Online Appendix for “Dissecting Bankruptcy Frictions”

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This Online Appendix presents additional results, mainly extensions of the theory, complementing the results presented in the paper. Most of the results presented here are summarized in the paper.

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A Inefficient Delay in Bargaining with Complete Information

In this section, we use two-period models to illustrate the key difference between our dynamic bargaining model and the seminal framework based on Rubinstein (1982), Merlo and Wilson (1995), and Merlo and Wilson (1998). In those models with complete information, there is no inefficient delay in equilibrium even in the presence of conflict of interests. Example (i) is a simplified version of Rubinstein (1982) and Bebchuk and Chang (1992) in which there is an efficient equilibrium with no delay. Example (ii) can be viewed as a simplified version of Merlo and Wilson (1995) and Merlo and Wilson (1998) in which efficient delay occurs in equilibrium. Example (iii) is a simplified version of our model in the main text. The key feature is that the “separation principle” of Merlo and Wilson (1998) is violated, and thus inefficient delay occurs in equilibrium even when the creditors face a complete-information environment.

A.1 Example (i): No Delay in Equilibrium

Suppose there are two periods: \( t = 0, 1 \). Consider senior and junior creditors who bargain over how to split the firm value. The senior and junior debt levels are denoted by \( D_S \) and \( D_J \), respectively. We assume that the firm value is equal to the senior debt level in period \( t = 0 \), that is, \( V_0 = D_S \). In period \( t = 1 \), there are two contingent scenarios with each occurring with equal probability \( \frac{1}{2} \). In one scenario, the firm value rises to a high level \( V_1 = D_S + \theta_u - \delta \); and in the other scenario, the firm value declines to a low level \( V_1 = D_S - \theta_d - \delta \). The depreciation rate of firm value \( \delta \) is positive. In the terminal period \( (t = 1) \), the judge uses the cram down provision, and distributes the outcome to creditors following the absolute priority rule (APR). The key feature is that the firm value in period \( t = 1 \) does not depend on who proposes the plan.

Suppose that \( \theta_u = \theta_d = \theta > \delta > 0 \). Thus, the size of the “cake” follows a supermartingale:

\[
E_0[V_1] \leq V_0. \tag{1}
\]

This is because 
\[
E_0[V_1] = (D_S + \theta - \delta)/2 + (D_S - \theta - \delta)/2 = D_S - \delta < D_S = V_0.
\]

Suppose the senior has the opportunity to propose in period \( t = 0 \). Before making decision, the senior needs to figure out the continuation value of both creditors if the deal moves to the next period. The continuation value is the average payoff in period \( t = 1 \). What are the contingent payoffs? If the world ends up in the high-level state with \( V_1 = D_S + \theta - \delta \) in period \( t = 1 \), the senior would get \( D_S \) and the junior would get \( \theta - \delta > 0 \); alternatively, if the world ends up in the low-level state with \( V_1 = D_S - \theta - \delta \) in period \( t = 1 \), the senior would get \( D_S - \theta - \delta \) and the junior would get 0. Thus, at the end of period \( t = 0 \), the continuation value of the senior creditor is 
\[
D_S/2 + (D_S - \theta - \delta)/2 = D_S - \delta < D_S = V_0.
\]

Therefore, the senior creditor would have to pay at least \( (\theta - \delta)/2 \) to the junior creditor if she would like to settle the deal in period \( t = 0 \), and the senior creditor would obtain the continuation value \( D_S - (\theta + \delta)/2 \) if she would like to settle the deal in period \( t = 1 \). The former choice (i.e. settling the deal in period \( t = 0 \) by paying off the
junior creditor) would pay the senior creditor with $V_0 - (\theta - \delta)/2 = D_S - (\theta - \delta)/2$, and the latter choice (i.e. settling the deal in period $t = 1$) will worth $D_S - (\theta + \delta)/2$ as present value to the senior creditor. It is obvious that the former choice is preferred by the senior creditor (i.e., $D_S - (\theta - \delta)/2 > D_S - (\theta + \delta)/2$). Fundamentally, this is a result of the supermartingale property in (1).

Therefore, there is no delay in equilibrium, and importantly, this is an efficient outcome since the firm value follows a supermartingale. This simple example illustrates the key economic insight of Rubinstein (1982).

### A.2 Example (ii): Efficient Delay in Equilibrium

Suppose there are two periods: $t = 0, 1$. Consider senior and junior creditors who bargain over how to split the firm value. The senior and junior debt levels are denoted by $D_S$ and $D_J$, respectively. We assume that the firm value is equal to the senior debt level in period $t = 0$, that is, $V_0 = D_S$. In period $t = 1$, there are two contingent scenarios with each occurring with equal probability $1/2$. In one scenario, the firm value rises to a high level $V_1 = D_S + \theta_u - \delta$; and in the other scenario, the firm value declines to a low level $V_1 = D_S - \theta_d - \delta$. The depreciation rate of firm value $\delta$ is positive. In the terminal period ($t = 1$), the judge uses the cram down provision, and distributes the outcome to creditors following the absolute priority rule (APR). The key feature is that the firm value in period $t = 1$ does not depend on who proposes the plan.

So far, the setup is exactly the same as the example in Section A.1. Now, we introduce the key difference: $\theta_u$ is much larger than $\theta_d$ such that $\theta_u - \theta_d > 2\delta$. Therefore, the total size of the “cake” follows a submartingale:

$$E_0[V_1] \geq V_0. \tag{2}$$

This is because $E_0[V_1] = (D_S + \theta_u - \delta)/2 + (D_S - \theta_d - \delta)/2 = D_S + (\theta_u - \theta_d - 2\delta)/2 > D_S = V_0$.

Suppose the senior has the opportunity to propose in period $t = 0$. Before making decision, the senior needs to figure out the continuation value of both creditors if the deal moves to the next period. The continuation value is the average payoff in period $t = 1$. What are the contingent payoffs? If the world ends up in the high-level state with $V_1 = D_S + \theta_u - \delta$ in period $t = 1$, the senior would get $D_S$ and the junior would get $\theta_u - \delta > 0$; alternatively, if the world ends up in the low-level state with $V_1 = D_S - \theta_d - \delta$ in period $t = 1$, the senior would get $D_S - \theta_d - \delta$ and the junior would get $0$. Thus, at the end of period $t = 0$, the continuation value of the senior creditor is $D_S/2 + (D_S - \theta_d - \delta)/2 = D_S - (\theta_d + \delta)/2$, and the continuation value of the junior creditor at the end of period $t = 0$ is $(\theta_u - \delta)/2 + 0/2 = (\theta_u - \delta)/2$. Therefore, the senior creditor would have to pay at least $(\theta_u - \delta)/2$ to the junior creditor if she would like to settle the deal in period $t = 0$, and the senior creditor would obtain the continuation value $D_S - (\theta_d + \delta)/2$ if she would like to settle the deal in period $t = 1$. The former choice (i.e. settling the deal in period $t = 0$ by paying off the junior creditor) would pay the senior creditor with $V_0 - (\theta_u - \delta)/2 = D_S - (\theta_u - \delta)/2$, and the latter choice (i.e. settling the deal in period $t = 1$) will worth $D_S - (\theta_d + \delta)/2$ as present value to the senior creditor. It
is obvious that the latter choice is preferred by the senior creditor (i.e., \( D_S - (\theta_u - \delta)/2 < D_S - (\theta_d + \delta)/2 \)). Fundamentally, this is a result of the submartingale property in (2).

Therefore, delay occurs in equilibrium, and importantly, this is an efficient outcome since the firm value follows a submartingale. This simple example illustrates the key economic insight discussed by Merlo and Wilson (1995) and Merlo and Wilson (1998).

A.3 Example (iii): Inefficient Delay in Equilibrium

Based on the insight of Coase Theorem, the bargaining cost is necessary (but not sufficient) to get inefficient delay. There are several ways to generate inefficient delay in dynamic bargaining games with complete information. One example is to incorporate non-stationary strategies like trigger strategies in supergames (e.g., Fernandez and Glazer, 1991; Busch and Wen, 1995). Another example is to consider the hold-up problem in multilateral bargaining (e.g., Cai, 2000). Our model is designed to capture key features of the bankruptcy bargaining process, including the key assumption that who proposes and leads the reorganization matters for the outcome (i.e., the violation of “separation principle”). More precisely, two creditors in our model have different reorganization skills, and they cannot propose using the counterparty’s plans. In other words, the size of the “cake” is proposer-dependent. The assumption is reasonable since the creditors of big bankruptcy cases are usually private equity funds and specialized distressed-asset hedge funds.

Suppose there are two periods: \( t = 0, 1 \). Consider senior and junior creditors who bargain over how to split the firm value. The senior and junior debt levels are denoted by \( D_S \) and \( D_J \), respectively. Suppose \( V_{J,0} \) and \( V_{S,0} \) are the size of the “cake” at \( t = 0 \) when junior and senior creditors propose, respectively. We assume that the firm value based on the senior creditor’s reorganization plan is equal to the senior debt level in period \( t = 0 \), that is, \( V_0 = D_S \). In period \( t = 1 \), there are two contingent scenarios with each occurring with equal probability \( 1/2 \). In one scenario, the firm value rises to a high level \( V_{i,1} = V_{i,0} + \theta - \delta \) for each \( i \in \{S, J\} \); and in the other scenario, the firm value declines to a low level \( V_{i,1} = V_{i,0} - \theta - \delta \) for each \( i \in \{S, J\} \). The depreciation rate of firm value \( \delta \) is positive.

Assume that \( V_{S,0} < V_{J,0} - \theta - \delta \). That is, the junior creditor has much higher initial reorganization skill. Suppose the senior has the opportunity to propose in period \( t = 0 \). Before making decision, the senior needs to figure out the continuation value of both creditors if the deal moves to the next period. The senior creditor needs to pay the junior creditor \( V_{J,0} - \delta \) to settle the deal at \( t = 0 \). However, the senior creditor finds it not worth to do it since \( V_{S,0} - (V_{J,0} - \delta) < -\theta < 0 \). Therefore, the deal will be settled at \( t = 1 \). The social planner only cares about \( V_{J,0} \) and \( V_{J,1} \) since they are always the higher values (i.e., \( V_{J,t} > V_{S,t} \) for \( t = 0, 1 \)). But, \( V_{J,t} \) is a supermartingale:

\[
E_0 [V_{J,1}] < V_{J,0}.
\]
This is because $E_0[V_{J,1}] = (V_{J,0} + \theta - \delta)/2 + (V_{J,0} - \theta - \delta)/2 = V_{J,0} - \delta < V_{J,0}$. Therefore, it is efficient to settle the deal at $t = 0$, and thus, inefficient delay occurs in equilibrium. This simple example illustrates the key economic insight of our model.

In summary, there are two frictions: (i) conflict of interests, and (ii) asymmetric information. Random proposing scheme also reflects a form of conflict of interests. Conflict of interests alone does not create inefficient delay in dynamic bargaining with complete information. The violation of “separation principle” due to proposer-dependent reorganization value leads to inefficient delay. Moreover, asymmetric information interacts with conflict of interests, which leads to further delay and efficiency loss.

B Extended Model I: Stochastic $V_t$

In the baseline model, we assume that $V_t = \rho^{-1}V_0$, decaying deterministically. Here, we shall assume that $V_t = \rho^{t-1}\hat{V}_t$ where $\hat{V}_t$ evolves as a two-state Markov process on $\{e^{-\nu}V_0, e^{\nu}V_0\}$ with $\nu > 0$.

At the very beginning of each period $t$ before everything else happens, there is a small probability $\pi \in (0, 1)$ by which $\hat{V}_{t-1}$ will jump from $\hat{V}_{t-1}$ randomly to a value among $\{e^{-\nu}V_0, e^{\nu}V_0\}$ with equal probability 1/2. That is, $\hat{V}_{t-1}$ updates to $\hat{V}_t$ at the beginning of every period $t$ before the proposer is randomly chosen. Taking senior creditor as an example, the continuation value right before the update of $\hat{V}_{t-1}$ (i.e. the continuation value at the very end of period $t - 1$) is denoted by $H_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, \hat{V}_{t-1})$, and the continuation value right after the realization of $\hat{V}_t$ is denoted by $W_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, \hat{V}_t)$. The relation between $H_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, \hat{V}_{t-1})$ and $W_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, \hat{V}_t)$ is

$$H_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, e^{-\nu}V_0) = (1 - \pi + \frac{\pi}{2})W_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, e^{-\nu}V_0) + \frac{\pi}{2}W_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, e^{\nu}V_0),$$

$$H_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, e^{\nu}V_0) = (1 - \pi + \frac{\pi}{2})W_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, e^{\nu}V_0) + \frac{\pi}{2}W_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, e^{-\nu}V_0).$$

How to calibrate the process $\hat{V}_t$? We calibrate $\nu$ and $\pi$ so that log $\hat{V}_t$ (in the model) and the log of industry-level Tobin’s Q (in the data) have the same persistence and conditional volatility. (Recall that $V_0$ in the baseline estimation equals a firm-specific constant times industry-level Tobin’s Q, so log $\hat{V}_t$ and the log of industry-level Tobin’s Q should share the same persistence and conditional volatility.) Using panel data by industry and year, and using the estimator of Han and Phillips (2010), we estimate a regression of log median Tobin’s Q on its lag and industry fixed effects. We choose $\nu$ and $\pi$ to match this regression’s estimated slope (0.603) and residual volatility (0.211), taking into account that one model period does not correspond to one year.

Now, we explain how to solve the bargaining game with stochastic firm’s potential reorganization value $V_t$. First, we describe the initial point of the dynamic programming procedure. The equilibrium, characterized by the Bellman equation, is solved recursively by backward induction. The “end period” is the first time $t$ such that $\rho^{t-1}e^{\nu}V_0 \leq L$. In equilibrium, there is certain probability that the bargaining
ends before the scenario $\rho^{-1}e^{-u}V_0 \leq \Lambda$ occurs. In the end period, both creditors will choose to quit the bargaining by liquidating the firm. The APR applies when splitting the liquidation value.

Next, we describe the Bellman equation for the senior creditor. Let us consider the continuation value function in period $t$ for any $t \geq 0$. The key is to establish the recursive Bellman equations for the continuation values at the beginning of the morning of period $t$, i.e., $W_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, \hat{\nu}_t)$ and $W_{J,t}(\theta_{J,t}, \ell_{S,t}, \ell_{J,t}, \hat{\nu}_t)$. The state variables include the endogenous state variables $(\ell_{S,t}, \ell_{J,t})$, the exogenous (private) state variable $\theta_{J,t}$ or $\theta_{S,t}$, and $\hat{\nu}_t$. The private information about $\theta_{S,t}$ and $\theta_{J,t}$ is learned by the senior and junior, respectively, at the very beginning of the afternoon of period $t-1$.

The continuation value of the senior creditor at the beginning of period $t$ follows the Bellman equation:

$$W_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, \hat{\nu}_t) = (1 - \lambda_J) \max \left\{ O_{S,t}, \max_{\xi_{S,t}} \mathbb{E}^S_t \left[ \tilde{M}_{S,t+1}(\xi_{S,t}) \right] \right\}$$

if $S$ proposes in the morning

$$+ \lambda_J \mathbb{E}^S_t \left[ \max_{\xi_{S,t+1} \in \{0,1\}} \tilde{A}_{S,t+1}(\xi_{S,t+1}) \bigg| \theta_{J,t} \geq \phi_{J,t} \right] \mathbb{P}^S_t \{ \theta_{J,t} \geq \phi_{J,t} \}$$

if $J$ proposes reorganization in the morning

$$+ \lambda_J \mathbb{E}^S_t \left[ \max \{ O_{S,t}, U_{t+1}(\theta_{S,t+1}, \hat{\nu}_{t+1}) - O_{J,t} \} \right] \mathbb{P}^S_t \{ \theta_{J,t} < \phi_{J,t} \},$$

if $J$ decides to liquid in the morning

where $\mathbb{E}^S_t$ is the conditional expectation of the senior creditor over $(\theta_{J,t}, \theta_{J,t+1}, \theta_{S,t+1}, \hat{\nu}_{t+1})$; namely, the junior creditor’s reorganization skills in the morning of periods $t$ and $t+1$, the senior’s reorganization skill in the morning of period $t+1$, and the potential reorganization value in the morning of period $t+1$, conditioning on $(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, \hat{\nu}_t)$. Here, $\xi_{S,t}$ is the offer made by the senior in the morning of period $t$. The indicator variable $\xi_{S,t+1} = 1$ means that the offer proposed by the junior creditor in the morning of period $t$ is accepted by the senior creditor in the afternoon of period $t$. Moreover, $\phi_{J,t}$ is the threshold for the junior creditor to choose reorganization over liquidation; that is, the junior creditor chooses to propose liquidation if and only if $\theta_{J,t} < \phi_{J,t}$.

If the senior creditor proposes in the morning of period $t$, the payoff to the senior creditor in the afternoon of period $t$, conditional on the choice $\xi_{S,t}$, is described as follows:

$$\tilde{M}_{S,t+1}(\xi_{S,t}) = \left[ U_{t+1}(\theta_{S,t+1}, \hat{\nu}_{t+1}) - \xi_{S,t} \right] 1\{ H_{J,t+1}(\theta_{J,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, \hat{\nu}_t) \leq \xi_{S,t} \}$$

if $J$ accepts the offer

$$+ H_{S,t+1}(\theta_{S,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, \hat{\nu}_t) 1\{ H_{J,t+1}(\theta_{J,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, \hat{\nu}_t) > \xi_{S,t} \}.$$
probability are
\[
\mathbb{P}(\hat{V}_{t+1} = e^{-\nu}V_0|\hat{V}_t = e^{\nu}V_0) = \pi/2
\]
\[
\mathbb{P}(\hat{V}_{t+1} = e^{\nu}V_0|\hat{V}_t = e^{-\nu}V_0) = (1 - \pi) + \pi/2.
\]

How do the endogenous state variables \(\ell_{S,t}\) and \(\ell_{J,t}\) evolve endogenously in this case? If the senior creditor receives the proposal opportunity in the morning of period \(t\), \(\ell_{S,t+1} = \theta_{S,t}\) and \(\ell_{J,t+1} = \max(\theta^*_J, \ell_{J,t})\) with \(\theta^*_J\) being determined by \(\xi_{S,t} = H_{J,t+1}(\theta^*_J, \theta_{S,t}, \theta^*_J, \hat{V}_t)\). The update of \(\ell_{S,t+1}\) takes place right after the junior creditor sees the proposal \(\xi_{S,t}\). The update is perfectly perceived and internalized by the senior creditor at the very beginning of period \(t\), when she makes the proposal decision right after receiving the proposing opportunity.

If the junior creditor proposes in the morning of period \(t\), the payoff to the senior creditor in the afternoon of period \(t\), conditional on the junior’s optimal choice \(\xi_{J,t}\) (which further depends on \((\theta_{J,t}, \ell_{S,t}, \ell_{J,t}, \hat{V}_t)\)), is described as follows:

\[
\max_{\zeta_{S,t+1} \in \{0, 1\}} \tilde{A}_{S,t+1}(\zeta_{S,t+1}) = \xi_{J,t} \mathbf{1}\left\{H_{S,t+1}(\theta_{S,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, \hat{V}_t) \leq \xi_{J,t}\right\}
\]

if \(S\) accepts the offer: \(\zeta_{S,t+1} = 1\)

\[
+ H_{S,t+1}(\theta_{S,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, \hat{V}_t) \mathbf{1}\left\{H_{S,t+1}(\theta_{S,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, \hat{V}_t) > \xi_{J,t}\right\}
\]

if \(S\) does not accept the offer: \(\zeta_{S,t+1} = 0\)

Finally, we describe the Bellman equation for the junior creditor. The continuation value of the junior creditor at the beginning of period \(t\) follows the Bellman equation:

\[
W_{J,t}(\theta_{J,t}, \ell_{S,t}, \ell_{J,t}, \hat{V}_t) = \lambda_J \max_{\xi_{J,t}} \left\{O_{J,t}, \max_{\ell_{J,t}} \mathbb{E}^J_t \left[\tilde{M}_{J,t+1}(\xi_{J,t})\right]\right\}
\]

if \(J\) proposes in the morning

\[
+ (1 - \lambda_J) \mathbb{E}^J_t \left[\max_{\zeta_{J,t+1} \in \{0, 1\}} \tilde{A}_{J,t+1}(\zeta_{J,t+1}) \bigg| \theta_{S,t} \geq \phi_{S,t}\right] \mathbb{P}_t^J\{\theta_{S,t} \geq \phi_{S,t}\}
\]

if \(S\) proposes reorganization in the morning

\[
+ (1 - \lambda_J) \mathbb{E}^J_t \left[\max\{O_{J,t}, U_{t+1}(\theta_{J,t+1}, \hat{V}_{t+1}) - O_{S,t}\}\right] \mathbb{P}_t^J\{\theta_{S,t} < \phi_{S,t}\},
\]

if \(S\) chooses to liquid in the morning

where \(\mathbb{E}^J_t\) is the conditional expectation of the junior creditor over \((\theta_{S,t}, \theta_{S,t+1}), \theta_{J,t+1}, \text{ and } \hat{V}_{t+1}; \text{ namely, the senior creditor’s reorganization skills in the morning of periods } t \text{ and } t + 1, \text{ the junior’s reorganization skill in the morning of period } t + 1, \text{ and the potential reorganization value in period } t + 1, \text{ conditioning on } (\theta_{J,t}, \ell_{S,t}, \ell_{J,t}, \hat{V}_t)\). Here, \(\xi_{J,t}\) is the offer made by the junior in the morning of period \(t\). The indicator variable \(\zeta_{J,t+1} = 1\) means that the offer proposed by the senior in the morning of period \(t\) is accepted by the junior in the afternoon of period \(t\). Moreover, \(\phi_{S,t}\) is the threshold for the senior creditor to choose
reorganization over liquidation.

If the junior creditor proposes in the morning of period $t$, the payoff to the junior creditor in the afternoon of period $t$, conditional on the choice $\xi_{J,t}$, is described as follows:

$$
\tilde{M}_{J,t+1}(\xi_{J,t}) = \left[ U_{t+1}(\theta_{J,t+1}, \tilde{V}_{t+1}) - \xi_{J,t} \right] \mathbf{1}\{H_{S,t+1}(\theta_{S,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, \tilde{V}_{t}) \leq \xi_{J,t}\}
$$

if $J$ accepts the offer

$$
+ H_{J,t+1}(\theta_{J,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, \tilde{V}_{t}) \mathbf{1}\{H_{S,t+1}(\theta_{S,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, \tilde{V}_{t}) > \xi_{J,t}\}.
$$

if $J$ does not accept the offer

In the afternoon of period $t$, the junior creditor observes $\xi_{J,t}$ and $\theta_{S,t+1}$, and she will choose to accept the offer with $\xi_{J,t}$ (i.e., the junior creditor chooses $\zeta_{S,t+1} = 1$) if and only if $H_{S,t+1}(\theta_{S,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, \tilde{V}_{t}) \leq \xi_{J,t}$.

How do the endogenous state variables $\ell_{S,t}$ and $\ell_{J,t}$ evolve endogenously in this case? If the junior creditor receives the proposal opportunity in the morning of period $t$, $\ell_{J,t+1} = \theta_{J,t}$ and $\ell_{S,t+1} = \max(\theta_{S,t}^{*}, \ell_{S,t})$ with $\theta_{S,t}^{*}$ being determined by $\xi_{J,t} = H_{S,t+1}(\theta_{S,t}^{*}, \theta_{J,t}, \tilde{V}_{t})$. The update of $\ell_{J,t}$ takes place right after the senior creditor sees the proposal $\xi_{J,t}$. The update is perfectly perceived and internalized by the junior creditor at the very beginning of period $t$, when she makes the proposal decision right after receiving the proposing opportunity.

If the senior creditor proposes in the morning of period $t$, the payoff to the junior creditor in the afternoon of period $t$, conditional on the senior’s optimal choice $\xi_{S,t}$ (which further depends on $(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, \tilde{V}_{t})$), is described as follows:

$$
\max_{\zeta_{J,t+1} \in \{0,1\}} \bar{M}_{J,t+1}(\zeta_{J,t+1}) = \xi_{S,t} \mathbf{1}\{H_{J,t+1}(\theta_{J,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, \tilde{V}_{t}) \leq \xi_{S,t}\}
$$

if $J$ accepts the offer: $\zeta_{J,t+1} = 1$

$$
+ H_{J,t+1}(\theta_{J,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, \tilde{V}_{t}) \mathbf{1}\{H_{J,t+1}(\theta_{J,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, \tilde{V}_{t}) > \xi_{S,t}\}.
$$

if $J$ does not accept the offer: $\zeta_{J,t+1} = 0$

C Extended Model II: Private Communication

In this extended model, we allow the creditors to communicate privately and learn each other’s type outside the court. We shall first describe the timeline of the model as follows.

Model Timeline. Figure OA.1 illustrates how bargaining works each period, including the pre-court period. Each period is divided into two subperiods, “morning” and “afternoon.” Proposals are made in the morning, responses in the afternoon. At the beginning of period $t$, the values of $\theta_{S,t}$ and $\theta_{J,t}$ are private information, and the counterparties’ beliefs about them are $F_{\beta}(\theta_{S,t} | \ell_{S,t})$ and $F_{\beta}(\theta_{J,t} | \ell_{J,t})$, respectively. The lower bounds $\ell_{S,t}$ and $\ell_{J,t}$ that characterize the beliefs are publicly known. One creditor, say creditor $k \in \{S, J\}$, is given the opportunity to make a proposal. The junior creditor receives this opportunity with
probability \( \lambda_J \), and the senior creditor receives it with probability \( 1 - \lambda_J \). Proposals are “take it or leave it,” so a higher \( \lambda_J \) increases the junior’s relative bargaining power. A creditor can propose reorganizing, liquidating, or waiting. In a reorganization proposal, creditor \( k \) (for example) proposes reorganizing the firm under her own plan and paying the counterparty \( \xi_{k,t} \), with the remaining value going back to herself. The subscript \( t \) on \( \xi_{k,t} \) means \( \xi_{k,t} \) depends on information up to the beginning of period \( t \). The reorganization proposal reveals creditor \( k \)’s reorganization skill \( \theta_{k,t} \), because, for example, the proposal includes a detailed business plan. Based on this information, the responding creditor \( \bar{k} \) updates his belief about \( \theta_{k,t+1} \) to

\[
F_\beta(\theta_{k,t+1}|\ell_{k,t+1}) \text{ with } \ell_{k,t+1} = \theta_{k,t}.
\]

Meanwhile, the proposing creditor \( k \) keeps the same belief about \( \theta_{k,t+1} \), characterized by \( F_\beta(\theta_{k,t+1}|\ell_{k,t}) \). Creditor \( k \) can also propose liquidating assets piecemeal for the total payout of \( L - C_t \), which is split according to APR. Liquidation proposals automatically end the bargaining, but they do not necessarily result in a liquidation. If the responding creditor \( \bar{k} \) prefers not to liquidate the firm, he can instead reorganize the firm under his own reorganization plan as long as he pays the proposing creditor \( k \) what she would receive upon a liquidation, under APR. Finally, creditor \( k \) can propose waiting by making a reorganization offer that will be rejected for sure (i.e., by proposing a very low \( \xi_{k,t} \)), effectively moving the game to the next period.

When the afternoon begins, the reorganization skills change from \( \theta_{S,t} \) and \( \theta_{J,t} \) to \( \theta_{S,t+1} \) and \( \theta_{J,t+1} \), respectively. Right after that, the updated reorganization skills \( \theta_{S,t} \) and \( \theta_{J,t} \) are observed privately. There is a probability \( p \) by which the updated skills are fully revealed to creditors right after their updates. When \( p \) is higher, the asymmetric information friction is weaker. As an extreme case, there is no asymmetric information when \( p = 1 \).

Based on his updated reorganization skill \( \theta_{k,t+1} \), the responding creditor \( \bar{k} \) weighs how much he would gain by accepting the proposal (i.e., the payment \( \xi_{k,t} \)) against how much he would get by declining the proposal and waiting (i.e., the continuation value, denoted \( W_{k,t+1} \)). If \( \xi_{k,t} \geq W_{k,t+1} \), the responding creditor accepts the offer and the game ends; otherwise, he rejects the offer and the game moves to the next period. When the responding creditor \( \bar{k} \) calculates \( W_{k,t+1} \), he takes into account that rejecting the offer would partially reveal his skill, \( \theta_{k,t+1} \). In other words, \( \bar{k} \) internalizes the “screening effect” of rejecting an offer. Specifically, if \( \bar{k} \) rejects the offer, the lower bound that characterizes the proposing creditor \( k \)’s belief about \( \theta_{k,t+1} \) is updated to \( \ell_{k,t+1} = \ell(\xi_{k,t}) \). In turn, the proposing creditor \( k \) internalizes the screening effect when choosing the proposal \( \xi_{k,t} \) at the beginning of the period, and she understands the equilibrium belief updating function \( \ell(\xi) \) for any proposal \( \xi \) she would make.

**Solution.** The equilibrium is solved recursively by backward induction. The “end” period is the first time such that \( \rho^t V_h \leq L \). In equilibrium, there is certain probability that the bargaining ends before the scenario \( \rho^t V_h \leq L \) occurs. In the terminal period, the creditors choose to quit the bargaining by liquidating the firm. The APR applies when splitting the liquidation value.

Now, we characterize the Bellman equations. Let’s consider period \( t \). The key is to establish the recursive Bellman equations for the afternoon continuation values \( W_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, \omega_t) \) and \( W_{J,t}(\theta_{J,t}, \ell_{S,t}, \ell_{J,t}, \omega_t) \)
Period $t$

Figure OA.1: Timeline of the model.

with the endogenous state variables $(\ell_{S,t}, \ell_{J,t})$ and (private) state variable $\theta_{J,t}$ or $\theta_{S,t}$. The private information about $\theta_{S,t}$ and $\theta_{J,t}$ are learned by the senior and junior, respectively, at the very beginning of the afternoon of period $t$; moreover, with probability $p$, the two private reorganization skills are fully revealed right after the creditors receive the private information.

The continuation value of the senior creditor at the end of period $t$ follows the Bellman equation:

$$W_{S,t}(\theta_{S,t}, \ell_{S,t}, \ell_{J,t}, \omega_t) = (1 - \lambda_J) \times \max_{\xi_{S,t}} \left\{ O_{S,t} \max_{\xi_{S,t}} \mathbb{E}_t^S \left[ \tilde{M}_{S,t+1}(\xi_{S,t}) \right] \right\}$$

if $S$ proposes in the morning of period $t$

$$+ \lambda_J \times \mathbb{E}_t^S \left[ \max_{\zeta_{S,t+1} \in \{0,1\}} \tilde{A}_{S,t+1}(\zeta_{S,t+1}) \mid \theta_{J,t} \geq \phi_{J,t} \right] \times \mathbb{P}_t^S \{ \theta_{J,t} \geq \phi_{J,t} \}$$

if $J$ proposes reorganization in the morning of period $t$

$$+ \lambda_J \times \mathbb{E}_t^S \left[ \max \{O_{S,t}, U_{t+1}(\theta_{S,t+1}) - O_{J,t} \} \right] \times \mathbb{P}_t^S \{ \theta_{J,t} < \phi_{J,t} \},$$

(4)

where $\mathbb{E}_t^S$ is the expectation of the senior creditor over $(\theta_{J,t}, \theta_{J,t+1}), \theta_{S,t+1},$ and $\omega_{t+1}$; namely, the junior creditor’s reorganization skills in the morning of periods $t$ and $t+1$, the senior creditor’s reorganization skill in the morning of period $t+1$, and private communication outcome $\omega_t$, conditional on $\theta_{S,t}, \ell_t = (\ell_{J,t}, \ell_{S,t})$, and $\omega_t$. The indicator variable $\zeta_{S,t+1} = 1$ means that the offer proposed by the junior creditor in the morning
of period $t$ is accepted by the senior creditor in the afternoon of period $t$. Here, $\phi_{J, t}$ is the threshold for the junior creditor to choose reorganization over liquidation.

The continuation value of the junior creditor follows the Bellman equation:

$$W_{J, t}(\theta_{J, t}, \ell_{S, t}, \ell_{J, t}, \omega_t) = \lambda_J \times \max \left\{ O_{J, t}, \max_{\xi_{J, t}} \mathbb{E}_{\xi_{J, t}} \left[ \tilde{M}_{J, t+1}(\xi_{J, t}) \right] \right\}$$

if $J$ proposes in the morning of period $t$

$$+ (1 - \lambda_J) \times \mathbb{E}_{\ell_{J, t+1}} \left[ \max_{\zeta_{J, t+1} \in \{0, 1\}} \tilde{A}_{J, t+1}(\zeta_{J, t+1}) \left| \theta_{S, t} \geq \phi_{S, t} \right. \right] \times \mathbb{P}_{I}^{J} \{ \theta_{S, t} \geq \phi_{S, t} \}$$

if $S$ proposes reorganization in the morning of period $t$

$$+ (1 - \lambda_J) \times \mathbb{E}_{\ell_{J, t+1}} \left[ \max \{ O_{J, t}, U_{t+1}(\theta_{J, t+1}) - O_{S, t} \} \right] \times \mathbb{P}_{I}^{J} \{ \theta_{S, t} < \phi_{S, t} \}$$

if $S$ chooses to liquid in the morning of period $t$.

Now, we provide more explanations on Bellman equations. Let’s focus on the Bellman equation for the senior creditor in (4). The details about the junior creditor’s Bellman equation in (5) can be explained in the same way.

We first describe the senior creditor’s belief about $\theta_{J, t}$. The only reason why $\omega_t$ serves as a state variable is that the senior’s belief about $\theta_{J, t}$, denoted by $\mathbb{P}_{I}^{S}(\theta_{J, t} \leq \theta)$, depends on $\omega_t$:

$$\mathbb{P}_{I}^{S}(\theta_{J, t} \leq \theta) = \begin{cases} F_\beta(\theta | \ell_{J, t}), & \text{if } \omega_t = 0 \\ F_\delta(\theta | \theta_{J, t}), & \text{if } \omega_t = 1. \end{cases}$$

(6)

Here $F_\beta(\theta | \ell)$ is the beta distribution, and $F_\delta(\theta | \ell)$ is the delta distribution:

$$F_\delta(\theta | \ell) = \begin{cases} 0, & \theta < \ell \\ 1, & \theta \geq \ell. \end{cases}$$

(7)

What are the senior creditor’s payoffs in the afternoon of period $t$? Let $\omega_{t+1}$ denote the indicator for whether the updated skills $\theta_{J, t+1}$ and $\theta_{S, t+1}$ are fully revealed in the afternoon of period $t$. More precisely, the updated skills are fully revealed if $\omega_{t+1} = 1$, and they are kept private otherwise. We assume that $\omega_{t+1}$ are random variables following i.i.d. Bernoulli distribution with probability $p$.

If the senior creditor proposes in the morning of period $t$, the payoff to the senior creditor in the
The continuation value functions $W_{S,t+1}(\cdot,1)$ and $W_{S,t+1}(\cdot,0)$ have different functional forms. At the moment right after the acceptance/rejection decision, 

$$\begin{align*}
(\ell_{J,t+1}, \ell_{S,t+1}) &= \begin{cases} 
(\theta^*_{J,t} \vee \ell_{J,t}, \theta_{S,t}) , & \text{with } \omega_{t+1} = 0 \\
(\theta_{J,t+1}, \theta_{S,t+1}) , & \text{with } \omega_{t+1} = 1 
\end{cases} \\
\xi_{S,t} &= W_{J,t+1}(\theta^*_{J,t}, \theta_{S,t}, \theta^*_{J,t}, 0).
\end{align*}$$

(9)

where $\theta^*_{J,t}$ is pinned down by the following equality:

$$\xi_{S,t} = W_{J,t+1}(\theta^*_{J,t}, \theta_{S,t}, \theta^*_{J,t}, 0).$$

(10)

Therefore, the belief $\theta^*_{J,t}$ depends on the information up to $t$, particularly on the decision variable $\xi_{S,t}$. The effect of the proposal $\xi_{S,t}$ on the belief formation, characterized by (9) and (10), is internalized by the senior creditor while making optimal decision on $\xi_{S,t}$. The optimal offer made by the senior creditor is

$$\xi^*_{S,t} = \arg\max_{\xi_{S,t}} E_S^S \left[ \tilde{M}_{S,t+1}(\xi_{S,t}) \right],$$

(11)

which is a function of the state variables $\theta_{S,t}$, $\ell_{J,t}$, and $\omega_t$.

If the junior creditor proposes in the morning of period $t$, the payoff to the senior creditor in the afternoon of period $t$, conditional on the choice $\xi_{S,t}$, is described as follows:

$$\tilde{M}_{S,t+1}(\xi_{S,t}) = \begin{cases} 
U_{t+1}(\theta_{S,t+1}) - \xi_{S,t} & \text{if updated skills are not revealed and } J \text{ accepts the offer} \\
+ W_{S,t+1}(\theta_{S,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, 0)1\{W_{J,t+1}(\theta_{J,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, 0) > \xi_{S,t}\}1\{\omega_{t+1} = 0\} & \text{if updated skills are not revealed and } J \text{ does not accept the offer} \\
+ [U_{t+1}(\theta_{S,t+1}) - \xi_{S,t}]1\{W_{J,t+1}(\theta_{J,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, 1) \leq \xi_{S,t}\}1\{\omega_{t+1} = 1\} & \text{if updated skills are revealed and } J \text{ accepts the offer} \\
+ W_{S,t+1}(\theta_{S,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, 1)1\{W_{J,t+1}(\theta_{J,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, 1) > \xi_{S,t}\}1\{\omega_{t+1} = 1\}. & \text{if updated skills are revealed and } J \text{ does not accept the offer}
\end{cases}$$

(8)
afternoon of period $t$, conditional on the choice $\xi_{J,t}^*$, and thus $\ell_{s,t+1}^*$, is described as follows:

\[
\max_{\zeta_{s,t+1} \in \{0,1\}} \tilde{A}_{s,t+1}(\zeta_{s,t+1}) = \max \left\{ \xi_{J,t}^*, W_{s,t+1}(\theta_{s,t+1}, \ell_{s,t+1}, \ell_{J,t+1}, \omega_{t+1}) \right\}
\]

\[
\begin{cases}
\xi_{J,t}^* 1\{W_{s,t+1}(\theta_{s,t+1}, \ell_{s,t+1}, \ell_{J,t+1}, 0) \leq \xi_{J,t}^*\} 1\{\omega_{t+1} = 0\} & \text{if } S \text{ accepts the offer: } \zeta_{s,t+1} = 1 \\
W_{s,t+1}(\theta_{s,t+1}, \ell_{s,t+1}, \ell_{J,t+1}, 0) 1\{W_{s,t+1}(\theta_{s,t+1}, \ell_{s,t+1}, \ell_{J,t+1}, 0) > \xi_{J,t}^*\} 1\{\omega_{t+1} = 0\} & \text{if } S \text{ does not accept the offer: } \zeta_{s,t+1} = 0 \\
\xi_{J,t}^* 1\{W_{s,t+1}(\theta_{s,t+1}, \ell_{s,t+1}, \ell_{J,t+1}, 1) \leq \xi_{J,t}^*\} 1\{\omega_{t+1} = 1\} & \text{if } S \text{ accepts the offer: } \zeta_{s,t+1} = 1 \\
W_{s,t+1}(\theta_{s,t+1}, \ell_{s,t+1}, \ell_{J,t+1}, 1) 1\{W_{s,t+1}(\theta_{s,t+1}, \ell_{s,t+1}, \ell_{J,t+1}, 1) > \xi_{J,t}^*\} 1\{\omega_{t+1} = 1\}. & \text{if } S \text{ does not accept the offer: } \zeta_{s,t+1} = 0 \\
\end{cases}
\]

(12)

The lower bounds in $\ell_{t+1}$ are updated according to the following rules, which is different from (9) and (10) since the junior creditor proposes in the morning of period $t$. The belief is updated as follows:

\[
(\ell_{J,t+1}, \ell_{s,t+1}) = \begin{cases}
(\theta_{J,t}, \theta_{s,t}^* \lor \ell_{s,t}), & \text{with } \omega_{t+1} = 0 \\
(\theta_{J,t+1}, \theta_{s,t+1}), & \text{with } \omega_{t+1} = 1
\end{cases}
\]

(14)

where the screening cutoff point $\theta_{s,t}^*$ is pinned down by

\[
\xi_{J,t}^* = W_{s,t+1}(\theta_{s,t}^*, \theta_{s,t}^*, \theta_{J,t}, 0).
\]

(15)

Here $\xi_{J,t}^*$ is the optimal proposal made by the junior creditor, depending on $\theta_{J,t}$, $\ell_{s,t}$, and $\omega_t$.

**More Explanations on the Expectations in Bellman Equations.** First, we consider the present value of proposing a reorganization plan $E_t^S[\tilde{M}_{s,t+1}(\xi_{s,t})]$. It equals to

\[
E_t^S[\tilde{M}_{s,t+1}(\xi_{s,t})] = (1-p)(E_1 + E_2) + p(E_3 + E_4),
\]

(16)

where

\[
E_1 = \int [U_{t+1}(\theta_{s,t+1}) - \xi_{s,t}] 1\{W_{J,t+1}(\theta_{J,t+1}, \theta_{s,t}, \theta_{s,t}^* \lor \ell_{J,t}, 0) \leq \xi_{s,t}\} \times dF(\theta_{J,t+1} | \theta_{s,t}) dF(\theta_{J,t} | \ell_{J,t}) \otimes F_{\beta}(\theta_{J,t+1} | \theta_{J,t}),
\]

(17)

and

\[
E_2 = \int W_{s,t+1}(\theta_{s,t+1}, \theta_{s,t}, \theta_{s,t}^* \lor \ell_{J,t}, 0) 1\{W_{J,t+1}(\theta_{J,t+1}, \theta_{s,t}, \theta_{s,t}^* \lor \ell_{J,t}, 0) > \xi_{s,t}\} \times dF(\theta_{J,t+1} | \theta_{s,t}) dF(\theta_{J,t} | \ell_{J,t}) \otimes F_{\beta}(\theta_{J,t+1} | \theta_{J,t}),
\]

(18)
and

\[ E_3 = \int [U_{t+1}(\theta_{S,t+1}) - \xi_{S,t}] \mathbf{1}\{W_{j,t+1}(\theta_{J,t+1}, \theta_{S,t+1}, \theta_{J,t+1}, 1) \leq \xi_{S,t}\} \times dF_\theta(\theta_{S,t+1}|\theta_{S,t})dF(\theta_{J,t}|\ell_{J,t}) \otimes F_\beta(\theta_{J,t+1}|\theta_{J,t}), \]  

(19)

and

\[ E_4 = \int W_{S,t+1}(\theta_{S,t+1}, \theta_{S,t+1}, \theta_{J,t+1}, 1) \mathbf{1}\{W_{j,t+1}(\theta_{J,t+1}, \ell_{S,t+1}, \ell_{J,t+1}, 1) > \xi_{S,t}\} \times dF_\theta(\theta_{S,t+1}|\theta_{S,t})dF(\theta_{J,t}|\ell_{J,t}) \otimes F_\beta(\theta_{J,t+1}|\theta_{J,t}), \]  

(20)

and

\[ F(\theta|\ell_{J,t}) = \begin{cases} 
F_\beta(\theta|\ell_{J,t}), & \text{if } \omega = 0 \\
F_\theta(\theta|\ell_{J,t}), & \text{if } \omega = 1.
\end{cases} \]  

(21)

Second, we consider \( \mathbb{E}_t^S \left[ \max_{\zeta_{S,t+1} \in \{0,1\}} A_{S,t+1}(\zeta_{S,t+1}) \mid \theta_{J,t} \geq \phi_{J,t} \right] \). It equals to

\[ \mathbb{E}_t^S \left[ \max_{\zeta_{S,t+1} \in \{0,1\}} A_{S,t+1}(\zeta_{S,t+1}) \mid \theta_{J,t} \geq \phi_{J,t} \right] = (1 - p)(A_1 + A_2) + p(A_3 + A_4), \]  

(22)

where

\[ A_1 = \int \xi^*_t \mathbf{1}\{W_{S,t+1}(\theta_{S,t+1}, \theta_{S,t}^* \lor \ell_{S,t}, \theta_{J,t}, 0) \leq \xi^*_t\} \times dF_\theta(\theta_{S,t+1}|\theta_{S,t})dG(\theta_{J,t}|\ell_{J,t}, \phi_{J,t}), \]  

(23)

and

\[ A_2 = \int W_{S,t+1}(\theta_{S,t+1}, \theta_{S,t}^* \lor \ell_{S,t}, \theta_{J,t}, 0) \mathbf{1}\{W_{S,t+1}(\theta_{S,t+1}, \theta_{S,t}^* \lor \ell_{S,t}, \theta_{J,t}, 0) > \xi^*_t\} \times dF_\theta(\theta_{S,t+1}|\theta_{S,t})dG(\theta_{J,t}|\ell_{J,t}, \phi_{J,t}), \]  

(24)

and

\[ A_3 = \int \xi^*_t \mathbf{1}\{W_{S,t+1}(\theta_{S,t+1}, \theta_{S,t+1}, \theta_{J,t+1}, 1) \leq \xi^*_t\} \times dF_\theta(\theta_{S,t+1}|\theta_{S,t})dG(\theta_{J,t}|\ell_{J,t}, \phi_{J,t}) \otimes F_\beta(\theta_{J,t+1}|\theta_{J,t}), \]  

(25)

and

\[ A_4 = \int W_{S,t+1}(\theta_{S,t+1}, \theta_{S,t+1}, \theta_{J,t+1}, 1) \mathbf{1}\{W_{S,t+1}(\theta_{S,t+1}, \theta_{S,t+1}, \theta_{J,t+1}, 1) > \xi^*_t\} \times dF_\theta(\theta_{S,t+1}|\theta_{S,t})dG(\theta_{J,t}|\ell_{J,t}, \phi_{J,t}) \otimes F_\beta(\theta_{J,t+1}|\theta_{J,t}), \]  

(26)

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and
\[
G(\theta | \ell_{J,t}, \phi_{J,t}) = \begin{cases} 
F_{\beta}(\theta | \ell_{J,t} \lor \phi_{J,t}), & \text{if } \omega_t = 0 \\
F_{\delta}(\theta | \theta_{J,t}) 1\{|\theta_{J,t} \geq \phi_{J,t}\}, & \text{if } \omega_t = 1.
\end{cases}
\] (27)

The setup for junior creditor’s payoffs in period \( t \) is similar to the senior creditor’s payoffs, which will not be repeated here.

D Signaling and Judge Cramdown

In the baseline model of Dou et al. (2019), we assume that the proposal fully reveals the proposer’s true reorganization skill. Here, we relax this assumption by prohibiting direct communication, even through the learning from detailed reorganization plan; instead, the proposer can signal her private information on the true reorganization skill through the amount of payoff she offers to the counterparty. In this section, we show that under some assumptions such as judge’s cramdown, a separating equilibrium exists. By definition (see Sobel, 2009), the hidden type of the signal sender is fully revealed by the signal embedded in actions in the separating equilibrium. We shall not provide a general proof for the existence of separating equilibria in our full model, because it is extremely complex and tedious, deviating too far away from the focus of this paper. Instead, we illustrate the main idea based on a special case of our baseline model in Dou et al. (2019).

Without loss of generality, we assume that \( \lambda_J = 0 \), which means that junior will not propose, and \( V \leq L - D_S \), which means that the bargaining will certainly be finished by the end of period \( t = 1 \) and the reorganization is not appealing. This simplification enables us to focus on discussing the signaling game by making strategic screening irrelevant. As a result, the strategic signaling will occur only in the first period \( t = 0 \). For simplicity, we further assume that \( \beta = 1 \), which means that the reorganization skills follow uniform distributions.

The proposed payoff is in the form of a fraction of the true reorganization skill level \( \theta_{S,0} \), say \( \xi_{S,0} = x \rho V \theta_{S,0} \). In other words, the proposer will propose the fraction \( x \) to the counterparty. This is an equivalent way to specify the proposed payoff as in the baseline model of Dou et al. (2019). The judge’s cramdown probability \( p(x) \) depends on the proposal fraction \( x \). We assume that \( p(x) \) satisfies the following conditions:

\[
p(x) > 0 \quad \text{and} \quad p'(x) > 0.
\]

We postulate the functional form \( p(x) = \gamma x \) with \( \gamma \in (0,1) \), without loss of generality.

Now, we describe the value functions of the creditors. In period \( t = 1 \), the deal would be liquidated. According to the APR, the gain of the senior and junior creditor from liquidation is \( O_{S,1} = D_S \) and \( O_{J,1} = L - D_S \), respectively. Then, it must hold that

\[
W_{S,1}(\theta_{S,1}, \ell_{S,1}, \ell_{J,1}) \equiv O_{S,1} = D_S \quad \text{and} \quad W_{J,1}(\theta_{J,1}, \ell_{S,1}, \ell_{J,1}) \equiv O_{J,1} = L - D_S.
\] (28)
In period $t = 0$, it holds that

$$W_{S,0}(\theta_{S,0}, \ell_{S,0}, \ell_{J,0}) = \max_{x \in [0,1]} \{ p(x) \rho V \{ \mathbb{E}_0 [\theta_{S,1}] - x\theta_{S,0} \} + [1 - p(x)] \rho V \mathbb{E}_0 [(\theta_{S,1} - x\theta_{S,0}) 1\{ O_{J,1} \leq x\rho V \theta_{S,0} \}] + [1 - p(x)] O_{S,1} \mathbb{E}_0 [1\{ O_{J,1} > x\rho V \theta_{S,0} \}] \}.$$  

The term $p(x) \rho V \{ \mathbb{E}_0 [\theta_{S,1}] - x\theta_{S,0} \}$ is the expected payoff to the senior creditor if the judge cramdown occurs. The term $[1 - p(x)] \rho V \mathbb{E}_0 [(\theta_{S,1} - x\theta_{S,0}) 1\{ O_{J,1} \leq x\rho V \theta_{S,0} \}]$ is the expected payoff to the senior creditor if the judge cramdown does not occur and the junior creditor accepts the proposal. The term $[1 - p(x)] O_{S,1} \mathbb{E}_0 [1\{ O_{J,1} > x\rho V \theta_{S,0} \}]$ is the expected payoff to the senior creditor if the judge cramdown does not occur and the junior creditor declines the proposal.

Because $O_{J,1} = L - D_S \geq V$, the Bellman equation above can be rewritten as follows:

$$W_{S,0}(\theta_{S,0}, \ell_{S,0}, \ell_{J,0}) = \max_{x \in [0,1]} \{ p(x) \rho V (1/2 + \theta_{S,0}/2 - x\theta_{S,0}) + [1 - p(x)] O_{S,1} \}.$$

The first-order condition is

$$\gamma [\rho V (1/2 + \theta_{S,0}/2 - x\theta_{S,0}) - O_{S,1}] = \gamma x\rho V \theta_{S,0}.$$  

Thus, after rearranging terms and plugging in $O_{S,1} = D_S$, the optimal proposal can be characterized by

$$x = \frac{\gamma (\rho V/2 - D_S) + \gamma \rho V \theta_{S,0}/2}{2\gamma \rho V \theta_{S,0}},$$

which is strictly monotonic in $\theta_{S,0}$. Therefore, the signaling using $x$ would fully reveal the private type $\theta_{S,0}$.

### E  Additional Empirical Robustness Exercises

This section contains details on additional robustness exercises discussed in Section 5 in the main paper:

(i) We re-estimate the model after replacing confirmation date with sale date for cases with a Section 363 sale outcome, which changes the duration measure for roughly 20% of cases. Re-estimation results are in Table OA.1, column “Alternative Duration Measure.” Parameter estimates are similar to those in our main analysis, and the estimated inefficiency is almost identical (0.079, compared to 0.078 in our main analysis).

(ii) We address concerns about potential underestimation of liquidation values, $L$. There is no evidence on the degree to which liquidation values are underestimated. To check whether underestimation of $L$
would affect our conclusions, we re-estimate the model after inflating all $L$ values by 20%. Results are in Table OA.1, column “Inflated Liquidation Value.” When $L$ is higher, there is a stronger incentive to liquidate. To offset this force and continue fitting the unchanged frequencies of liquidation and reorganization observed in the data, the model needs a faster learning speed (i.e., lower $\beta$). A higher liquidation value also disproportionately increases payoffs to the junior creditor. To offset this effect and continue fitting the unchanged payoff data, the model needs a lower estimated probability that the junior proposes ($\lambda_J$). The estimated inefficiency increases to 0.097, compared to 0.078 in the main analysis. This result suggests that our conclusion would be even stronger if liquidation values were underestimated.

(iii) We compare results in cases with and without DIP financing. Results from estimating in these two subsamples are in the last columns of Table OA.1. Consistent with DIP financing reducing junior creditors’ relative bargaining power, we find that cases with DIP financing feature a lower estimated value of $\lambda_J$, the junior’s probability of proposing. The estimated inefficiency, however, is quite similar across the two subsamples (0.077 and 0.070).

(iv) We remove 22 cases (roughly 7% of the sample) in which the reported filing reason is tort or fraud. Removing these cases has virtually no effect on the data moments, so parameter estimates and the implied inefficiency would be almost identical to our main results.

(v) We remove 13 cases (roughly 4% of the sample) in which the equity holders are not fully wiped out. Removing these cases has virtually no effect on the data moments, so parameter estimates and the implied inefficiency would be almost identical to our main results.
Table OA.1: Additional Empirical Robustness Exercises

This table contains results from estimating the model with alternative measures or subsamples. Details are in the text above.

<table>
<thead>
<tr>
<th>Alternative Duration Measure</th>
<th>Inflated Liquidation Value</th>
<th>Cases Without DIP Financing</th>
<th>Cases With DIP Financing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Parameter Estimates</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Months per Period ($\mu$)</td>
<td>4.381</td>
<td>4.189</td>
<td>4.474</td>
</tr>
<tr>
<td>Senior’s Initial Skill ($\theta_{S,0}$)</td>
<td>0.281</td>
<td>0.249</td>
<td>0.270</td>
</tr>
<tr>
<td>Junior’s Initial Skill ($\theta_{J,0}$)</td>
<td>0.364</td>
<td>0.391</td>
<td>0.360</td>
</tr>
<tr>
<td>Inverse Speed of Creditor Learning ($\beta$)</td>
<td>9.801</td>
<td>7.993</td>
<td>9.526</td>
</tr>
<tr>
<td>Persistence of Reorganization Value ($\rho$)</td>
<td>0.884</td>
<td>0.879</td>
<td>0.882</td>
</tr>
<tr>
<td>Fixed Cost of Going to Court ($c_0$, %)</td>
<td>4.669</td>
<td>3.883</td>
<td>4.161</td>
</tr>
<tr>
<td>Junior’s Probability of Proposing ($\lambda_J$)</td>
<td>0.346</td>
<td>0.168</td>
<td>0.441</td>
</tr>
</tbody>
</table>

**Panel B: Model Implications (Social Planner Model Minus Estimated Model)**

<table>
<thead>
<tr>
<th></th>
<th>Alternative</th>
<th>Inflated</th>
<th>Cases Without DIP Financing</th>
<th>Cases With DIP Financing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Total Recovery Rate</td>
<td>0.079</td>
<td>0.097</td>
<td>0.077</td>
<td>0.07</td>
</tr>
<tr>
<td>Avg. Reorganization Value</td>
<td>0.081</td>
<td>0.089</td>
<td>0.07</td>
<td>0.033</td>
</tr>
<tr>
<td>Fraction Resolved Pre-Court</td>
<td>0.136</td>
<td>0.172</td>
<td>0.092</td>
<td>0.666</td>
</tr>
<tr>
<td>Avg. Duration of Court Cases (Months)</td>
<td>-11.8</td>
<td>-13.0</td>
<td>-10.5</td>
<td>-12.6</td>
</tr>
</tbody>
</table>

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F Additional Identification Analysis

Andrews, Gentzkow and Shapiro (2017, 2020) proposed a local measure of the relationship between parameter estimates and moments for enhancing the transparency of structural identification and estimation. In this spirit, we present the Jacobian matrix of moments with respect to parameter values in the main text. Here, we show that similar identification results are obtained using the sensitivity matrix proposed by Andrews, Gentzkow and Shapiro (2017).

Table OA.2 shows the sensitivity matrix of seven parameters to nine simulated moments. It shows a transparent identification of our model, which is consistent with the Jacobian matrix of moments with respect to parameter values in the main text.

To be more precise, the first moment is helpful for identifying \( \mu \), the number of months per period. The second moment is informative about \( \beta \), which governs the speed of learning. The third moment is important to identify \( \rho \), capturing the decay speed of reorganization value. The toughest identification challenge is to disentangle \( \beta \) and \( \rho \) since both affect the costs and benefits of waiting in the bargaining. Moments (1) – (3) together provide clear identification for \( \beta \) and \( \rho \) since they all move in different directions when perturbing \( \beta \) and \( \rho \). Intuitively, the fourth moment strongly identifies \( c_0 \), the fixed cost of entering the court. The fifth and sixth moments significantly and positively affect the parameter estimates \( \theta_{S,0} \) and \( \theta_{J,0} \), respectively. As a result, \( \theta_{S,0} \) and \( \theta_{J,0} \) are clearly identified by these two moments. The seventh moment is chosen to identify the ex-ante bargaining power of the junior creditor, captured by \( \lambda_J \). The eighth and ninth moments are over-identification moment restrictions. Each of them is highly informative to several parameters at the same time, and thus provides valid and useful disciplines for parameter estimation.
Table OA.2: Sensitivity of Parameters to Moments

This table shows the sensitivity of model parameters (in columns) with respect to model-implied moments (in rows) proposed by Andrews, Gentzkow and Shapiro (2017). Moments are defined in detail in the main text. Parameter \( \mu \) is the months per model period, \( \beta \) is the (inverse) speed of creditor learning, \( \rho \) is the persistence of reorganization value, \( c_0 \) is the fixed cost of going to court, \( \theta_{S,0} \) and \( \theta_{J,0} \) are the initial skill levels of the senior and junior creditor, respectively, and \( \lambda_J \) is the probability that the junior proposes in a given period.

<table>
<thead>
<tr>
<th>Moments</th>
<th>( \mu )</th>
<th>( \beta )</th>
<th>( \rho )</th>
<th>( c_0 )</th>
<th>( \theta_{S,0} )</th>
<th>( \theta_{J,0} )</th>
<th>( \lambda_J )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>0.5633</td>
<td>-0.2794</td>
<td>-0.9548</td>
<td>0.0253</td>
<td>-0.0045</td>
<td>0.0532</td>
<td>0.0964</td>
</tr>
<tr>
<td>(2)</td>
<td>-0.3506</td>
<td>-0.6398</td>
<td>0.3424</td>
<td>0.4961</td>
<td>0.0727</td>
<td>0.6454</td>
<td>-0.3258</td>
</tr>
<tr>
<td>(3)</td>
<td>-0.0700</td>
<td>0.1723</td>
<td>1.0098</td>
<td>0.0533</td>
<td>0.0167</td>
<td>0.0850</td>
<td>-0.1507</td>
</tr>
<tr>
<td>(4)</td>
<td>-0.1107</td>
<td>0.0294</td>
<td>0.2758</td>
<td>-0.0845</td>
<td>-0.1509</td>
<td>-0.3265</td>
<td>0.2769</td>
</tr>
<tr>
<td>(5)</td>
<td>0.2588</td>
<td>0.0904</td>
<td>-0.2076</td>
<td>-0.0572</td>
<td>0.4658</td>
<td>0.1481</td>
<td>0.1746</td>
</tr>
<tr>
<td>(6)</td>
<td>-0.3787</td>
<td>0.0839</td>
<td>0.5236</td>
<td>0.1623</td>
<td>-0.5813</td>
<td>0.1137</td>
<td>-0.0465</td>
</tr>
<tr>
<td>(7)</td>
<td>-0.2118</td>
<td>0.3329</td>
<td>0.7017</td>
<td>-0.2676</td>
<td>-0.7919</td>
<td>0.2014</td>
<td>0.5074</td>
</tr>
<tr>
<td>(8)</td>
<td>0.3250</td>
<td>0.0360</td>
<td>0.2595</td>
<td>0.3169</td>
<td>0.9651</td>
<td>0.5290</td>
<td>0.1955</td>
</tr>
<tr>
<td>(9)</td>
<td>0.5029</td>
<td>0.5246</td>
<td>-0.3748</td>
<td>-0.4024</td>
<td>-0.0637</td>
<td>-0.7065</td>
<td>0.2888</td>
</tr>
</tbody>
</table>

Panel B: Description of Moments

1. Average log number of months between observed proposals for in-court cases.
2. Fraction of cases that result in a reorganization, conditional on resolving in court.
3. Average log duration of in-court cases, in months.
4. Fraction of cases resolved in court.
5. Senior creditor’s average recovery rate in pre-court reorganizations.
6. Junior creditor’s average recovery rate in pre-court reorganizations.
7. Junior creditor’s average fraction of gain, conditional on an in-court reorganization.
8. Total recovery rate averaged across all in-court reorganizations.
9. Regression slope coefficient of log total recovery rate on case duration across all in-court reorganizations.

References


