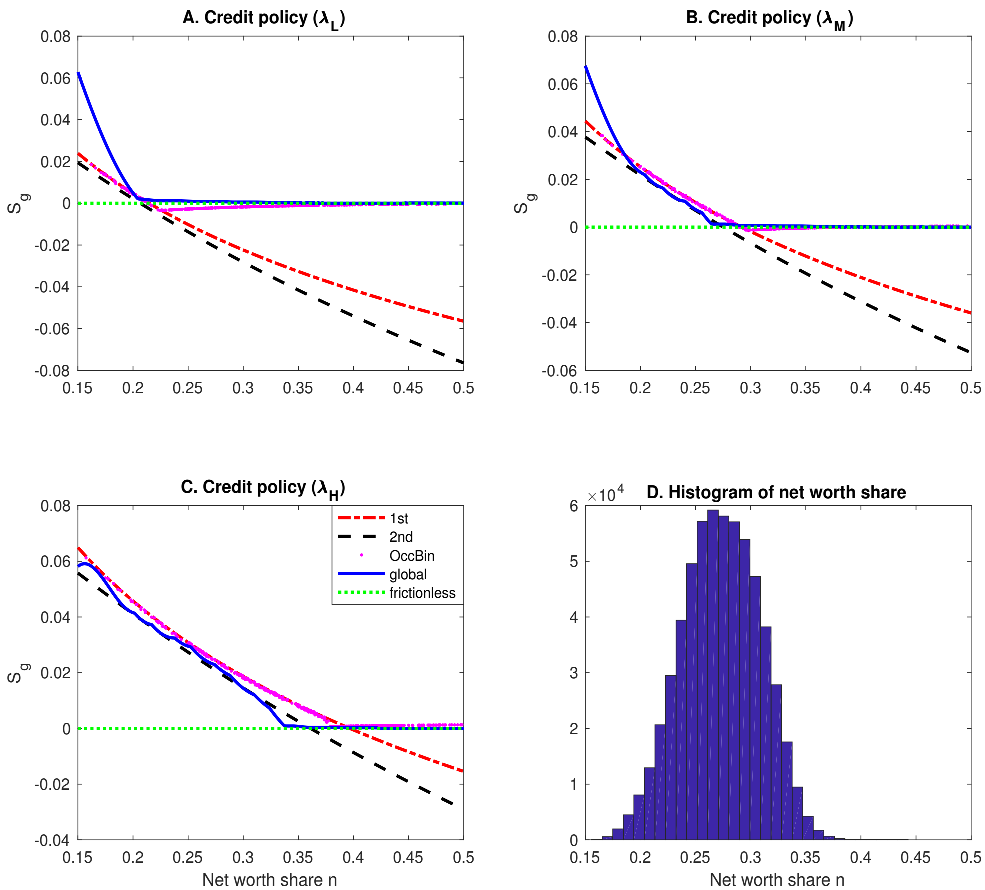
NOTE: This figure shows the policy function of financial variables (risk premium Ξ and capital price q) as a function of intermediary net worth share n for different states λ_L, λ_M, and λ_H. The dot-dashed red line shows the policy function obtained through first order perturbation method.The dashed black line displays the policy function obtained through second order perturbation method with pruning.The dotted pink line shows the policy function obtained through the OccBin method. The solid blue line plots the policy function obtained using global method. The green dots are values of corresponding variables in a frictionless economy.

Figure 4: Policy Functions of Financial Constraint Variables



NOTE: This figure shows the policy function of variables of the constraint (value of net worth Ω and multiplier of the constraint μ) as a function of intermediary net worth share n for different states λ_L, λ_M, and λ_H. The dot-dashed red line shows the policy function obtained through first order perturbation method.The dashed black line displays the policy function obtained through second order perturbation method with pruning. The dotted pink line shows the policy function obtained through the OccBin method. The solid blue line plots the policy function obtained using global method. The green dots are values of corresponding variables in a frictionless economy.

Figure 5: Credit Policy and Stationary Distribution



NOTE: This figure shows the policy function of credit policy S_g as a function of intermediary net worth share n for different states λ_L, λ_M, and λ_H. The dot-dashed red line shows the policy function obtained through first order perturbation method.The dashed black line displays the policy function obtained through second order perturbation method with pruning. The dotted pink line shows the policy function obtained through the OccBin method. The solid blue line plots the policy function obtained using global method. The green dots are values of corresponding variables in a frictionless economy. We also show the histogram of the stationary distribution of net worth share.

We first seek to understand the dynamics of the key state variable, net worth share n after the shock. n is determined by the response of both intermediary net worth and the total value of capital in the economy. Upon a positive shock on λ, both intermediary net worth and total value of λ decreases, but intermediary net worth decreases more due to leverage, so n declines at first. When the economy recovers from the shock, intermediaries accumulate net worth faster than the recovery of price of capital due to leverage for the same reason, so the net worth share exceeds its ergodic mean (stochastic steady state). As time goes by, n converges back to its ergodic mean.

Other variables' direction of impulse responses are then intuitive, given that we have already derived policy functions as a function of n and λ. When λ is hit by a positive shock, the constraint gets tightened, so risk premium rises, price of capital drops, and investment rate drops. Consumption increases due to labor reallocation from the investment good sector to the consumption good sector, and wage decreases as a result. The government credit policy is more aggressive because of the higher risk premium. Their dynamics track the dynamics of n.

When λ was given a negative shock, all variables move in the opposite direction. However, the impulse responses are asymmetric. The constraint loosens such that it does not bind, so the risk premium and constraint multiplier do not fall by the same magnitude as in response to a positive λ shock. From Figure 7, we find that first and second order pertubation methods deliver exaggerating the impulse responses, because they force the constraint to be always binding. For example, in Panel A, the risk premium in first order pertubation solution is more than twice as much as the response from global solution method. OccBin method captures the switch of regime from binding constraint to slack constraint, so that the impulse responses are much closer to the global ones. However, the impulse responses from OccBin method still do not coincide with the global solution.

To sum up, we compare the behavior of economic variables in response to a positive and a negative shock on λ using different solution methods. We find that when λ tightens, first and second order pertubation method deliver accurate impulse responses, but when λ loosens such that the constraint does not bind, the pertubation method overstates the response of economic variables. OccBin method can partially improve the impulse responses to a negative λ shock. Our findings highlight the asymmetry and shed light on the role of constraint slackness in determining the impulse responses of economic variables.

3.4 Error Analysis

In this section, we evaluate the accuracy of different solution methods by calculating the errors of equilibrium conditions using our policy functions. The key informative equations are intertemporal Euler equations, as in Judd (1992) and Aruoba et al. (2006). Figure 8 to Figure 10 present the errors to the following equations: intertemporal Euler equations (one for household and two for intermediaries), intratemporal price equations (wage and price of capital), credit policy, and market clearing conditions (consumption good, investment good, and security) as a function of n. Each row represents one value of λ, as shown in the caption of each plot. Specifically, the errors to the three Euler equations are computed as follows:

$$EE_{hh} = \mathbb{E}_t \mathcal{M}_{t,t+1}(1 + R_{f,t}) - 1 \tag{50}$$

$$EE_{int,rf} = \mathbb{E}_t \mathcal{M}_{t,t+1} \frac{1 - \theta + \theta\Omega_{t+1}}{\Omega_t}(1 + R_{f,t}) - (1 - \mu_t) \tag{51}$$

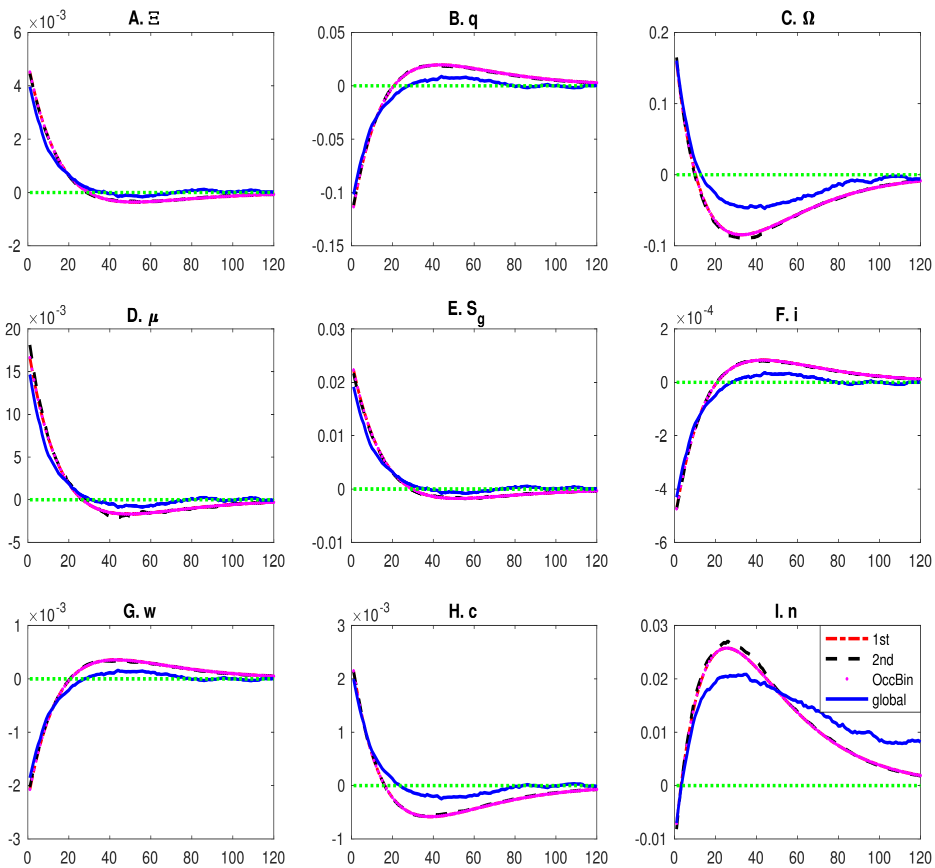
$$EE_{int,rk} = \mathbb{E}_t \mathcal{M}_{t,t+1} \frac{1 - \theta + \theta\Omega_{t+1}}{\Omega_t}(1 + R_{k,t+1}) - (1 - \mu_t + \frac{\lambda_t \mu_t}{\Omega_t}) \tag{52}$$

where:

$$\mathcal{M}_{t,t+1} \equiv \beta \left[\left(\frac{c_{t+1}}{c_t} \right) \exp(\sigma_a \epsilon_{a,t+1}) \right]^{-\gamma} (i_t + 1 - \delta)^{-\gamma} \tag{53}$$

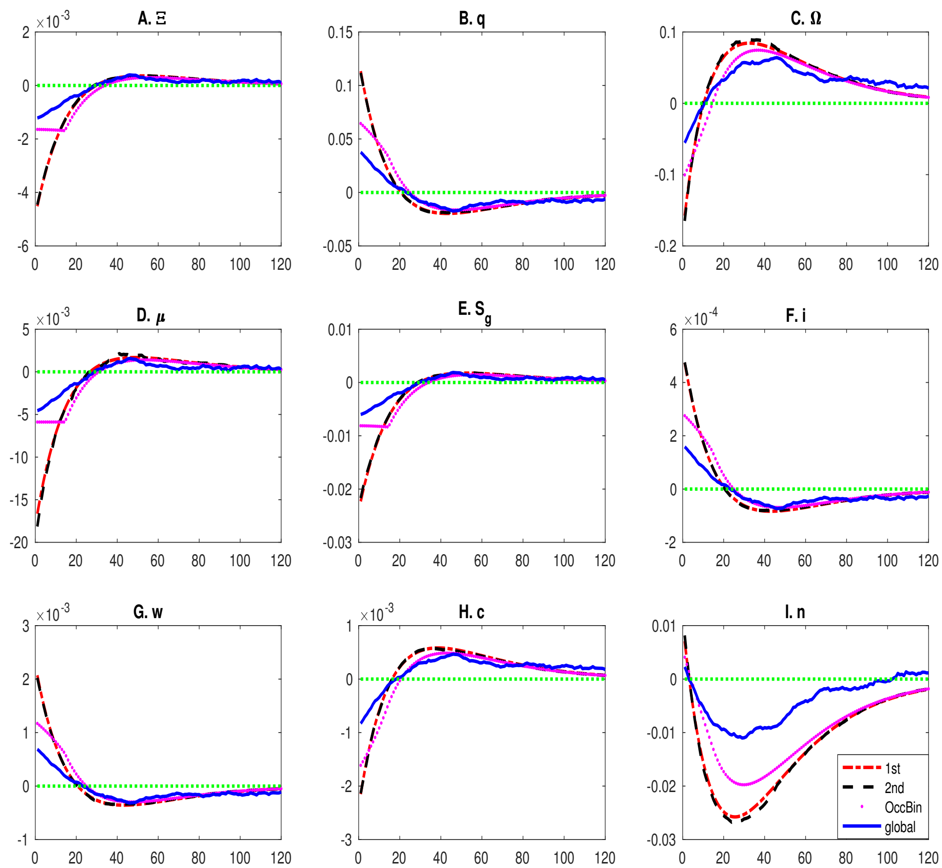
We show the errors to three Euler equations in the most relevant range of $n \sim [0.2, 0.4]$ in Figure 8. We find that first order perturbation solution has significantly larger approximation error than global solution, while second order pertubation solution is close to the global solution when λ take the medium and high value. The slackness of the constraint determines how accurate the perturbation method is. When λ takes the low value, the constraint is less likely to bind so the approximation error is larger

Figure 6: Impulse Response Functions to a Positive Shock on λ



NOTE: This figure shows the impulse response functions of various variables upon a positive shock on λ, from λ_M to λ_H. The dot-dashed red line shows the impulse response function obtained through first order pertubation method.The dashed black line displays the impulse response function obtained through second order pertubation method with pruning. The dotted pink line shows the impulse response function obtained through the OccBin method. The solid blue line plots the impulse response function obtained using global method.

Figure 7: Impulse Response Functions to a Negative Shock on λ



NOTE: This figure shows the impulse response functions of various variables upon a negative shock on λ, from λ_M to λ_L. The dot-dashed red line shows the impulse response function obtained through first order pertubation method.The dashed black line displays the impulse response function obtained through second order pertubation method with pruning. The dotted pink line shows the impulse response function obtained through the OccBin method. The solid blue line plots the impulse response function obtained using global method.

with perturbation method. Also, we find that the approximation to households Euler equation is more accurate than intermediary Euler equations, as whether the constraint binds or not plays a more important role in the two intermediary Euler equations.

The errors to Euler equations to OccBin method are not included in the figure. Due to the nonlinear feature of the policy function, it is hard to back out a parametric decision rule of all variables as a function of n and λ, which can be applied to evaluating expectations in the Euler equations. Linear or polynomial approximations will introduce extra errors that makes the comparison less meaningful. Simulation-based evaluation of Euler errors does not work well here either, because we have two state variables. Among the 50,000 simulations, only about 1,000 observations lie in the range where λ takes around these three specific values. Evaluation the conditional expectation for every n value is very inaccurate. Expanding simulation lengths with OccBin method greatly lengthens computation time.

Therefore, we do not directly compare the conditional Euler equation errors for each value of n and λ. Instead, we simulate the model for 50,000 periods with each method, and evaluate the error squares as in (50), (51), and (52) using the simulated sample. Then we average these error squares and obtain an unconditional Euler equation errors. Panel A of Table 2 presents the comparison of unconditional Euler errors with different methods. Further, we split the simulation sample into three regions of n: $n < 0.25$, $0.25 < n < 0.35$, and $n > 0.35$, and evaluate the unconditional Euler errors in each n range. Results are shown in Panel B to D in Table 2.

From Table 2 Panel A, we see that the households Euler equation errors are similar using the four method, while errors to the two Euler equations are much larger using first order and second order pertubation methods, as these methods ignore the slackness of the constraint. OccBin method reduces the errors, but is still outperformed by the global solution method. A further comparison in Panel B to D shows that for all values of n, the Euler errors to the two Euler equations associated with intermediaries are the smallest using the global method.

In the meanwhile, we also evaluate how well perturbation method can approximate the three market clearing conditions (consumption, investment, and security) in Figure

9. The three equations are:

$$y_t = c_t + \bar{g}y_t + \tau S_{g,t}q_t(i_t + 1 - \delta) \tag{54}$$

$$Z_i l_{i,t} = i_t + \frac{\vartheta}{2} i_t^2 \tag{55}$$

$$1 = S_{p,t} + S_{g,t} \tag{56}$$

In Figure 10, we show the errors to two intratemporal pricing equations (price of capital and wage) and the credit policy equation. These equations are:

$$\frac{q_t}{p_t} = 1 + \vartheta i_t \tag{57}$$

$$w_t = (1-\alpha) l_{c,t}^{-\alpha} \tag{58}$$

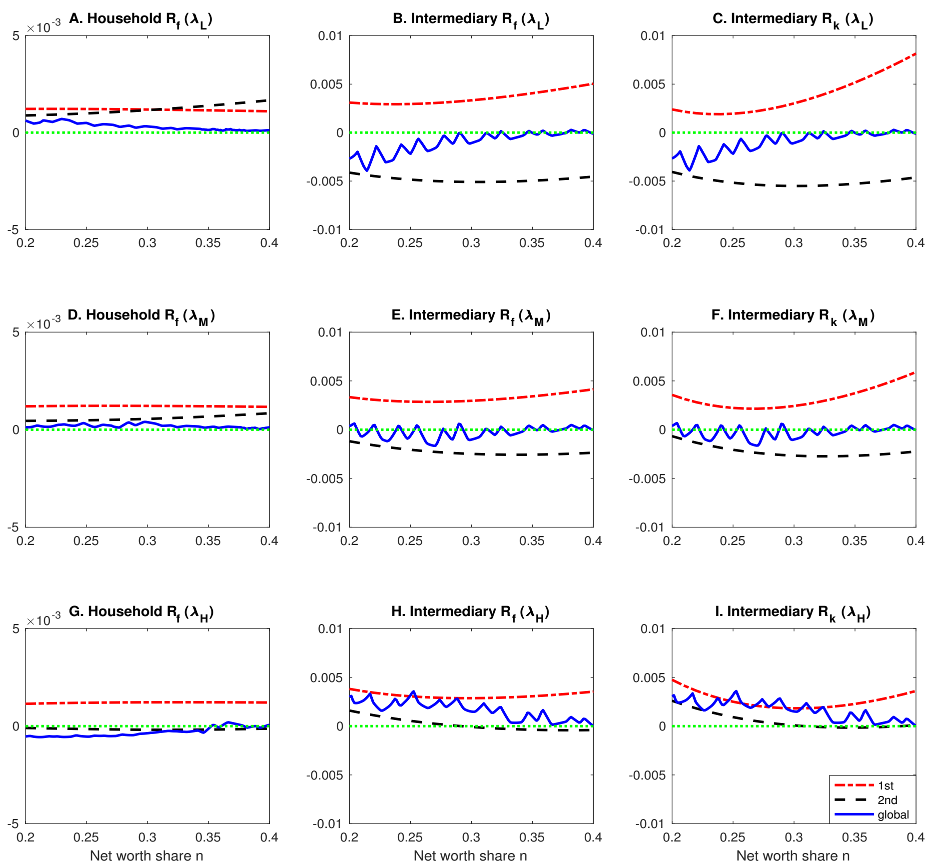
$$1 = S_{p,t}(1 + \nu_{g,t}(\Xi_t - \Xi^*)) \tag{59}$$

Errors to all equations are scaled by the deterministic steady state of the left-hand-side of these equations. From the two figures, we see that the approximation is good in general, while the errors are relatively larger for the investment market clearing condition, price of capital, and the credit policy equation. These equations display less log-linearity so that log-linearization is less accurate. Adding second-order terms can substantially reduce the approximation errors. OccBin method reduces approximation errors when n is large. Overall, the perturbation approximation to static equations are reasonably good.

3.5 Slackness of the Constraint

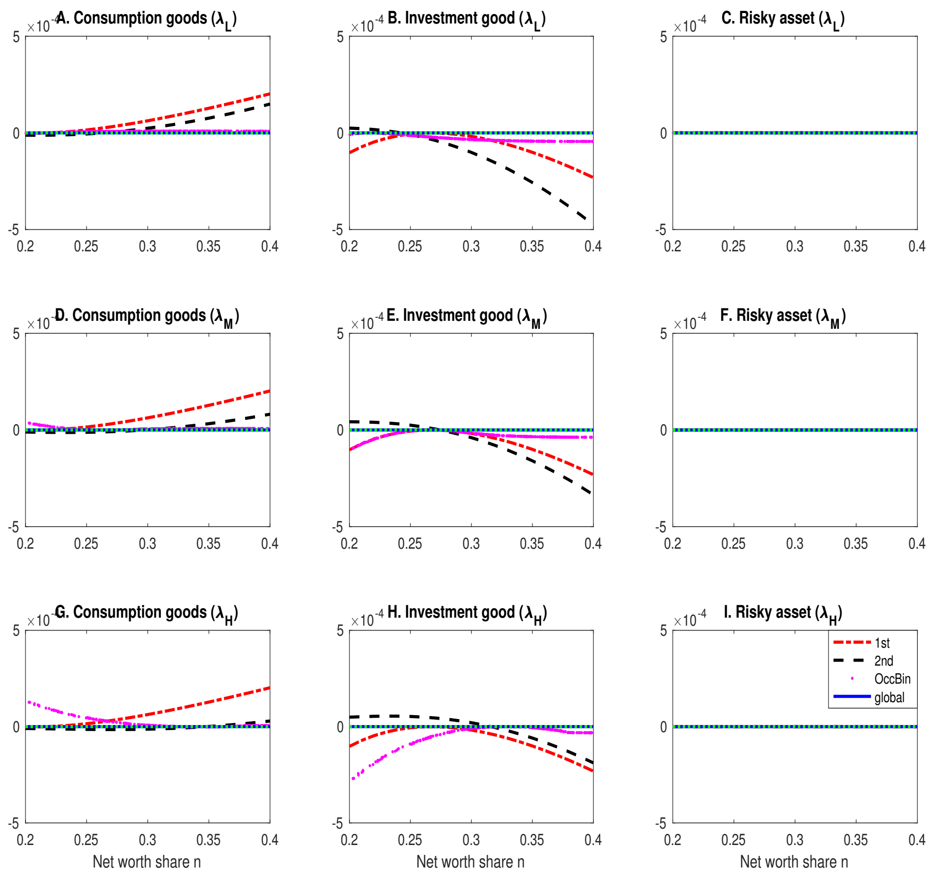
As we discuss in the previous subsections, the main difference between using local perturbation method and global solution method is how we deal with the slackness of the constraint. We assume the constraint always binds when using perturbation method, while we allow for occasionally binding constraint when solving globally. To

Figure 8: Error to Euler Equations



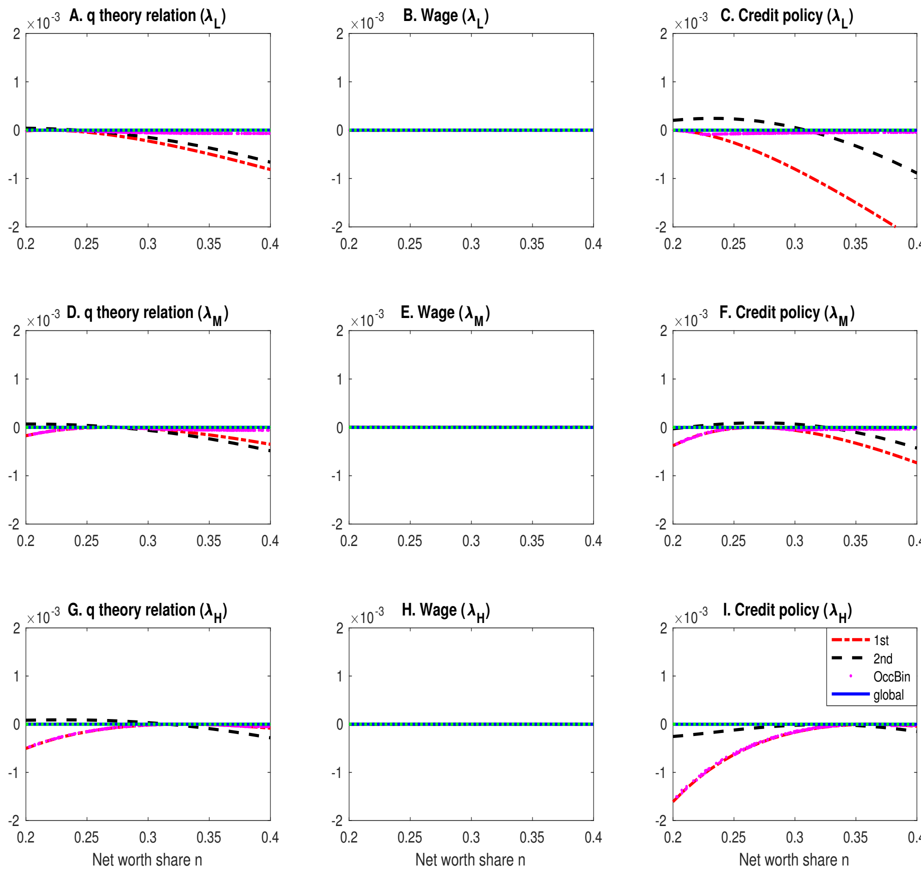
NOTE: This figure shows the errors to three Euler equations: households Euler equations, intermediary Euler equation with risk free rate, and intermediary Euler equation with capital. Each column represents a state of λ. Column 1 corresponds to λ_L, column 2 corresponds to λ_M, and column 3 corresponds to λ_H. The dot-dashed red line shows the errors of first order perturbation solution.The dashed black line displays the errors of second order perturbation method with pruning. The solid blue line plots the errors of global solution.

Figure 9: Error to Market Clearing Conditions



NOTE: This figure shows the errors to three market clearing conditions: consumption good, investment good, and security. Each column represents a state of λ. Column 1 corresponds to λ_L, column 2 corresponds to λ_M, and column 3 corresponds to λ_H. The dot-hased red line shows the errors of first order perturbation solution.The dashed black line displays the errors of second order perturbation method with pruning. The dotted pink line shows the errors of OccBin method. The solid blue line plots the errors of global solution.

Figure 10: Error to Intratemporal Price and Credit Policy



NOTE: This figure shows the errors to intratemporal price (price of capital and wage) and policy equations. Each column represents a state of λ. Column 1 corresponds to λ_L, column 2 corresponds to λ_M, and column 3 corresponds to λ_H. The dot-dashed red line shows the errors of first order perturbation solution.The dashed black line displays the errors of second order perturbation method with pruning. The dotted pink line shows the errors of OccBin method. The solid blue line plots the errors of global solution.

Table 2: Comparison of Euler Equation Errors

	1st order	2nd order	OccBin	Global
Panel A: Unconditional Euler Errors				
Household risk free rate	0.0026	0.0026	0.0026	0.0025
Intermediary risk free rate	0.0091	0.0095	0.0058	0.0051
Intermediary return to capital	0.0069	0.0073	0.0044	0.0038
Panel B: Euler Errors for $n < 0.25$				
Household risk free rate	0.0026	0.0025	0.0026	0.0025
Intermediary risk free rate	0.0092	0.0127	0.0069	0.006
Intermediary return to capital	0.007	0.0101	0.0053	0.0046
Panel C: Euler Errors for $0.25 < n < 0.35$				
Household risk free rate	0.0026	0.0026	0.0027	0.0025
Intermediary risk free rate	0.0089	0.0078	0.0063	0.0047
Intermediary return to capital	0.0068	0.0058	0.0048	0.0035
Panel D: Euler Errors for $n > 0.35$				
Household risk free rate	0.0026	0.0028	0.0026	0.0026
Intermediary risk free rate	0.0092	0.0047	0.005	0.0036
Intermediary return to capital	0.007	0.0032	0.0037	0.0036

NOTE: Panel A presents the unconditional average Euler error squares using first-order pertubation, second-order pertubation, OccBin, and global solution method. Panel B shows the average Euler error squares for samples with $n < 0.25, 0.25 < n < 0.35, n > 0.35$ with the four solution methods, respectively.

conclude this section, we define a metric of the slackness of the constraint:

$$\text{Slackness} = \frac{\Omega(n,\lambda)n - \lambda \times S_p(n,\lambda)}{\Omega(n,\lambda)n} \tag{60}$$

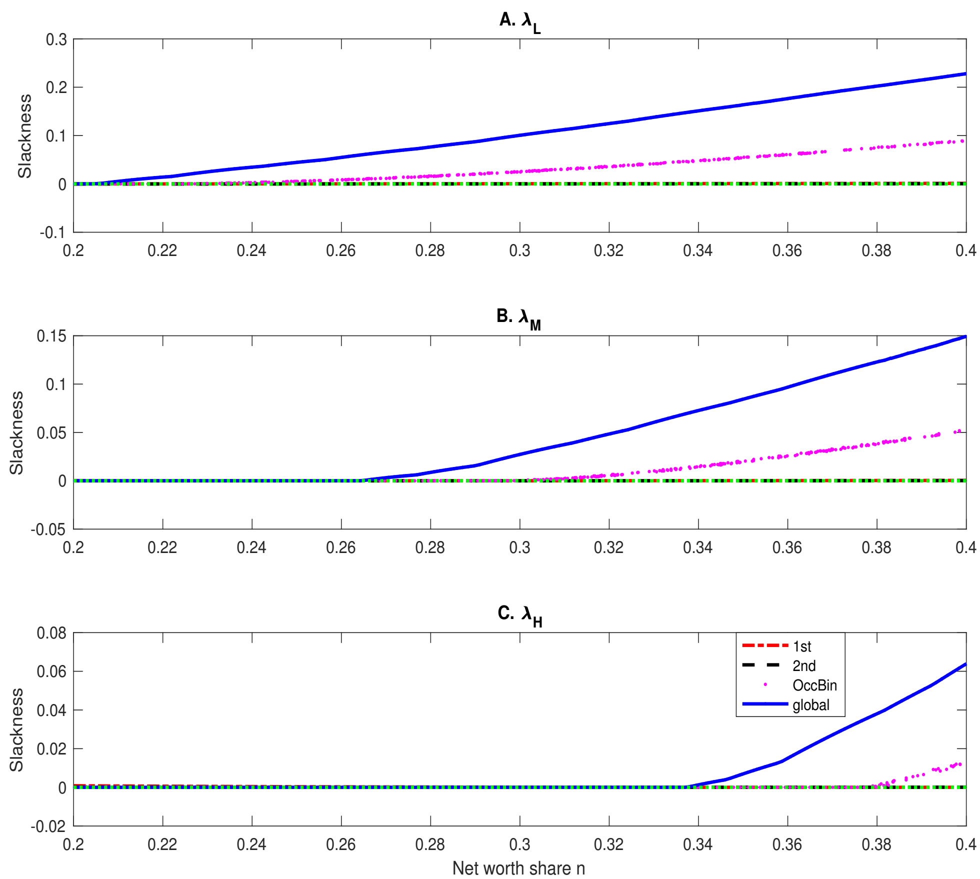
When the constraint binds, the slackness measure is 0. When the constraint is slack, the slackness measure is positive.

Figure 11 shows the slackness measure as a function of n, for different states of λ. The slackness metric of the first and second order perturbation solution are always 0, as the constraint is always forced to bind. The OccBin method is able to capture the occasionally binding feature of the constraint, able to obtain a slack constraint when n is large. However, the cutoff value of n still differs from the global solution. For global solution, the slackness metric is 0 for small values of n, in which cases the constraint binds. When n gets larger, the constraint becomes slack and the metric turns positive and keeps increasing. The cutoff value of n slightly differs for global method and OccBin method, since OccBin method ignores the role of risk. Comparing the metric across different values of λ, the cutoff value of binding constraint for n increases with λ. If λ takes the value λ_H, the constraint binds more easily than the case when $\lambda = \lambda_L$.

4 The Local Perturbation Method Can Fail

The previous section compares the different solution methods in solving our model in the main text. When applying perturbation method, it is crucial to select a deterministic steady state around which to perturb the policy functions. In this section, we show an economy in which there does not exist a deterministic steady state with a binding constraint. The case is relevant to our purpose of credit policy analysis as a realistic description of the macro economy. From mid 1980's, we experienced a "Great Moderation" period until the recent financial crisis, while in 2007-08 there was a sharp "sudden stop". In this section, we compare two cases of calibration: Section 4.1 analyzes a case close to the economy in the previous section, in which

Figure 11: Slackness of the Constraint



NOTE: This figure shows the slackness metric for the constraint for as a function of n and λ. Each subplot represents a state of λ. The dot-dashed red line shows the slackness metric of first order perturbation solution.The dashed black line displays the slackness metric of second order perturbation method with pruning. The dotted pink line shows the slackness metric of OccBin method. The solid blue line plots the slackness metric of global solution.

the equilibrium investment rate is high. Section 4.2 analyzes a case in which the equilibrium investment rate is low so that there does not even exist a steady state. In this case, global method is the only choice for us to solve the model. In both cases, we fix λ at a constant $\bar{\lambda} = 0.381$.

4.1 The Economy with High Investment Rate

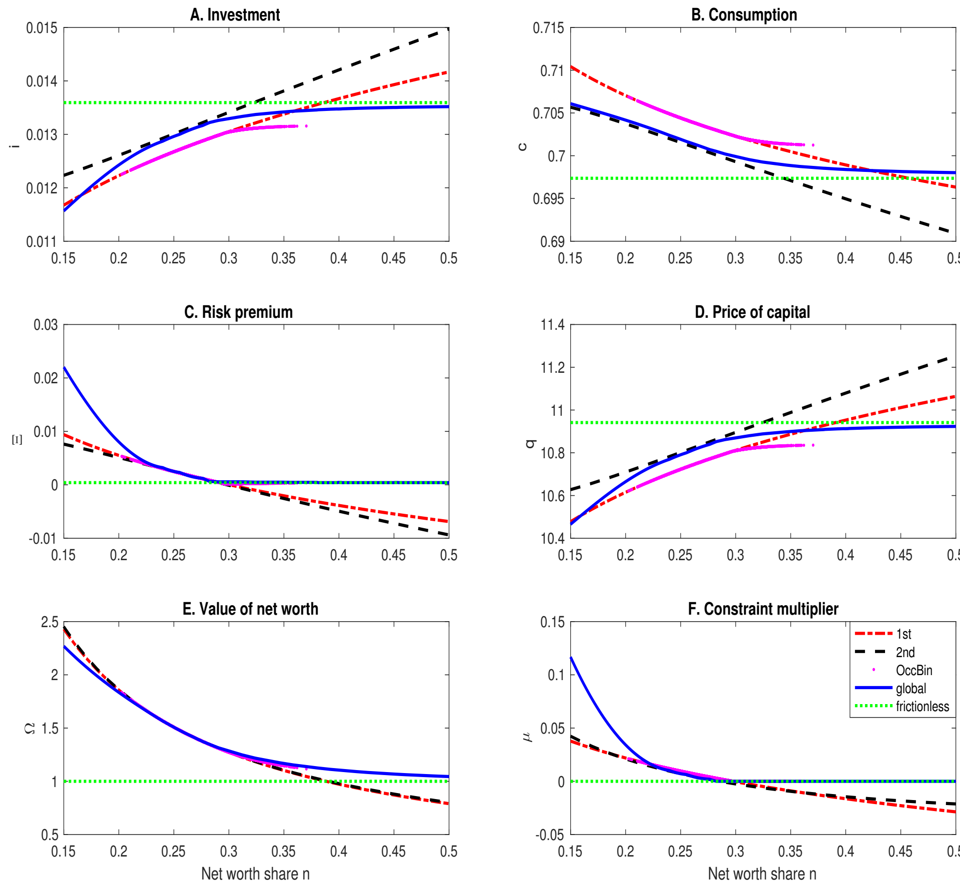
In this subsection, we analyze a case with high equilibrium investment rate. We calibrate production efficiency $Z_i = 1 \times \frac{1}{12}$. All other parameters are identical to Table 1.

Figure 12 displays the policy functions of consumption, investment, risk premium, price of capital, value of net worth, and constraint multiplier as functions of net worth ratio. The policy function has similar properties with what we showed in section 3.2. As we discussed in the previous section, pertubation and global method deliver similar results when the constraint binds, while they diverge with high levels of n when the constraint does not bind. OccBin method improves the approximation by capturing the regime switch from binding to slack constraint, but ignore the impact of higher order terms.

Figure 13 shows the credit policy as a function of n. Credit policy tracks the policy function of risk premium by construction. From the histogram, we select the net worth ratio range of 0.2 to 0.4 as the most relevant region for the economy.

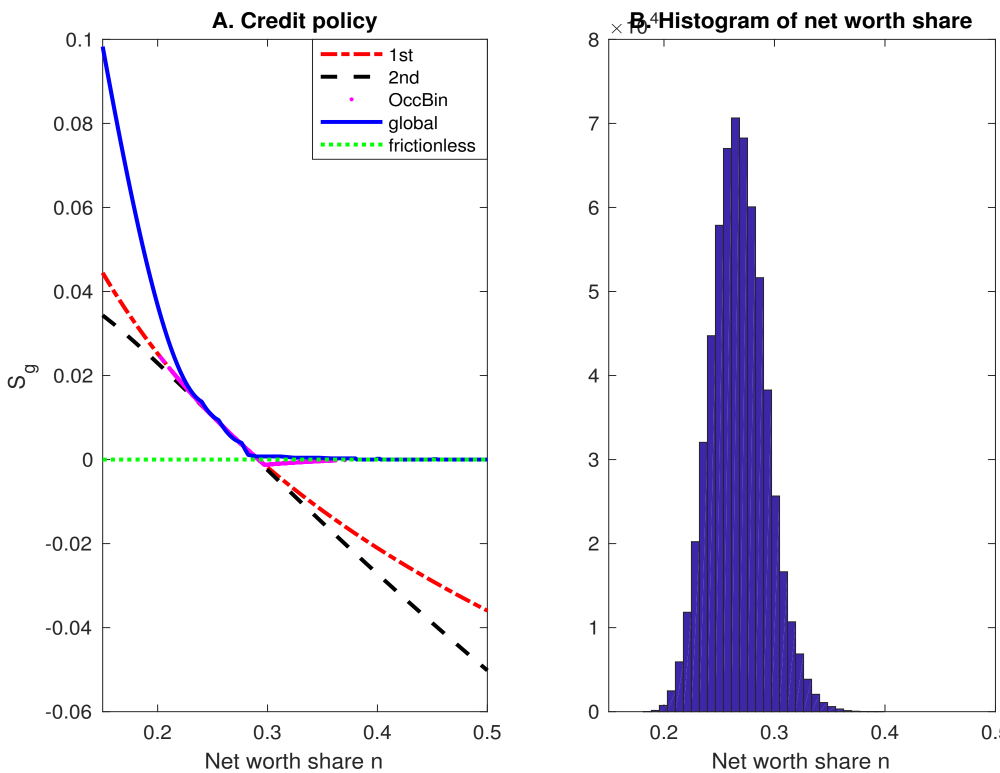
We also show the slackness metric of this economy as in section 3.5 in Figure 14. As in our economy with fluctuating λ, the slackness metric is always 0 for the first or second order perturbation solutions.The OccBin method can capture the occasionally binding feature of the constraint, but the cutoff is slightly different from the one delivered by global solution. For global solutions, the slackness metric turns positive when n exceeds a cutoff value (0.28 in this economy), as the constraint becomes slack.

Figure 12: Policy Functions: High Investment Rate Economy



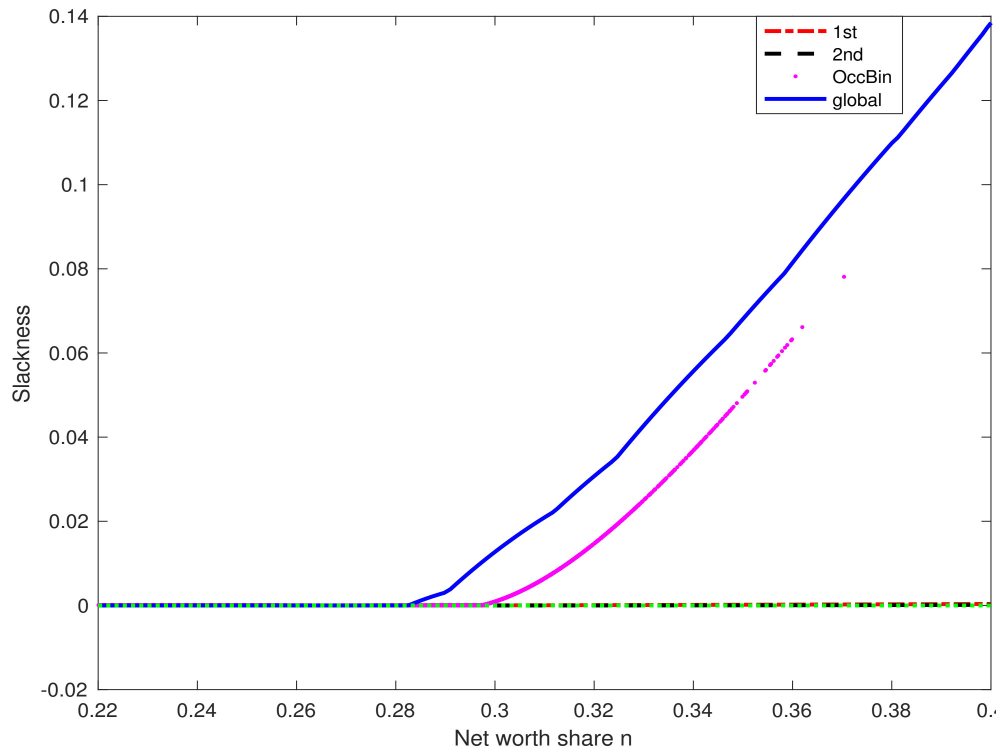
NOTE: This figure shows the policy functions of variables in an economy with high Z_i and high investment rate. The dot-dashed red line shows the policy function obtained through first order perturbation method.The dashed black line displays the policy function obtained through second order perturbation method with pruning. The dotted pink line shows the policy functions obtained through OccBin method. The solid blue line plots the policy function obtained using global method. The green dots are values of corresponding variables in a frictionless economy.

Figure 13: Credit Policy and Histogram: High Investment Rate Economy



NOTE: This figure shows the credit policy and a histogram of net worth ratio in an economy with high Z_i and high investment rate. The dot-dashed red line shows the credit policy obtained through first order perturbation method.The dashed black line displays the credit policy obtained through second order perturbation method with pruning. The dotted pink line shows the credit policy obtained through OccBin method. The solid blue line plots the credit policy obtained using global method. The green dots are values of corresponding variables in a frictionless economy.

Figure 14: Slackness metric to the High Investment Rate Economy



NOTE: This figure shows the slackness metric of solutions in an economy with high Z_i and high investment rate. The dot-dashed red line shows slackness metric obtained through first order perturbation method.The dashed black line displays the slackness metric obtained through second order perturbation method with pruning. The dotted pink line shows the slackness measure obtained using OccBin method. The solid blue line plots the slackness metric obtained using global method.

4.2 The Economy with Low Investment Rate

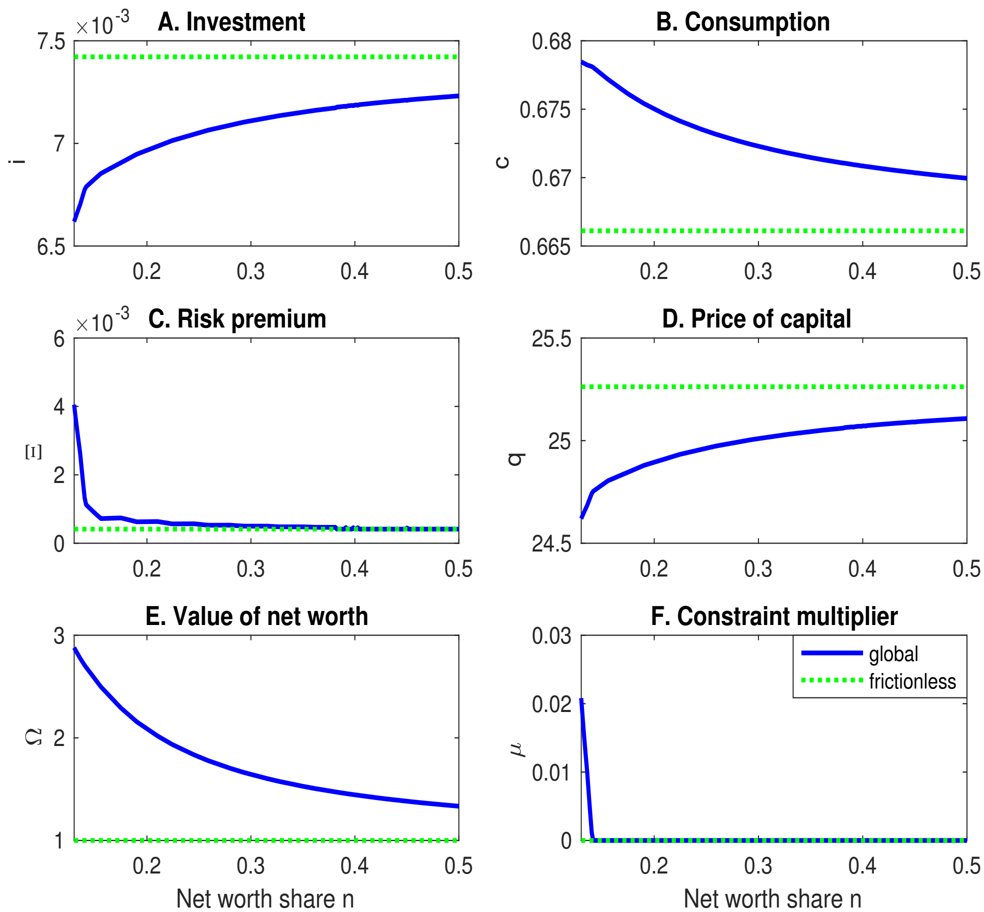
Next we look at another economy with $Z_i = 0.4$. In this economy, investment good production is less efficient and equilibrium investment rate is lower. There does not exist a deterministic steady state in which the constraint binds. In fact, the economy does not have a deterministic steady state, so that pertubation method fails.

Although the constraint binds only occasionally in this economy, the policy function still stays away from the ones in a frictionless economy. Appendix A lays out the equilibrium conditions of a frictionless economy. Figure 15 and 16 compare the policy function of the same set of variables using global method and the frictionless economy as in Figure 12 and 13. We also show a histogram of net worth ratio of this economy in figure 16. The constraint in this economy binds only in a small range of n. Note that the constraint holds in the frictionless economy if $n > \bar{\lambda} = 0.381$, but in our economy, n fluctuates below 0.3. When the constraint is expected to be binding in some small value of n, the expected value of net worth Ω will be greater than 1 even when the constraint does not bind at the current state and the risk premium is higher than the frictionless case. Recall that the constraint takes the form $\Omega n \geq \lambda S_p$, the constraint does not bind even when n is smaller than $\bar{\lambda}$, if $\Omega > 1$ and $S_p < 1$. Consequently, investment is lower than the frictionless case even when the constraint does not bind. Global solution method is necessary to solve this model.

5 Conclusion

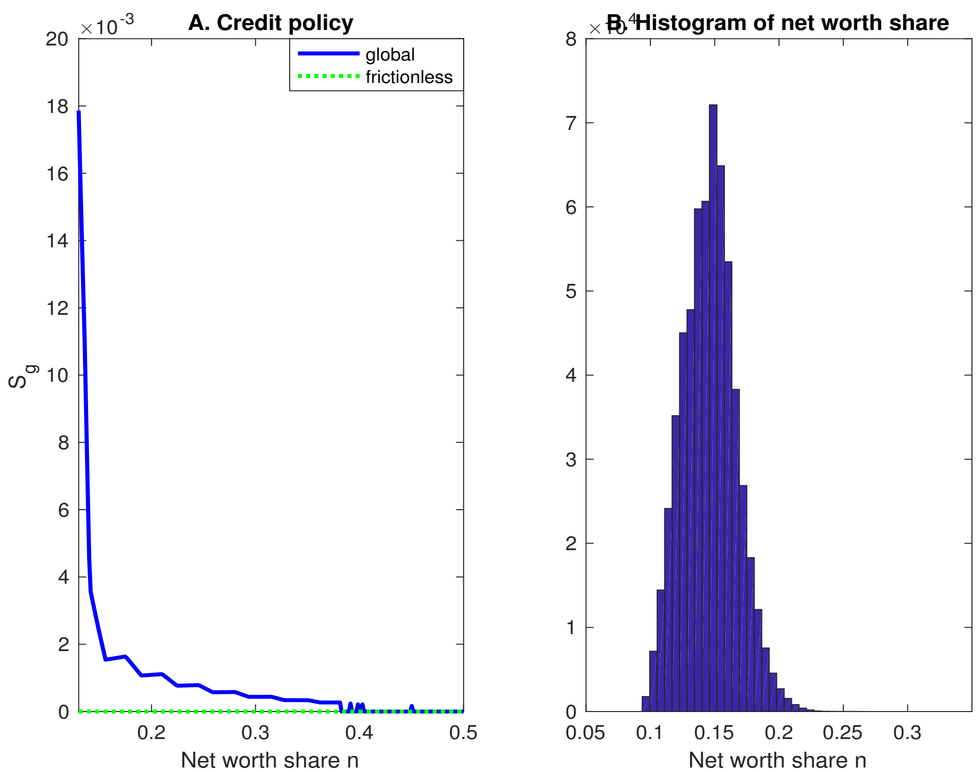
This paper presents a fully specified simple model with financial intermediaries subject to finanicial frictions and solves the model globally. We show that the global solution outperforms various local peturbation methods when the constraint binds occasionally and risks play important roles in the economy. Furthermore, we show that global solutions are necessary if we do not have a deterministic steady state with binding constraint. Our global method can be extended to richer models and used to study credit policies and other financial policies.

Figure 15: Policy Functions: Low Investment Rate Economy



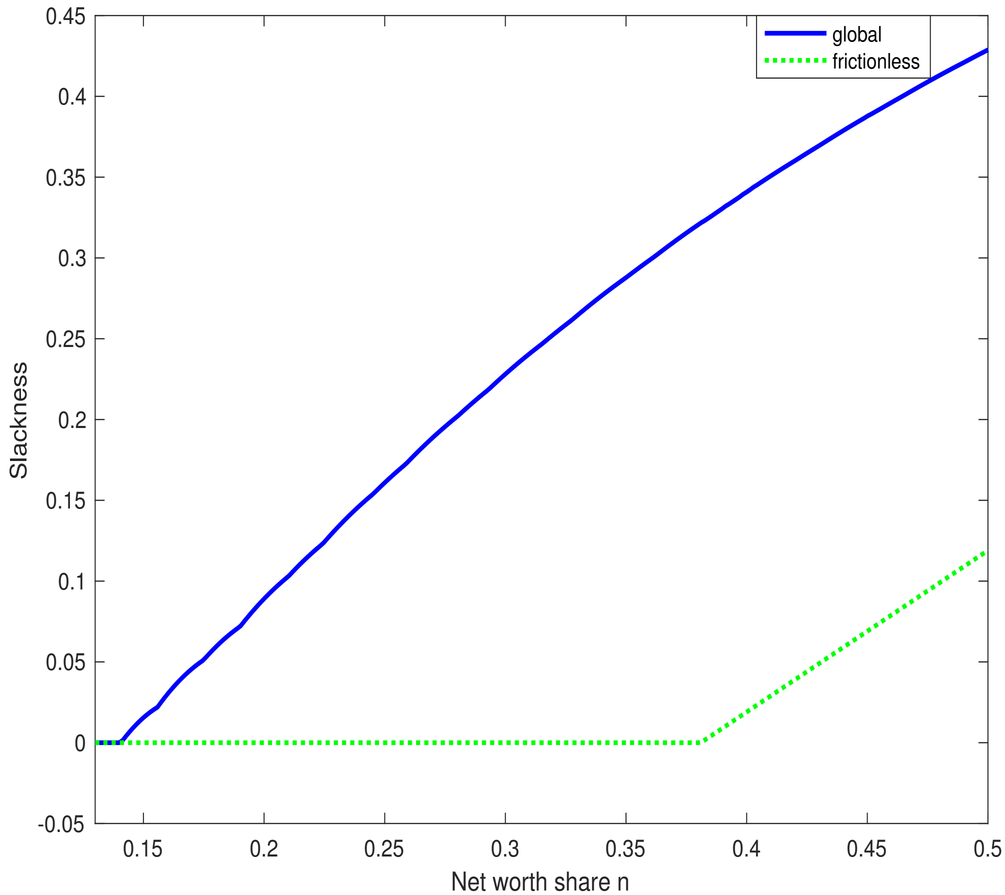
NOTE: This figure shows the policy functions of variables in an economy with low Z_i and low investment rate. The solid blue line plots the policy function obtained using global method. The green dots are values of corresponding variables in a frictionless economy.

Figure 16: Credit Policy and Histogram: Low Investment Rate Economy



NOTE: This figure shows the credit policy and a histogram of net worth ratio in an economy with low Z_i and low investment rate. The solid blue line plots the credit policy obtained using global method. The green dots are values of corresponding variables in a frictionless economy.

Figure 17: Slackness Metric to the Low Investment Rate Economy



Note: This figure shows the slackness metric of solutions in an economy with low Z_i and high investment rate.

Appendix

A Frictionless Benchmark

A frictionless economy is used as a benchmark (1) for government policy in the larger model, (2) to calibrate the parameters, (3) as a starting point of the time-iteration algorithm, (4) as a check of the solution method for the full model, and (5) for economic analysis.

The equilibrium is the balanced growth path with stochastic trend $A_{c,t}K_t^{\alpha}$. Because $\tau > 0$, the government should not conduct credit policy in the frictionless economy, i.e. $\nu_g \equiv 0$.

The prices are

$$Q_t = qA_{c,t}K_t^{\alpha-1} \quad \text{and} \quad P_t = pA_{c,t}K_t^{\alpha-1}. \tag{61}$$

The optimal investment rate satisfies

$$q/p = 1 + \vartheta i. \tag{62}$$

The optimal wage is

$$W_t = (1-\alpha)A_{c,t}K_t^{\alpha}L_{c,t}^{-\alpha}. \tag{63}$$

Taking out the trend in W_t, the optimal normalized wage is

$$w = (1-\alpha)\ell_c^{-\alpha}, \tag{64}$$

where $\ell_c \equiv L_{c,t}$. Denote $\ell_\iota \equiv L_{\iota,t}$. The labor market clearing condition implies

$$\ell_c + \ell_\iota = 1. \tag{65}$$

The dividend is

$$D_t = \alpha A_{c,t}K_t^{\alpha}\ell_c^{1-\alpha} - P_t\left(iK_t + \frac{\vartheta}{2}i^2K_t\right). \tag{66}$$

We characterize the equilibrium dividend and consumption as respectively

$$D_t = dA_{c,t}K_t^{\alpha} \quad \text{and} \quad C_t = cA_{c,t}K_t^{\alpha}. \tag{67}$$

The market clearing condition for consumption goods implies

$$y = d + w\ell_c + w\ell_\iota. \tag{68}$$

And the relationship (66) can be rewritten as

$$d = \alpha\ell_c^{1-\alpha} - p\left(i + \frac{\vartheta}{2}i^2\right).$$

The equilibrium resource constraint implies that the consumption goods output y and the household consumption c are

$$y = \ell_c^{1-\alpha} \quad \text{and} \quad c = (1-\overline{g})\ell_c^{1-\alpha}. \tag{69}$$

The zero-profit condition in the investment goods sector is

$$W_t L_{\iota,t} = P_t K_t L_{\iota,t} \tag{70}$$

which implies that $w = p$, and thus (64) can be rewritten as

$$p = (1-\alpha)\ell_c^{-\alpha}. \tag{71}$$

The market clearing condition for the investment goods is

$$i + \frac{\vartheta}{2}i^2 = \ell_\iota. \tag{72}$$

The IMRS of household is

$$\mathcal{M}_{t,t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} = \beta\left(\frac{A_{t+1}K_{t+1}^{\alpha}}{A_{c,t}K_t^{\alpha}}\right)^{-\gamma} = \beta e^{-\gamma\sigma_a\epsilon_{a,t+1}}\left(i+1-\delta\right)^{-\gamma\alpha} \tag{73}$$

The equilibrium interest rate satisfies

$$1 = \mathbb{E}_t \left[\mathcal{M}_{t,t+1} \right] (1 + R) \tag{74}$$

It implies that

$$\ln(1 + R) = -\ln \beta + \gamma\alpha \ln(i + 1 - \delta) - M(\gamma, \sigma_a), \tag{75}$$

where $M(\gamma, \sigma_a) \equiv \ln \left(\frac{1}{2} e^{-\gamma\sigma_a} + \frac{1}{2} e^{\gamma\sigma_a} \right)$. [We need to add a note, this is not Gaussian.]. As in the standard Lucas-tree economy, there are three components to the equilibrium interest rate. The first is the time value captured by the subjective discount rate $-\ln \beta$. The second is the sensitivity to the growth of consumption $\gamma \times \alpha \ln(i + 1 - \delta)$. The third component is the precautionary saving motive term $M(\gamma, \sigma_a)$. The interest rate depends heavily on the growth rate $i + 1 - \delta$, where the investment rate i is mainly governed by the adjustment cost coefficient ϑ.

The equity return satisfies the following Euler equation:

$$1 = \mathbb{E}_t[\mathcal{M}_{t,t+1}](1 + R_{k,t+1}) \tag{76}$$

The equilibrium stock return is

$$1 + R_{k,t+1} = \frac{D_{t+1} + Q_{t+1} K_{t+2}}{Q_t K_{t+1}} = e^{\sigma_a \epsilon_{a,t+1}} (i + 1 - \delta)^{\alpha - 1} \left(\frac{d}{q} + i + 1 - \delta \right), \tag{77}$$

with log return

$$\ln(1 + R_{k,t+1}) = (\alpha - 1) \ln(i + 1 - \delta) + \ln \left(\frac{d}{q} + i + 1 - \delta \right) + \sigma_a \epsilon_{a,t+1}. \tag{78}$$

Therefore, the conditional expected log return is

$$\mathbb{E}_t \left[\ln(1 + R_{k,t+1}) \right] = (\alpha - 1) \ln(i + 1 - \delta) + \ln \left(\frac{d}{q} + i + 1 - \delta \right). \tag{79}$$

The Euler equation for equity return can be rewritten as:

$$0 = \ln \beta + (\alpha - 1 - \gamma\alpha) \ln (i + 1 - \delta) + \ln \left(\frac{d}{q} + i + 1 - \delta \right) + M_k(\gamma, \sigma_a), \tag{80}$$

where $M_k(\gamma, \sigma_a) \equiv \ln \left(\frac{1}{2} e^{(\gamma-1)\sigma_a} + \frac{1}{2} e^{-(\gamma-1)\sigma_a} \right)$. Combining (75), (78), and (80), the equilibrium risk premium can be derived without solving for the other equilibrium variables. The equilibrium risk premium is

$$\Xi^* \equiv \mathbb{E} \left[\ln \left(1 + R_k^* \right) \right] - \ln \left(1 + R^* \right) = M(\gamma, \sigma_a) - M_k(\gamma, \sigma_a) \approx \gamma \sigma_a^2 - \frac{1}{2} \sigma_a^2. \tag{81}$$

This coincides with the equilibrium risk premium in the Lucas-tree economy with Jensen's term $\frac{1}{2}\sigma_a^2$. Similarly, the risk premium is independent of the growth rate of the economy.

Other equilibrium variables c^*, i^*, ℓ_c^*, ℓ_ι^*, p^*, q^*, and d^* can be solved from the system of equations including (62), (65), (68), (69), (71), (72), and (80).

A.1 A Nonlinear Equation

Equation (80) can be solved analytically from a nonlinear equation. We conjecture i^* is a constant, and plug in d^* and q^*, we have derived a nonlinear equation for i^*:

$$0 = \ln \beta + (\alpha - 1 - \gamma\alpha) \ln(i^* + 1 - \delta) + \ln(\frac{\alpha(1 - i^* - \frac{\vartheta}{2}(i^*)^2) - (1 - \alpha)}{(1 - \alpha)(1 + \vartheta i^*)} + i^* + 1 - \delta) + M_k(\gamma, \sigma_a)$$

We solve this nonlinear equation with parameters shown in the main text. We compare the analytical result with what we obtain by solving the model with log-linearization.

A.2 Solution with Log-Linear Approximation

We can also solve the equilibrium of this economy by log-linearization approximation. The system of equations include static equations (62), (65), (66), (68), (69), (71), (72), and the two Euler equations:

$$\mathbb{E}_t \underbrace{\beta e^{-\gamma g_{a,t+1}} (i_t + 1 - \delta)^{-\gamma\alpha}}_{\mathcal{M}_{t,t+1}} (1 + R_{f,t}) = 1 \tag{82}$$

$$\mathbb{E}_t \underbrace{\beta e^{-\gamma g_{a,t+1}}(i_t+1-\delta)^{-\gamma\alpha}}_{\mathcal{M}_{t,t+1}} \underbrace{e^{g_{a,t+1}}(i_t+1-\delta)^{\alpha-1}(\frac{d_{t+1}}{q_{t+1}}+i_{t+1}+1-\delta)}_{1+R_{k,t+1}} = 1 \tag{83}$$

where $g_{a,t+1} \equiv \sigma_a \epsilon_{t+1}$.

To solve the model with log-linearization approximation, we need to solve for the deterministic steady state first. The deterministic steady state for g_a is $g_{a,ss}=0$. Plug it into equation (83), we have:

$$\beta(i+1-\delta)^{-\gamma\alpha}(i+1-\delta)^{\alpha-1}(\frac{d}{q}+i+1-\delta)=1$$

Plug in d and q as a function of i, we can solve for the steady state value for investmet rate, i_{ss}. Hence, we can solve for $c_{ss}, \ell_{c,ss}, \ell_{\iota,ss}, p_{ss}, q_{ss}, d_{ss}$ in the same way as described above.

The equilibrium investment rate obtained from first order log-linear approximated solution is a constant, being smaller than the analytical solution. The difference is due to the omission of higher order term $\frac{1}{2}(\gamma-1)^2\sigma_a^2$ when expanding the Euler equation (83) around the steady state. This error can be used as a benchmark for solution comparison to assess the inaccuracy caused by omission of higher order term.

Furthermore, if we use second order approximation to solve the same system of equations, we will get exactly the same result as the analytical solution. In this economy, only a constant second order term $\frac{1}{2}(\gamma-1)^2\sigma_a^2$ shows up in the exact solution, so second order approximation suffices to characterize the behavior of the system.

A.3 Social Planner's Problem

We look at the social planner's problem in this economy. Suppose the economy consists of a continuum of households and firms (both consumption good firms and investment good firms), and the social planner can choose directly how much households consume, and how many consumption goods each consumption good producer produces, and how many investment goods the investment good producer produces. The social planner solves the following problem:

$$\max_{C_t, K_t^j, I_t^j, \tilde{I}_t^j L_{c,t}^j, L_{\iota,t}^j} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \tag{84}$$

subject to:

$$C_t \leq \int_j [A_{c,t}(K_t^j)^\alpha (L_{c,t}^j)^{1-\alpha}]dj \tag{85}$$

$$K_{t+1}^j = (1-\delta)K_t^j + \tilde{I}_t^j \tag{86}$$

$$I_t^j + \frac{\vartheta}{2}(\frac{I_t^j}{K_t^j})^2 K_t^j \leq Z_\iota K_t^j L_{\iota,t}^i \tag{87}$$

$$\int_j (L_{c,t}^j + L_{\iota,t}^j)dj = 1 \tag{88}$$

$$\int_j I_t^j dj = \int_j \tilde{I}_t^j dj \tag{89}$$

$\tilde{I}_t^j$ is the actual investment of firm j, while I_t^j is the production of investment good in firm j. The two do not have to be equal to each other, as the planner can freely reallocate investment goods across firms. We can rewrite equation (86), (87), and (89) as:

$$K_{t+1}^j = (1-\delta + i_t^j)K_t^j \tag{90}$$

$$i_t^j + \frac{\vartheta}{2}(i_t^j)^2 \leq Z_\iota L_{\iota,t}^j \tag{91}$$

$$\int_j i_t^j dj = \int_i \tilde{i}_t^j dj \tag{92}$$

where we define $i_t^j \equiv \frac{I_t^j}{K_t^j}, \tilde{i}_t^j \equiv \frac{\tilde{I}_t^j}{K_t^j}$.

Assign $\beta^t\zeta_{1,t}, \beta^t\zeta_{2,t}^j, \beta^t\zeta_{3,t}^j$, and $\beta^t\zeta_{4,t}, \beta^t\xi_t$ to be the multipliers to the five constraints, and take first order conditions with respect to $C_t, K_{t+1}^j, i_t^j, \tilde{i}_t^j, L_{c,t}^j$, and $L_{\iota,t}^j$

that characterizes the optimality conditions in the economy:

$$C_t^{-\gamma} = \zeta_{1,t} \tag{93}$$

$$\zeta_{2,t}^j = \beta \mathbb{E}_t (1 - \delta + i_{t+1}^j) \zeta_{2,t+1}^j + \zeta_{1,t+1} \frac{\alpha Y_{t+1}^j}{K_{t+1}^j} \tag{94}$$

$$\zeta_{2,t}^j = \xi_t \tag{95}$$

$$\zeta_{3,t}^j (1 + \vartheta i_t^j) = \xi_t \tag{96}$$

$$\zeta_{1,t} \frac{(1-\alpha) Y_t^j}{L_{c,t}^j} = \zeta_{4,t} \tag{97}$$

$$\zeta_{3,t} Z_\iota = \zeta_{4,t} \tag{98}$$

All firms are identical, we can get rid of the superscript j. $\zeta_{2,t}$ is the shadow value of capital, corresponding to $u'(C_t)Q_t$ in the decentralized economy. $\zeta_{3,t}$ is the shadow value of investment goods, corresponding to $u'(C_t)P_t$ in the decentralized economy. $\zeta_{4,t}$ is the shadow value of labor, corresponding to $u'(C_t)W_t$ in the decentralized economy.

Thus, equation (94) corresponds to the Euler equation, equation (96) corresponds to the q relation, equation (98) corresponds to the wage equation.

The difference between the planner's solution and the decentralized equilibrium lies in the Euler equation, i.e., equation (94). We rewrite (94) as:

$$1 = \beta \mathbb{E}_t (\frac{C_{t+1}}{C_t})^{-\gamma} [\frac{(1-\delta+i_{t+1})Q_{t+1} + \frac{\alpha Y_{t+1}}{K_{t+1}}}{Q_t}] = \beta \mathbb{E}_t (\frac{C_{t+1}}{C_t})^{-\gamma} \frac{Q_{t+1} K_{t+2} + \alpha Y_{t+1}}{Q_t K_{t+1}} \tag{99}$$

Recall in our decentralized equilibrium, the Euler equation is:

$$1 = E_t (\frac{C_{t+1}}{C_t})^{-\gamma} \frac{Q_{t+1} K_{t+2} + \alpha Y_{t+1} - P_{t+1} \Gamma(I_{t+1}, K_{t+1})}{Q_t K_{t+1}} \tag{100}$$

The difference comes out of the setup in our decentralized equilibrium that only the cash flow from final goods are capitalized, but not the cash flow from investment goods. Therefore, compared to the Euler equation (99), the return to capital in the decentralized equilibrium substracts the cost of producing investment.

B Frictional Economy

B.1 Equilibrium Conditions

There is a leverage constraint for each intermediary. The Lagrangian multiplier is $\mu(\lambda, n)$ and is associated with the financial constraint. It is dual to the slackness in the balance sheet, and thus we define $\mu(\lambda, n) \equiv \max\{0, -\mu_b(\lambda, n)\}^2$. The slackness of the balance sheets of the intermediaries is

$$\Omega(\lambda, n)n - \lambda S_p(\lambda, n) = \max\{0, \mu_b(\lambda, n)\}^2 \tag{101}$$

These two conditions imply that

$$\mu(\lambda, n)\left[\Omega(\lambda, n)n - \lambda S_p(\lambda, n)\right] = 0. \tag{102}$$

The resource constraint of the economy follows from (47) and (48) that

$$(1 - \overline{g})y(\lambda, n) = c(\lambda, n) + \tau S_g(\lambda, n)q(\lambda, n)\left[1 + i(\lambda, n) - \delta\right]. \tag{103}$$

Importantly, the expectation correspondence characterizes how the next period's aggregate normalized net worth n' as an endogenous state variable depends on the current states (λ, n) and exogenous state variables and shocks in the next period. The expectation correspondence is also an equilibrium to be solved:

$$n' = \Gamma(\lambda, n; \lambda', \epsilon_a'). \tag{104}$$

Following (39), the expectation correspondence can be expressed as:

$$\begin{aligned} n' &= \Gamma(\lambda, n; \lambda', \epsilon_a') \\ &= \frac{\theta\left\{G_r(\lambda, n; \lambda', \epsilon_a') - [1 + R_f(\lambda, n)]\right\} S_p(\lambda, n) + \theta\left[1 + R_f(\lambda, n)\right] n}{G_k(\lambda, n; \lambda', \epsilon_a')} + \aleph. \end{aligned}$$

Here $G_r(\lambda, n; \lambda', \epsilon_a') \equiv 1 + R_k(\lambda, n; \lambda', \epsilon_a')$ is the total stock return, and $G_k(\lambda, n; \lambda', \epsilon_a') \equiv Q'K'/(QK)$ is the total return of capital gain on stocks whose expression can be found in (106). The stock return described in (11) can be rewritten as

$$1 + R_k(\lambda, n; \lambda', \epsilon_a') = \frac{d(\lambda', n')}{q(\lambda, n)} e^{\sigma_a \epsilon_a'} \left[i(\lambda, n) + 1 - \delta \right]^{\alpha - 1} + G_k(\lambda, n; \lambda', \epsilon_a') \tag{105}$$

where the capital gain return is

$$G_k(\lambda, n; \lambda', \epsilon_a') = \frac{q(\lambda', n')}{q(\lambda, n)} e^{\sigma_a \epsilon_a'} \left[i(\lambda, n) + 1 - \delta \right]^{\alpha - 1} \left[i(\lambda', n') + 1 - \delta \right] \tag{106}$$

According to (32), the Euler equation for stock returns is

$$\begin{aligned} &\Omega(\lambda, n) c(\lambda, n)^{-\gamma} - \mu(\lambda, n) c(\lambda, n)^{-\gamma} \left[\Omega(\lambda, n) - \lambda \right] \\ &\qquad = \beta \left[i(\lambda, n) + 1 - \delta \right]^{-\gamma\alpha} \mathbb{E} \left\{ c(\lambda', n')^{-\gamma} e^{-\gamma \sigma_a \epsilon_a'} \left[1 - \theta + \theta \Omega(\lambda', n') \right] G_r(\lambda, n; \lambda', \epsilon_a') \middle| \lambda, n \right\} \end{aligned}$$

Define

$$\widetilde{G}_r(\lambda, n; \lambda', \epsilon_a') = G_r(\lambda, n; \lambda', \epsilon_a') / q(\lambda, n). \tag{107}$$

To obtain the Euler equation above, the key intermediate step is

$$\frac{\Lambda'}{\Lambda} = \beta \left[\frac{c(\lambda', n')}{c(\lambda, n)} \right]^{-\gamma} e^{-\gamma \sigma_a \epsilon_a'} \left[i(\lambda, n) + 1 - \delta \right]^{-\gamma\alpha}. \tag{108}$$

According to (31), the intermediary Euler equation for the risk-free rate is

$$\begin{aligned} &\left[1 - \mu(\lambda, n) \right] \Omega(\lambda, n) c(\lambda, n)^{-\gamma} \\ &\qquad = \beta \left[i(\lambda, n) + 1 - \delta \right]^{-\gamma\alpha} \left[1 + R_f(\lambda, n) \right] \mathbb{E} \left\{ c(\lambda', n')^{-\gamma} e^{-\gamma \sigma_a \epsilon_a'} \left[1 - \theta + \theta \Omega(\lambda', n') \right] \middle| \lambda, n \right\} \end{aligned}$$

The household Euler equation for the risk-free rate is

$$1 = \beta \left[i(\lambda, n) + 1 - \delta \right]^{-\gamma\alpha} \left[1 + R_f(\lambda, n) \right] \mathbb{E}_t \left\{ e^{-\gamma \sigma_a \epsilon_a'} \middle| \lambda, n \right\} \tag{109}$$

The investment goods production is

$$u(\lambda, n) = \ell_\iota(\lambda, n). \tag{110}$$

The investment goods sector market clearing condition is

$$u(\lambda, n) = i(\lambda, n) + \frac{\vartheta}{2} i(\lambda, n)^2. \tag{111}$$

And the labor market clearing condition is

$$\ell_c(\lambda, n) + \ell_\iota(\lambda, n) = 1. \tag{112}$$

The zero-profit condition for investment good firms is

$$w(\lambda, n) = p(\lambda, n). \tag{113}$$

The optimal demand of labor in consumption goods sector is

$$w(\lambda, n) = (1 - \alpha)\ell_c(\lambda, n)^{-\alpha}. \tag{114}$$

The total dividend paid out from the consumption goods sector is

$$d(\lambda, n) = \alpha \ell_c(\lambda, n)^{1-\alpha} - p(\lambda, n)u(\lambda, n), \tag{115}$$

where $\alpha \ell_c(\lambda, n)^{1-\alpha}$ is the total consumption goods minus the labor cost and $p(\lambda, n)u(\lambda, n)$ is the expenditure of purchasing investment goods. The optimal investment decision is characterized by the traditional q theory relationship:

$$q(\lambda, n) = [1 + \vartheta i(\lambda, n)]\, p(\lambda, n). \tag{116}$$

The consumption goods production is

$$y(\lambda, n) = \ell_c(\lambda, n)^{1-\alpha}. \tag{117}$$

The log risk premium $\Xi(\lambda, n)$ is defined as

$$\Xi(\lambda, n) + \log\left[1 + R_f(\lambda, n)\right] = \mathbb{E}_t\left\{G_r(\lambda, n; \lambda', \epsilon'_a)|\lambda, n\right\}. \tag{118}$$

The credit policy can be written as

$$S_p(\lambda, n)\left[1 + \nu_g(\Xi(\lambda, n) - \Xi^*)\right] = 1, \tag{119}$$

where $\nu_g = \nu_{g,0} + \nu_{g,1}\left(\frac{1}{n_t} - 1\right)$ is state-dependent.

C Solution with Perturbation Method

This section provides the details of how to solve the model in the main text with first and second order approximation using Dynare 4.4.5. First, we list explicitly the system of equations we need to solve the model in dynare. Second, details of computing the deterministic steady state are provided. Third, we show how to convert the policy function reported by Dynare to functions of state variables $n, \log\lambda$ for comparison with global solution. Finally, we show how to evaluate Euler errors with policy functions in hand.

C.1 System of Equations

We assume that the constraint always binds, so we have:

$$\Omega_t n_t = \lambda_t S_{p,t} \tag{120}$$

The households' Euler equation for risk free rate is the same as (82). The two Euler equations for the intermediary are (31) and (32):

$$E_t \mathcal{M}^{\mathcal{I}}_{t,t+1}(1 + R_{f,t}) = 1 - \mu_t \tag{121}$$

$$E_t \mathcal{M}^{\mathcal{I}}_{t,t+1}(1 + R_{k,t+1}) = 1 - \mu_t + \frac{\lambda_t \mu_t}{\Omega_t} \tag{122}$$

where:

$$\mathcal{M}^{\mathcal{I}}_{t,t+1} = \beta(\frac{c_{t+1}}{c_t})^{-\gamma} e^{-\gamma g_{a,t+1}} (i_t + 1 - \delta)^{-\gamma\alpha} \frac{1 - \theta + \theta\Omega_{t+1}}{\Omega_t} \tag{123}$$

$g_{a,t+1} \equiv \sigma_a \epsilon_{a,t+1}$ as in the frictionless economy. The derivation of these two Euler equations (31) and (32) are in the main text. $1 + R_{k,t+1}$ is defined in (105) ane (106).

$$1+R_{k,t+1} = \frac{d_{t+1}}{q_t} \exp(g_{a,t+1})(i_t+1-\delta)^{\alpha-1} + \underbrace{\frac{q_{t+1}}{q_t} \exp(g_{a,t+1})(i_t + 1 - \delta)^{\alpha-1}(i_{t+1} + 1 - \delta)}_{G_{k,t+1}} \tag{124}$$

Aggregate net worth evolves as in equation (39):

$$n_{t+1} = \theta[(R_{k,t+1} - R_{f,t})S_{p,t} + n_t(1 + R_{f,t})]/G_{k,t+1} + \aleph \tag{125}$$

Risk premium is defined in (118) and government policy is characterized by (119).

The equilibrium of this economy is characterized by static equations (103), (110), (111), (112), (113), (114), (115), 116), (117), (119), two intermediary Euler equations (121),(122) and intermediary constraint (120), and net worth evolution (125).

C.2 Deterministic Steady State

The deterministic steady state of this economy is characterized by a set of nonlinear equations. We solve the deterministic steady state numerically using nonlinear solver as follows:

- Make a guess on i_{ss}, and solve for $c_{ss}, \ell_{c,ss}, \ell_{\iota,ss}, p_{ss}, q_{ss}, d_{ss}$ as in the frictionless economy.
- Solve for the deterministic steady state of $1 + R_{ss}$ and $1 + R_{k,ss}$:

$$R_{ss} = \frac{1}{\beta}(i_{ss} + 1 - \delta)^{\gamma\alpha} \tag{126}$$

$$R_{k,ss} = \frac{d_{ss}}{q_{ss}}(i_{ss} + 1 - \delta)^{\alpha-1} + (i_{ss} + 1 - \delta)^{\alpha} \tag{127}$$

- Solve for the steady state government policy from:

$$S_{p,ss}\left[1+\nu_g(\ln(1+R_{k,ss})-\ln(1+R_{ss})-\Xi^*)\right]=1$$

- The deterministic steady state of Ω_{ss}, μ_{ss} can be solved by (121) and (122) using a nonlinear solver.
- The steady state of net worth share is $n_{ss}=\frac{\lambda_{ss}}{\Omega_{ss}}$, according to (120).
- Check whether equation (125) holds. If not, iterate on i_{ss} until it holds.

C.3 Policy Function

When we solve the model using first order pertubation method, dynare delivers output in the form:

$$X_t = A_0 + A_1 X_{t-1} + A_2 \epsilon_t \tag{128}$$

where X_t includes all variables in the economy, and ε_t includes all the primitive shocks in the economy.

We need to convert the policy function in VAR(1) form into a function of n and λ. Suppose we have a variable $s(n,\lambda)$ (it can be any variable in the economy which only depends on the two state variables n,λ), we make a (log)-linear approximation [7] as:

$$s_t = C_{0,s} + C_{1,s}\log n_t + C_{2,s}\log\lambda_t \tag{129}$$

We extract the related rows in the VAR(1) form:

$$s_t = A_{0,s} + A_{1,s}X_{t-1} + A_{2,s}\epsilon_t \tag{130}$$

And the row for n and λ:

$$\log n_t = A_{0,n} + A_{1,n}X_{t-1} + A_{2,n}\epsilon_t \tag{131}$$

$$\log\lambda_t = A_{0,\lambda} + A_{1,\lambda}X_{t-1} + A_{2,\lambda}\epsilon_t \tag{132}$$

[7] Variables of S_g, μ, and Ξ are in level, not in logs.

Plug (131) and (132) into (129), we have:

$$s_t = (C_{0,s} + C_{1,s}A_{0,n} + C_{2,s}A_{0,\lambda}) + (C_{1,s}A_{1,n} + C_{2,s}A_{1,\lambda})X_{t-1} + (C_{1,s}A_{2,n} + C_{2,s}A_{2,\lambda})\epsilon_t \tag{133}$$

We solve for $C_{1,s}$ and $C_{2,s}$ from:

$$C_{1,s}A_{1,n} + C_{2,s}A_{1,\lambda} = A_{1,s}, C_{1,s}A_{2,n} + C_{2,s}A_{2,\lambda} = A_{2,s} \tag{134}$$

The mean $C_{o,s}$ can be pinned down by:

$$C_{0,s} = \bar{s} - C_{1,s}\log\bar{n} - C_{2,s}\log\bar{\lambda} \tag{135}$$

where $\bar{s}, \log\bar{n}, \log\bar{\lambda}$ are the mean of respective variables.

In practice, it is easier to get the coefficients $C_{0,s}, C_{1,s}$, and $C_{2,s}$ by regressing the simulated variables on simulated $\log n$ and $\log\lambda$. To get the dynamics of net worth share, we regress n' on $\log n, \log\lambda, \log\lambda'$ and ϵ_a. The nature of linear VAR system guarantees that the regression delivers a perfect fit.

Policy functions can be obtained in the same way when we solve the model with second order pertubation. We regress the simulated variables on $\log n$, $\log\lambda$, $(\log n)^2$, $(\log\lambda)^2$, and $\log n \log\lambda$. There is no guarantee that the fit is perfect, but we can check the R^2 of each regression. In our model, each regression has R^2 very close to 1. To get the dynamics of net worth share n', we include $\log n, \log\lambda, \log\lambda', \epsilon_a$, and all quadratic and interactive terms.

C.4 Evaluation of Euler Errors

To evaluate the pertubation solutions' Euler errors, it is crucial to evaluate the expectation in the Euler equations. For each time t with state n_t, λ_t, there are six possible states in time $t+1$: $\epsilon_{a,t+1} = 1$ or -1 to approximate the standard normal distribution, interacting with λ_{t+1} taking high, medium or low value. With the policy function of $n'(n, \lambda, \lambda', \epsilon_a)$ at hand, we get derive n' in each state. Since we express all variables as a function of n and λ, we already obtain the value of all variables in each state.

D The Method of Guerrieri and Iacoviello (2015)

This section introduces the Guerrieri and Iacoviello (2015) toolkit that can solve dynamic models with occasionally binding constraint easily (We refer to the method as

"OccBin" hereafter) [8]. The policy function plots, error analysis and impulse response in the main text labeled as "OccBin" are obtained in the procedures shown below.

The main idea of the OccBin method is to (log-)linearize the model in two regimes: one with the constraint binding and the other with the constraint slack. In our example, the key equations that differ across two regimes are leverage constraint and intermediary Euler equation for holding capital. In the constrained economy, we have the constraint binding:

$$\Omega_t n_t = \lambda_t S_{p,t} \tag{136}$$

To characterize the equilibrium conditions for an economy with slack constraint, we replace the binding constraint with:

$$\mu_t = 0 \tag{137}$$

Combine the above two equations with other equilibrium conditions specified in the text, and use (log-)linear approximation, we will obtain two sets of equations for both regimes. Suppose for the regime with slack constraint, we have:

$$\mathcal{A}E_t X_{t+1} + \mathcal{B}X_t + \mathcal{C}X_{t-1} + \mathcal{D} + \mathcal{E}\epsilon_t = 0 \tag{138}$$

For the regime with binding constraint, we have:

$$\mathcal{A}^* E_t X_{t+1} + \mathcal{B}^* X_t + \mathcal{C}^* X_{t-1} + \mathcal{E}^*\epsilon_t = 0 \tag{139}$$

Suppose the decision rules in the constrained regime are:

$$X_t = \mathcal{P}^* X_{t-1} + \mathcal{Q}^*\epsilon_t \tag{140}$$

It can be directly obtained from solving the economy in the constrained regime. The economy starts from the constrained regime at the deterministic steady state at time 0 and there is a one-time unexpected shock in time 1. Our goal is to solve for the policy function of the economy:

$$X_t = \mathcal{P}_t X_{t-1} + \mathcal{R}_t + \mathcal{Q}_t\epsilon_t \tag{141}$$

[8] The description and algorithm follows section 2 of Guerrieri and Iacoviello (2015), except that here we choose the regime with binding constraint as the base regime.

where $\mathcal{P}_t$, $\mathcal{Q}_t$, and $\mathcal{R}_t$ are time-varying. We have the constant $\mathcal{R}_t$ because when the economy switches between regimes, the steady state also changes. We only have a one-time unexpected shock at time 1 and no more shocks afterwards. The algorithm to solve the policy function proceeds in six steps:

(i) Conjecture that after T periods the economy converts back to the initial binding regime. Therefore, the dynamics of X_t at time T follows:

$$X_T = \mathcal{P}^* X_{T-1} + \mathcal{Q}^* \epsilon_t \tag{142}$$

where $\mathcal{P}^*$, $\mathcal{Q}^*$ are obtained from solving the model in which the constraint always binds.

(ii) Plug equation (142) into equation (138) and get:

$$X_{T-1} = -(\mathcal{A}\mathcal{P}^* + \mathcal{B})^{-1}\mathcal{C}X_{T-2} - (\mathcal{A}\mathcal{P}^* + \mathcal{B})^{-1}\mathcal{D} \tag{143}$$

which gives rise to:

$$\mathcal{P}_{T-1} = -(\mathcal{A}\mathcal{P}^* + \mathcal{B})^{-1}\mathcal{C}, \mathcal{R}_{T-1} = -(\mathcal{A}\mathcal{P}^* + \mathcal{B})^{-1}\mathcal{D} \tag{144}$$

(iii) Using $X_{T-1} = \mathcal{P}_{T-1}X_{T-2} + \mathcal{R}_{T-1}$, repeat the procedure, iterate backward and solve for $\mathcal{P}_{T-2}, \mathcal{R}_{T-2}$.

(iv) Repeat this procedure backward and solve for $\mathcal{P}_{T-t}, \mathcal{R}_{T-t}$ with $t < T$.

(v) Use $\mathcal{P}_2$ to solve for $\mathcal{Q}_1$:

$$\mathcal{Q}_1 = -(\mathcal{A}\mathcal{P}_2 + \mathcal{B})^{-1}\mathcal{E} \tag{145}$$

(vi) Using the guess for solutions to compute the response of X_t to the one-time shock, and check whether the guess is consistent with the solution. If consistent, stop; if not, update the guess and redo steps (i) to (v).

There are two requirements for this method to be applicable. First of all, the base regime economy has to have a deterministic steady state and satisfy the usual Blanchard-Kahn condition, but not necessarily for the alternative regime economy. Secondly, after the shock, the economy will ultimately revert back to the base regime.

The OccBin method has its limitations. First of all, the use of OccBin is only limited to first order approximation, thus failing to capture the second and higher order terms, which play important roles in financial variables. Secondly, the OccBin heavily depends on the existence of a deterministic steady state in the base regime economy, so that the method cannot be used to solve our model with low investment rate. In the low investment rate economy, there is no deterministic steady state when the constraint binds, and the deterministic steady state in the frictionless economy does not satisfy the constraint. Thirdly, the method does not take into account the uncertainty about whether future constraint will bind or not, which kills the effect of higher-order terms of the constrained multiplier.

E Algorithm

The baseline idea of the algorithm is value function iteration method (i.e. the time-iteration method), which is a standard generic method. The implementation is what matters when the researcher tries to apply this generic idea to solve various classes of macroeconomic models. We develop a particular implementation for the class of macroeconomic models with financial intermediaries whose borrowing constraints are occasionally binding. We start from the "end" of the economy and then iteratively solve the equilibrium until convergence according to a pre-specified small threshold. The algorithm consists of three steps:

(1) Solve the equilibrium in the terminal period (i.e. the end of the economy):

- In the end of the economy, the asset prices become zero, there is no need for investment, and the intermediaries become irrelevant.
- Deriving the solution is very straightforward. See the beginning of the code "MacroReview-FrictionalEconomy-MonthlyFrequency-TwoPeriodEnd.m".

(2) Solve the equilibrium in a two-period model with the last period to be the end of the economy:

- Start with frictionless case because its solution is close to the last period solution. The financial friction parameter λ increases gradually step by step. For each step, the solution in the previous step is used as the initial value in solving the equilibrium in the current step.
- The final step gives the solution for the equilibrium of the two-period frictional economy.
- The code is "MacroReview-FrictionalEconomy-MonthlyFrequency-TwoPeriodEnd.m".

(3) Solve the equilibrium for the infinite-period model as the limit of backward induction:

- Start with the solving the equilibrium by using the results of (2) for the two-period model as the next-period equilibrium. And then, iterate backwards until the equilibrium converges.
- Use the previous period's equilibrium as the initial value for solving the current period equilibrium.
- The key feature is to solve the correspondence mapping of the endogenous state variable $n' = \Gamma(\lambda, n, \lambda', \epsilon_a')$ simultaneously with all the endogenous variables. To make it possible, the projection method is incorporated into each iteration to interpolate the equilibrium in the next period.
- The marginal value of net worth in the next period in crucial for computing the prices in the current period based on the Euler equations. Within each iteration, the marginal value of net worth is derived using the solved equilibrium consumption and the Envelope condition.
- The financial friction levels λ_L, λ_M, and λ_H are kept constant over iterations.
- The code is "MacroReview-FrictionalEconomy-MonthlyFrequency-TimeIteration.m".

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