Misallocation and Asset Prices

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Motivation

Misallocation affects the aggregate productivity level and growth rate

- Aggregate productivity levels
  e.g., Olley_Pakes (1996); Hsieh_Klenow (2009); Bartelsman_Haltiwanger_Scarpetta (2013)
- Economic growth rates in short-run transitions
  e.g., Buera_Shin (2013); Moll (2014)
- Economic growth rates in the long run
  e.g., Jones (2013); Acemoglu_Akcigit_Bloom_Kerr (2018); Peters (2020); König_Storesletten_Song_Zilibotti (2022)

Misallocation fluctuates over time with cyclical patterns

e.g., Eisfeldt_Rampini (2006); Eisfeldt_Shi (2018)

Question: Can misallocation lead to fluctuations in economic growth, especially in its low-frequency components?

- Yes, endogenous slow-moving misallocation is pivotal
- So what? It has significant implications for asset prices (and welfare)
  i.e., "misallocation-driven endogenous long-run risk"

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Overview of the theoretical framework

A general-equilibrium model with heterogeneous firms and endogenous stochastic growth

- Built on Moll (2014)
  - Endogenous misallocation in capital due to financial frictions
  - Persistent firm-level idiosyncratic productivity
- Extended in four ways, while preserving analytial tractability:
  - Public firms operated by managers with agency frictions
  - Endogenous growth through R&D (Romer, 1987, 1990)
  - Transitory aggregate shocks that drive misallocation
  - EZW preferences and the marginal q of intangible capital

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Overview of the model ingredients

Model = Romer + Moll

Three sectors:

- **Final goods sector:**
  - production capital + labor + a variety of intermediate goods
    $\Rightarrow$ final goods
  - **subject to financial frictions**
- **Intermediate goods sector:**
  - final goods $\Rightarrow$ differentiated intermediate goods
  - blueprints $\Rightarrow$ monopoly power
- **R&D sector:** final goods $\Rightarrow$ blueprints

Primitive shocks:

- One aggregate shock to the quality of capital (i.e., "liquidity shock")
- An idiosyncratic productivity shock for each final goods firm

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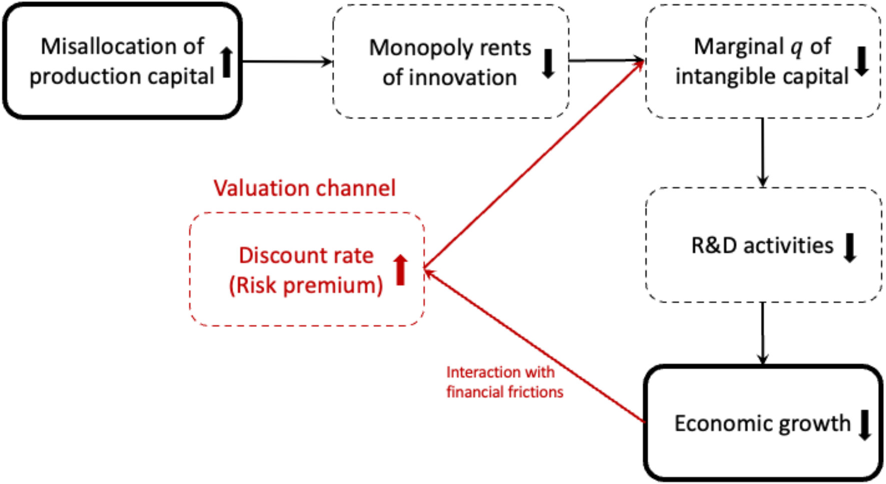
Overview of the mechanism



Valuation channel due to endogenous long-run risk

Main results & contributions

Misallocation-driven fluctuations in growth are important for asset prices

- Misallocation-driven low-frequency growth fluctuations

$\Longrightarrow$ Uncover the “dark matter” in long-run risk models

e.g., Chen_Dou_Kogan (2024); Cheng_Dou_Liao (2022)

Analytical tractability: Misallocation is a key endogenous state variable

- Our theory motivates a covariance-type measure of misallocation

e.g., Olley_Pakes (1996); Bartelsman_Haltiwanger_Scarpetta (2009, 2013)

Empirical findings support the model

- Misallocation negatively predicts R&D/consumption/output growth
- Causal effect of misallocation in production capital on R&D intensity
- Various asset pricing tests

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Outline

1. **Model**

2. **Solution and mechanism**

3. **Quantitative analysis**

Final goods sector

A continuum of heterogeneous firms with productivity $z_{i,t}$ and capital $a_{i,t}$

- Firms are indexed by $i \in \mathcal{I}$
- The distribution of firms is $\varphi_t(z, a)$, endogenous and varying over time

CRS technology:

$$y_{i,t} = \left[(z_{i,t} u_{i,t} k_{i,t})^{\alpha} \ell_{i,t}^{1-\alpha} \right]^{1-\varepsilon} x_{i,t}^{\varepsilon}, \quad \text{with} \quad x_{i,t} = \left(\int_0^{N_t} x_{i,j,t}^{\nu} \mathrm{d}j \right)^{\frac{1}{\nu}}$$

- intermediate goods $x_{i,t}$, knowledge stock N_t (intangible capital), labor $\ell_{i,t}$
- $k_{i,t} = a_{i,t} + \widehat{a}_{i,t}$, with leased capital $\widehat{a}_{i,t}$
- $u_{i,t} \in [0, 1]$ is the utilization intensity, with costs

$$u_{i,t} k_{i,t} \left(\delta_k \mathrm{d}t + \sigma_k \mathrm{d}W_t \right), \quad \text{where } \mathrm{d}W_t = \text{capital quality shock}$$

- $z_{i,t}$ is firm-level productivity,

$$\mathrm{d} \ln z_{i,t} = -\theta \ln z_{i,t} \mathrm{d}t + \sigma \sqrt{\theta} \mathrm{d}W_{i,t}$$

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Final goods sector (continued)

Capital stock accumulation

$$\mathrm{d}a_{i,t} = a_{i,t}\left(-\delta_a \mathrm{d}t + \sigma_a \mathrm{d}W_t\right) + \mathrm{d}I_{i,t}$$

$$\mathrm{d}I_{i,t} = \underbrace{(y_{i,t} - p_t x_{i,t} - w_t \ell_{i,t} - r_{f,t}\widehat{a}_{i,t})\mathrm{d}t}_{\text{production profits}} - \underbrace{d_{i,t}\mathrm{d}t}_{\text{dividend}} - \underbrace{u_{i,t}k_{i,t}(\delta_k \mathrm{d}t + \sigma_k \mathrm{d}W_t)}_{\text{depreciation}}$$

where p_t = price of immediate goods, w_t = wage rate, and $r_{f,t}$ = riskfree rate

The capital quality shock, $\mathrm{d}W_t$, is the only aggregate shock

- More productive firms choose higher $u_{i,t} \Longrightarrow$ more exposed to $\mathrm{d}W_t$
  - $\Rightarrow$ Misallocation varies over time, driven by $\mathrm{d}W_t$

Final goods sector (continued)

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Final goods sector (continued)

Capital stock accumulation

$$da_{i,t} = a_{i,t}\left(-\delta_a dt + \sigma_a dW_t\right) + dI_{i,t}$$

$$dI_{i,t} = \underbrace{(y_{i,t} - p_t x_{i,t} - w_t \ell_{i,t} - r_{f,t}\widehat{a}_{i,t})dt}_{\text{production profits}} - \underbrace{d_{i,t}dt}_{\text{dividend}} - \underbrace{u_{i,t}k_{i,t}(\delta_k dt + \sigma_k dW_t)}_{\text{depreciation}}$$

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Intermediate goods sector

A continuum of homogeneous producers, indexed by $j \in [0, N_t]$

A blueprint secures monopoly power for a specific intermediate good, and producing one intermediate good requires one final good:

$$\pi_{j,t} = \max_{p_{j,t}} \underbrace{p_{j,t} e_t(p_{j,t})}_{\text{revenue}} - \underbrace{e_t(p_{j,t})}_{\text{cost}},$$

subject to the downward-sloping demand curve:

$$e_t(p_{j,t}) \equiv \left(\frac{p_{j,t}}{p_t}\right)^{\frac{1}{\nu-1}} X_t, \quad \text{with} \quad X_t \equiv \int_{i \in \mathcal{I}} x_{i,t} \mathrm{d}i \quad \text{and} \quad p_t = \left(\int_0^{N_t} p_{j,t}^{\frac{\nu}{\nu-1}} \mathrm{d}j\right)^{\frac{\nu-1}{\nu}}$$

The value of a blueprint $q_{j,t}$ is the marginal q of innovation, given by

$$0 = \Lambda_t(\pi_{j,t} - \delta_b q_{j,t})\mathrm{d}t + \mathbb{E}_t\left[\mathrm{d}\left(\Lambda_t q_{j,t}\right)\right],$$

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R&D sector

A continuum of innovators, each with a success rate $\vartheta_t > 0$ for experiments

- Each R&D experiment requires the use of final goods with unity intensity

If S_t inventors do experiments, the knowledge stock N_t evolves according to:

$$\mathrm{d}N_t = \vartheta_t S_t \mathrm{d}t - \delta_b N_t \mathrm{d}t, \quad \text{with } \vartheta_t \equiv \chi \left(\frac{N_t}{S_t}\right)^h \text{ and } h \in (0, 1)$$

Free-entry condition $\Longrightarrow$ marginal return = marginal cost:

$$q_t \vartheta_t = 1$$

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Agents and financial frictions

Representative agent $=$ workers $+$ managers

- Identical EZW recursive preferences and perfect risk sharing
- Each manager controls and operates a final-good firm

Limited enforcement problem $\Longrightarrow$ financial frictions

- Manager i extracts rents $\tau a_{i,t}$, subject to shareholders' costly invention

  $\Longrightarrow$ equity market constraint on the dividend flow:

$$d_{i,t} = \rho a_{i,t}, \quad \text{with} \quad \rho \in (\tau, 1)$$

- Manager i can divert $\widehat{a}_{i,t}/\lambda$, subject to lenders' costly asset repossession

  $\Longrightarrow$ collateral constraint on borrowing:

$$\widehat{a}_{i,t} \leq \lambda a_{i,t}, \quad \text{with} \quad \lambda \in [1, \infty)$$

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$$\widehat{a}_{i,t} \leq \lambda a_{i,t}, \quad \text{with} \quad \lambda \in [1, \infty)$$

Managers' problem

The manager of firm *i* maximizes the value of his own rents $\tau a_{i,t}$

$$J_{i,t} = \max_{u_{i,s}, \widehat{a}_{i,s}, \ell_{i,s}, x_{i,j,s}} \mathbb{E}_t \left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} \tau a_{i,s} ds \right],$$

subject to the financial frictions and the budget constraint

"Bang-bang" and linear solutions (similar to Moll (2014))

$$u_{i,t} = \begin{cases} 1, & z_{i,t} \geq \underline{z}_t \\ 0 & z_{i,t} < \underline{z}_t, \end{cases} \qquad k_{i,t} = \begin{cases} (1+\lambda) a_{i,t}, & z_{i,t} \geq \underline{z}_t \\ 0 & z_{i,t} < \underline{z}_t, \end{cases}$$

$$\ell_{i,t} = \left[\frac{(1-\alpha)(1-\varepsilon)}{\omega_t} \right]^{\frac{1}{\alpha}} \left(\frac{\varepsilon}{p_t} \right)^{\frac{\varepsilon}{\alpha(1-\varepsilon)}} z_{i,t} u_{i,t} k_{i,t},$$

$$x_{i,j,t} = \left(\frac{p_t}{p_{j,t}} \right)^{\frac{1}{1-\nu}} \left(\frac{\varepsilon}{p_t} \right)^{\frac{1-(1-\alpha)(1-\varepsilon)}{\alpha(1-\varepsilon)}} \left[\frac{(1-\alpha)(1-\varepsilon)}{\omega_t} \right]^{\frac{1-\alpha}{\alpha}} z_{i,t} u_{i,t} k_{i,t}$$

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Aggregation

The aggregate output is

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha},$$

where $L_t \equiv 1$ and aggregate TFP is (similar to Kung_Schmid (2015)):

$$Z_t = (\varepsilon\nu)^{\frac{\varepsilon}{1-\varepsilon}} H_t N_t^{1-\alpha} \;\; \text{with} \;\; H_t = \left[\frac{\int_{\underline{z}_t}^{\infty} z\omega_t(z)\mathrm{d}z}{\int_{\underline{z}_t}^{\infty} \omega_t(z)}\right]^{\alpha},$$

and, the capital share density $\omega_t(z)$ is

$$\omega_t(z) \equiv \frac{1}{A_t}\int_0^{\infty} a\varphi_t(z,a)\mathrm{d}a$$

The productivity cutoff $\underline{z}_t$ is pinned down by

$$\frac{K_t}{A_t} = (1+\lambda)\int_{\underline{z}_t}^{\infty} \omega_t(z)\mathrm{d}z$$

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Outline

1. **Model**

2. **Solution and mechanism**

3. **Quantitative analysis**

Parametric approximation

Challenge: $\omega_t(z)$ is an infinite-dimensional "endogenous state variable"

Parametric approximation: $(\ln z_{i,t}, \ln a_{i,t}) \sim$ **Bivariate Normal**

- $\ln z_{i,t} \sim N(0, \sigma^2/2)$
- $\ln a_{i,t} \approx$ Normal, if $\theta \not\approx 0$, due to Berry-Esseen bound

Connection to standard global-solution methods based on numerical approximation

- Similarity: Use the first few moments to approximate a distribution
  e.g., Krusell_Smith (1997)
- Difference: Impose a parametric functional form, not "numerically fit"

Benefits: Closed-form characterization of aggregate dynamics

- Higher-order approximations yield similar results

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Closed-form solution for distribution and productivity

Under our approximation, $\omega_t(z)$ has the closed-form expression:

$$\omega_t(z) = \frac{1}{z\sigma\sqrt{\pi}} \exp\left[-\frac{\left(\ln z + M_t\sigma^2/2\right)^2}{\sigma^2}\right],$$

where $M_t \equiv -\frac{\text{Cov}(\ln z_{i,t}, \ln a_{i,t})}{\text{var}(\ln z_{i,t})}$ is the misallocation measure

The aggregate TFP H_t is expressed in a closed-form:

$$\ln H_t = [...] - \frac{\alpha\sigma^2}{2} M_t$$

The aggregate R&D intensity $\frac{S_t}{A_t}$ satisfies

$$\ln\left(\frac{S_t}{A_t}\right) = [...] - \frac{\alpha\sigma^2}{2h} M_t$$

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Covariance-type measure of misallocation

Misallocation (M_t) also reflects the distribution of MRPK:

$$M_t \equiv -\frac{\text{Cov}(\ln z_{i,t}, \ln a_{i,t})}{\text{var}(\ln z_{i,t})} = -\frac{\text{Cov}(\ln v_{i,t}, \ln a_{i,t})}{\text{var}(\ln v_{i,t})}.$$

where MRPK is $v_{i,t} = (\varepsilon/p_t)^{\frac{\varepsilon}{\alpha(1-\varepsilon)}} \left[(1-\alpha)(1-\varepsilon)/w_t\right]^{\frac{1-\alpha}{\alpha}} z_{i,t}$

Misallocation (M_t) motivates a covariance-type empirical measure

- Similar to the size-and-productivity covariance

  e.g., Olley_Pakes (1996); Bartelsman_Haltiwanger_Scarpetta (2009, 2013)

- Different but quite related to measures based on dispersion

  e.g., Foster_Haltiwanger_Syverson (2008); Hsieh_Klenow (2009)

- More robust against multiplicative measurement errors compared to dispersion-based misallocation measures

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Evolution of misallocation

The economy is characterized by the evolution of M_t:

$$\mathrm{d}M_t = -\theta M_t \mathrm{d}t - \frac{\mathrm{Cov}(\ln z_{i,t}, \mathrm{d}\ln a_{i,t})}{\mathrm{var}(\ln z_{i,t})},$$

where

$$-\frac{\mathrm{Cov}(\ln z_{i,t}, \mathrm{d}\ln a_{i,t})}{\mathrm{var}(\ln z_{i,t})} = [\cdots]\mathrm{d}t + \underbrace{[\cdots]}_{>0}\mathrm{d}W_t$$

- Misallocation M_t is countercyclical
- Misallocation M_t is slow moving, with its persistence dependent on θ

$\Longrightarrow$ Uncover the "dark matter" in long-run risk models

e.g., Chen_Dou_Kogan (2024); Cheng_Dou_Liao (2022)

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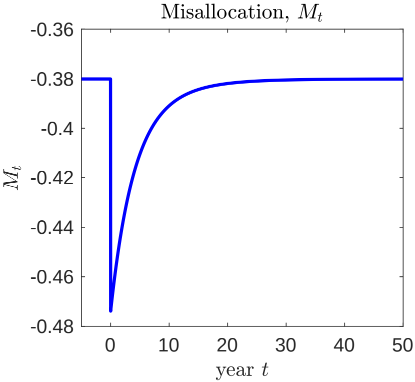
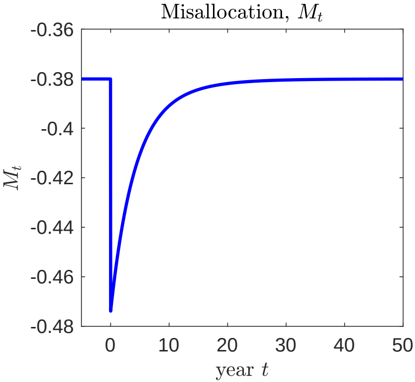
Impulse responses

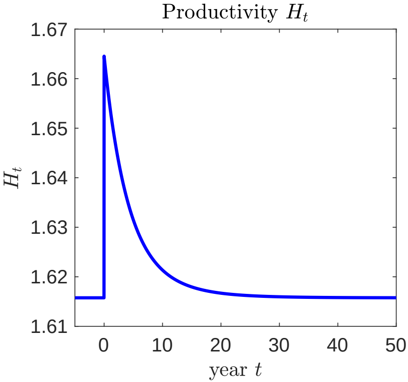
Consider a one-time shock to M at $t = 0$.



Intensive-margin effect

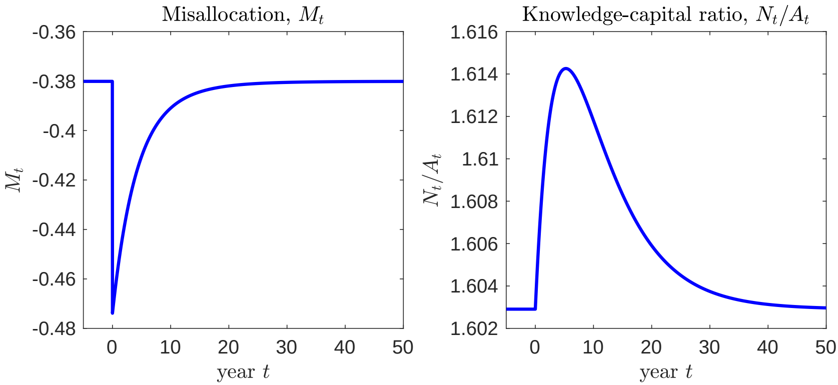
M_t determines the final-goods sector's productivity H_t





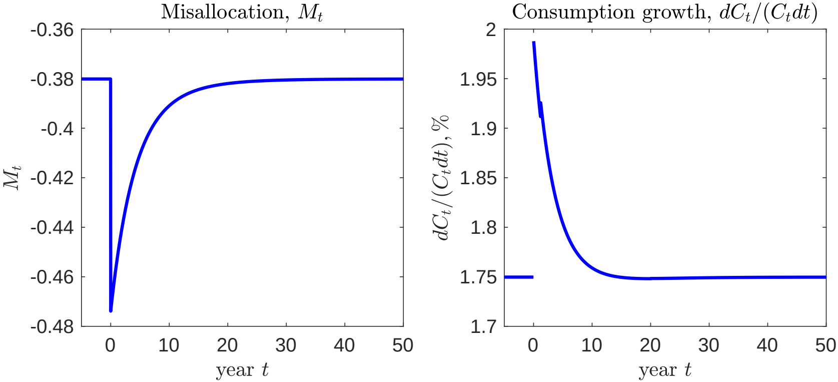
Extensive-margin effect

M_t determines growth through R&D, which produces N_t



Slow-moving misallocation and growth

A one-time shock to M generates a persistent effect on growth



Comparative dynamics

Persistence of $z_{i,t}$ $\Rightarrow$ Persistence of M_t $\Rightarrow$ Persistence of growth

$$\mathrm{d}M_t = -\theta M_t \mathrm{d}t - \frac{\mathrm{Cov}(\ln z_{i,t}, \mathrm{d} \ln a_{i,t})}{\mathrm{var}(\ln z_{i,t})},$$

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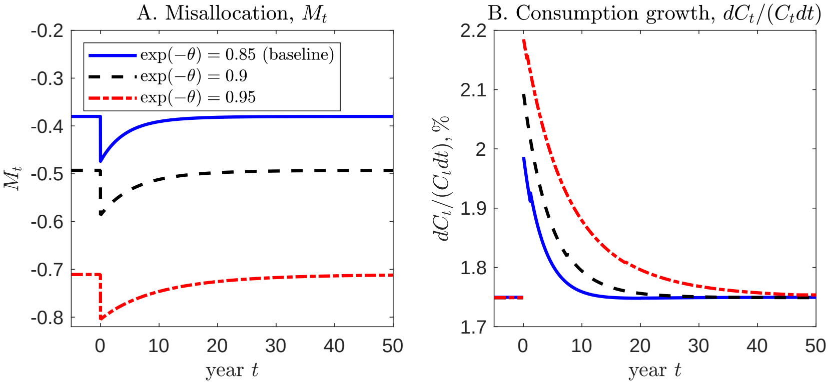
Persistence of $z_{i,t}$ ⇒ Persistence of M_t ⇒ Persistence of growth

$$dM_t = -\theta M_t dt - \frac{\text{Cov}(\ln z_{i,t}, d\ln a_{i,t})}{\text{var}(\ln z_{i,t})},$$



The half-life of the growth rate's transition is 3.0, 4.2, and 6.9 years

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Outline

1. **Model**

2. **Solution and mechanism**

3. **Quantitative analysis**

Misallocation measure

A measure of misallocation directly motivated by the model:

$$a_{i,t} = \alpha_t + \beta_t^{\text{Alloc}} \times z_{i,t} + \varepsilon_{i,t},$$

where

$$a_{i,t} = T^{-1} \sum_{\tau=1}^{T} \ln(ppent_{i,t+1-\tau}) \text{ and } z_{i,t} = T^{-1} \sum_{\tau=1}^{T} \ln\left(\frac{sales_{i,t+1-\tau}}{\widehat{ppent}_{i,t+1-\tau}}\right)$$

Discussions:

- We use $\widehat{ppent}_{i,t+1-\tau}$ to account for leased capital
  e.g., Rauh-Sufi (2011); Rampini-Viswanathan (2013)
- We also use "tangible net worth" to construct $a_{i,t}$ as robustness results
  e.g., Chava-Roberts (2008); Roberts-Sufi (2009); Sufi (2009)

The misallocation measure is

$$\widehat{M}_t = \text{the HP filtered time series of } -\beta_t^{\text{Alloc}}$$

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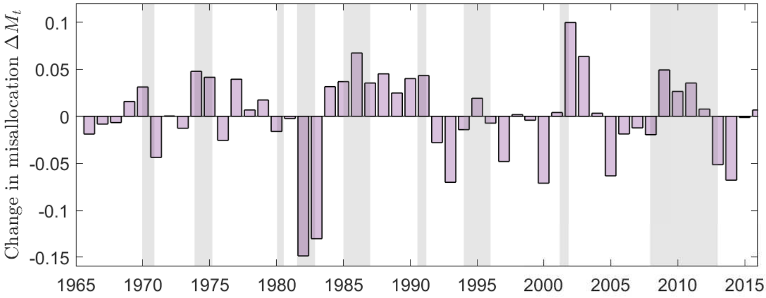
$\widehat{M}_t$ is countercyclical

The value of $\widehat{M}_t$ increases sharply in 7 of the 9 economic downturns



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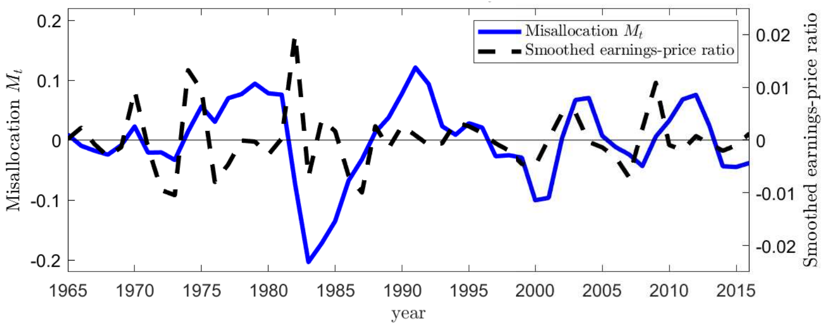
$\widehat{M}_t$ is slow moving

Yearly autocorrelation of $\widehat{M}_t$ is 0.75,

- In line with Bansal_Yaron (2004)'s calibration for the persistence of expected growth rates



Calibration

Majority of standard parameters are set following the literature

Set $\exp(-\theta) = 0.85$, following Asker_Collard-Wexler_De Loecker (2014)

Four parameters are internally calibrated to match four moments

Parameter	Symbol	Value	Moments	Data	Model
Subjective discount rate	δ	0.01	Real risk-free rate (%)	1.11	1.58
R&D productivity	χ	1.35	Consumption growth rate (%)	1.76	1.75
Capital depreciation shock	σ_k	0.19	Consumption growth vol. (%)	1.50	1.67
Dividend payout rate	ρ	0.037	Dividend yield (%)	2.35	2.14

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Untargeted moments in data and model

Moments	Data	Model	Moments	Data	Model
		Panel A: Consumption moments			
$AC1(\Delta \ln C_t)$ (%)	0.44	0.46	$AC2(\Delta \ln C_t)$ (%)	0.08	0.28
$AC5(\Delta \ln C_t)$ (%)	-0.01	0.00	$AC10(\Delta \ln C_t)$ (%)	0.06	-0.06
$VR2(\Delta \ln C_t)$ (%)	1.52	1.46	$VR5(\Delta \ln C_t)$ (%)	2.02	2.21
		Panel B: Other moments			
$AC1(\Delta \ln S_t)$ (%)	0.30	0.42	$AC1(M_t)$ (%)	0.75	0.73
$SR[R_{w,t}]$	0.36	0.39	$\sigma[r_{f,t}]$ (%)	2.06	0.47

Misallocation, R&D, and growth

	R&D intensity	
	$t+1$	
	Data	Model
β	-0.106^{***}	-0.039^{***}
	[-3.793]	[-9.065]

Misallocation, R&D, and growth

	R&D intensity		Consumption growth	
	$t+1$		$t \rightarrow t+5$	
	Data	Model	Data	Model
β	-0.106^{***}	-0.039^{***}	-0.227^{***}	-0.276^{***}
	[−3.793]	[−9.065]	[−3.781]	[−3.436]

Misallocation, R&D, and growth

	R&D intensity		Consumption growth		Output growth	
	$t+1$		$t \rightarrow t+5$		$t \rightarrow t+5$	
	Data	Model	Data	Model	Data	Model
β	-0.106^{***}	-0.039^{***}	-0.227^{***}	-0.276^{***}	-0.218^{**}	-0.233^{***}
	[-3.793]	[-9.065]	[-3.781]	[-3.436]	[-2.492]	[-3.123]

Asset pricing implications

	(1) Baseline
$\mathbb{E}[R^e_{w,t}]$ (%)	0.54
$\sigma[R^e_{w,t}]$ (%)	1.39
$SR[R_{w,t}]$	0.39
$\mathbb{E}[r_{f,t}]$ (%)	1.58
$\sigma[r_{f,t}]$ (%)	0.47
$\frac{\sigma[\Lambda_{t+1}/\Lambda_t]}{\mathbb{E}[\Lambda_{t+1}/\Lambda_t]}$	0.61

Asset pricing implications

	(1) Baseline	(2) $M_t \equiv \mathbb{E}[M_t]$
$\mathbb{E}[R^e_{w,t}]$ (%)	0.54	0
$\sigma[R^e_{w,t}]$ (%)	1.39	0
$SR[R_{w,t}]$	0.39	–
$\mathbb{E}[r_{f,t}]$ (%)	1.58	1.87
$\sigma[r_{f,t}]$ (%)	0.47	0
$\frac{\sigma[\Lambda_{t+1}/\Lambda_t]}{\mathbb{E}[\Lambda_{t+1}/\Lambda_t]}$	0.61	0

Asset pricing implications

	(1) Baseline	(2) $M_t \equiv \mathbb{E}[M_t]$	(3) $dN_t \equiv 0$
$\mathbb{E}[R^e_{w,t}]$ (%)	0.54	0	0.02
$\sigma[R^e_{w,t}]$ (%)	1.39	0	0.72
$SR[R_{w,t}]$	0.39	–	0.02
$\mathbb{E}[r_{f,t}]$ (%)	1.58	1.87	0.98
$\sigma[r_{f,t}]$ (%)	0.47	0	0.34
$\frac{\sigma[\Lambda_{t+1}/\Lambda_t]}{\mathbb{E}[\Lambda_{t+1}/\Lambda_t]}$	0.61	0	0.03

Asset pricing implications

	(1) Baseline	(2) $M_t \equiv \mathbb{E}[M_t]$	(3) $dN_t \equiv 0$	(4) $e^{-\theta} = 0.2$	(5) $e^{-\theta} = 0.45$
$\mathbb{E}[R^e_{w,t}]$ (%)	0.54	0	0.02	0.01	0.08
$\sigma[R^e_{w,t}]$ (%)	1.39	0	0.72	1.17	1.09
$SR[R_{w,t}]$	0.39	–	0.02	0.01	0.08
$\mathbb{E}[r_{f,t}]$ (%)	1.58	1.87	0.98	1.93	1.88
$\sigma[r_{f,t}]$ (%)	0.47	0	0.34	0.33	0.41
$\frac{\sigma[\Lambda_{t+1}/\Lambda_t]}{\mathbb{E}[\Lambda_{t+1}/\Lambda_t]}$	0.61	0	0.03	0.06	0.10

Asset pricing implications

	(1) Baseline	(2) $M_t \equiv \mathbb{E}[M_t]$	(3) $dN_t \equiv 0$	(4) $e^{-\theta}$ $= 0.2$	(5) $= 0.45$	(6) CRRA ($\gamma = 1/\psi$) $= 1.5$	(7) $= 3$
$\mathbb{E}[R^e_{w,t}]$ (%)	0.54	0	0.02	0.01	0.08	0.02	0.02
$\sigma[R^e_{w,t}]$ (%)	1.39	0	0.72	1.17	1.09	1.01	0.57
$SR[R_{w,t}]$	0.39	–	0.02	0.01	0.08	0.02	0.04
$\mathbb{E}[r_{f,t}]$ (%)	1.58	1.87	0.98	1.93	1.88	3.60	6.17
$\sigma[r_{f,t}]$ (%)	0.47	0	0.34	0.33	0.41	0.47	0.57
$\frac{\sigma[\Lambda_{t+1}/\Lambda_t]}{\mathbb{E}[\Lambda_{t+1}/\Lambda_t]}$	0.61	0	0.03	0.06	0.10	0.03	0.05

Key: low-frequency growth fluctuations + recursive preferences

Asset pricing implications

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Baseline	$M_t \equiv \mathbb{E}[M_t]$	$dN_t \equiv 0$	$e^{-\theta}$		CRRA ($\gamma = 1/\psi$)	
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Welfare implications

In the model, all growth fluctuations are driven by misallocation fluctuations

- Estimate welfare gains from eliminate fluctuations

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	Baseline	$dN_t \equiv 0$	$e^{-\theta}$		CRRA ($\gamma = 1/\psi$)	
			$= 0.2$	$= 0.45$	$= 1.5$	$= 3$
Welfare gains (%)	10.34	0.33	0.24	0.98	0.58	0.65

Key: low-frequency growth fluctuations + recursive preferences

Tight connection between asset prices and welfare costs

e.g., Alverez_Jermann (2004, 2005)

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Conclusions

A tractable model to link misallocation, growth, and asset prices

- Agency conflicts and the resultant financial frictions are crucial
- A valuation channel is pivotal in the quantitative relationships

Misallocation drives low-frequency growth fluctuations

- Cross-section is informative for long-term time-series evolution
- Misallocation uncovers the “dark matter” in long-run risk models
- Misallocation explains asset returns as a powerful macro factor
- Shocks that lead to misallocation fluctuations have large welfare costs